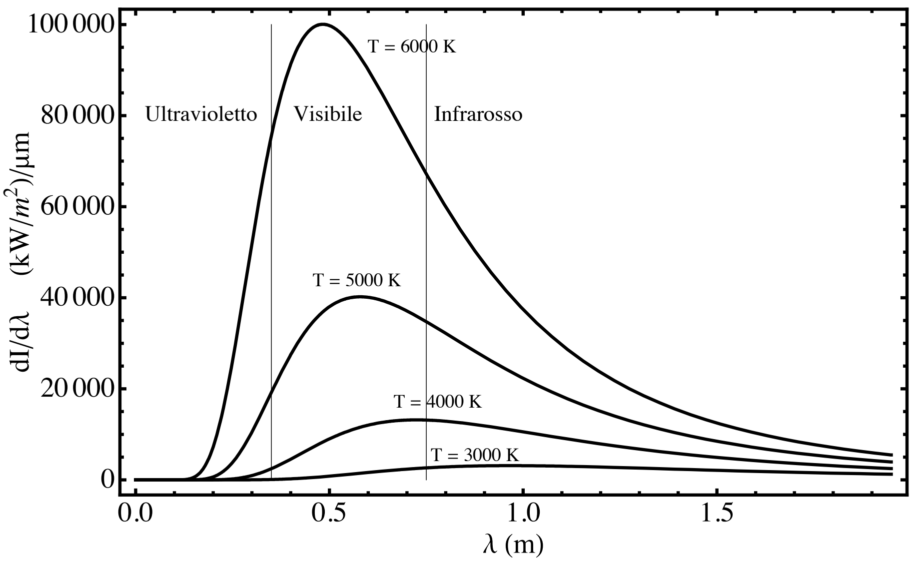
Misura della costante di Planck dallo spettro di corpo nero di una lampada ad incandescenza.

L'esperimento che segue consente di ottenere una stima della costante di Planck dallo spettro di corpo nero. Si tratta di una misura con degli errori abbastanza grandi, ma che permette di ottenere almeno l'ordine di grandezza della costante di Planck. L'idea alla base dell'esperimento è quella di utilizzare lo spettro di corpo nero a due temperature diverse e a lunghezza d'onda fissata per ottenere una stima della costante di Planck a partire dal rapporto delle irradianze misurate.



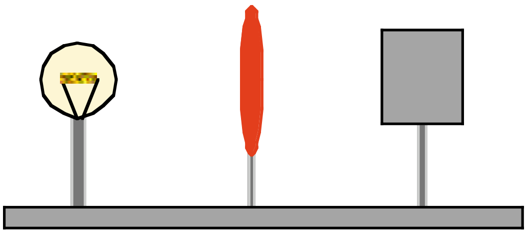
Irradianza del corpo nero a diverse lunghezze d'onda. Ad una lunghezza d'onda fissata l'irradianza aumenta all'aumentare della temperatura. Dal rapporto di irradianza a due diverse temperature si può ottenere una stima della costante di Planck.

Materiali necessari:

- una lampadina ad incandescenza con supporto
- un fotometro con supporto
- due voltmetri
- un amperometro
- un generatore a tensione variabile
- un filtro colorato
- una lente convergente
- un metro

Descrizione:

Il montaggio di questo esperimento è estremamente semplice ed è mostrato in figura

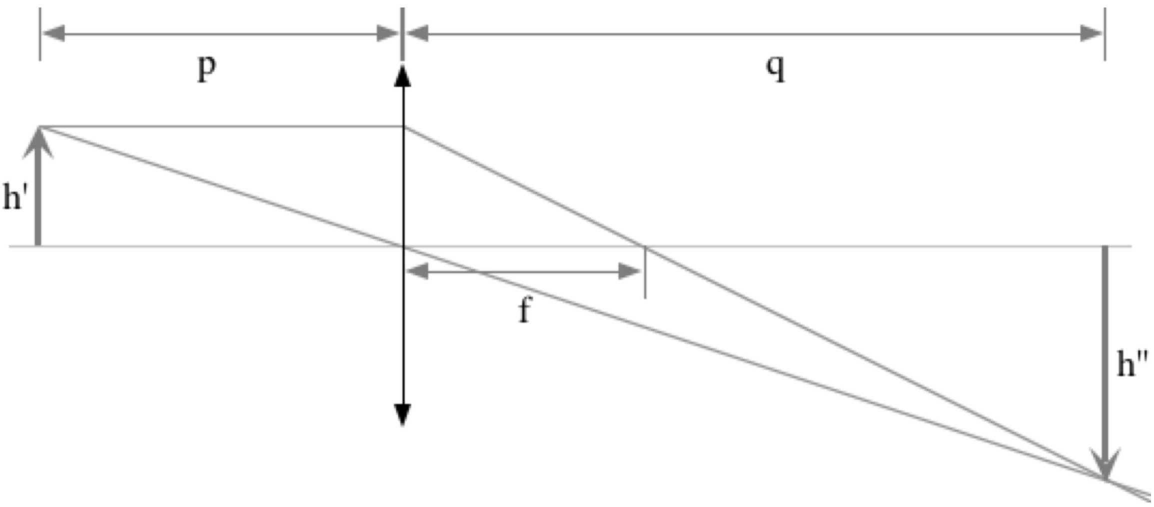


la lampada ad incandescenza va alimentata dal generatore, ed inoltre il circuito della lampada deve comprendere anche un voltmetro (in parallelo alla lampada) e l'amperometro (in serie), in modo da misurare la tensione v ai capi della lampada e la corrente i nel circuito e calcolare quindi la potenza $W = vi$ dissipata dalla lampada.

Se assumiamo che la potenza assorbita dal generatore sia convertita totalmente in energia radiante, allora possiamo utilizzare la legge di Stefan-Boltzmann e scrivere la relazione

$$W = Aa\sigma T^4$$

dove A è la superficie totale di emissione del filamento. A si può misurare assumendo che il filamento abbia una forma regolare (ad esempio cilindrica) e quindi proiettando l'immagine del filamento su uno schermo, utilizzando per questo scopo la lente convergente, per misurarne le dimensioni. Le misure geometriche si fanno direttamente sullo schermo di proiezione e quindi si convertono in dimensioni reali del filamento conoscendo l'ingrandimento della lente (si veda la figura seguente)



Questa figura mostra come calcolare la dimensione reale del filamento dalle misure fatte: si vede subito che deve valere la relazione $\frac{h'}{p} = \frac{h"}{q}$, e quindi $h' = \frac{p}{q} h"$.

Ci si ricordi ora che l'irradianza che arriva sul diodo è proporzionale alla seguente espressione

$$I(\lambda) \propto \frac{1}{\lambda^5 (e^{hc/\lambda kT} - 1)}$$

La potenza totale emessa dal filamento dipende solo dalla sua temperatura che si può ottenere dalla formula

$$T = \left(\frac{vi}{Aa\sigma}\right)^{1/4}$$

e quindi l'inverso dell'irradianza a lunghezza d'onda λ è

$$\frac{1}{I(\lambda)} \propto \lambda^5 \left\{ \exp\left[\frac{hc}{\lambda k} \cdot \left(\frac{Aa\sigma}{vi}\right)^{1/4}\right] - 1 \right\}$$

e quindi quando si utilizzano due diverse tensioni di alimentazione della lampadina si può costruire un rapporto di irradianze a lunghezza d'onda fissata:

$$\frac{I_2(\lambda)}{I_1(\lambda)} = \frac{\exp\left[\frac{hc}{\lambda k} \cdot \left(\frac{Aa\sigma}{v_1 i_1}\right)^{1/4}\right] - 1}{\exp\left[\frac{hc}{\lambda k} \cdot \left(\frac{Aa\sigma}{v_2 i_2}\right)^{1/4}\right] - 1} \approx \exp\left[\frac{hc}{\lambda k} \cdot \left(\frac{Aa\sigma}{v_1 i_1}\right)^{1/4} - \frac{hc}{\lambda k} \cdot \left(\frac{Aa\sigma}{v_2 i_2}\right)^{1/4}\right]$$

dove si è supposto che gli esponenziali siano molto maggiori di 1 (possiamo verificare questa ipotesi a posteriori, sapendo che per la luce verde $h\nu \approx 2.5$ eV, mentre alla temperatura di una lampadina 2000 - 3000 K il prodotto kT vale circa 0.25 eV, e quindi si vede che in l'esponenziale vale circa $e^{10} \approx 2.2 \cdot 10^4 >> 1$, e l'ipotesi è dunque valida nella regione del picco dello spettro di corpo nero emesso da una lampadina ad incandescenza).

Questo rapporto di irradianze corrisponde a un rapporto r di tensioni v_{PH} misurate con il fotometro, e quindi, prendendo il logaritmo, si ottiene

$$\ln \frac{I_2(\lambda)}{I_1(\lambda)} = \ln \frac{v_{PH,2}}{v_{PH,1}} = \ln r \approx \frac{hc}{\lambda k} \cdot \left(\frac{Aa\sigma}{v_1 i_1}\right)^{1/4} - \frac{hc}{\lambda k} \cdot \left(\frac{Aa\sigma}{v_2 i_2}\right)^{1/4}$$

e infine

$$h \approx \frac{\lambda k \ln r}{c(Aa\sigma)^{1/4}} \left[\left(\frac{1}{v_1 i_1}\right)^{1/4} - \left(\frac{1}{v_2 i_2}\right)^{1/4}\right]^{-1}$$

L'emissività a del filamento di tungsteno ad alta temperatura si trova tabulata in letteratura[1], e per questa misura prendiamo $a \approx 0.3$. Non è un valore molto preciso, ma sufficiente a dare almeno l'ordine di grandezza della costante di Planck.

[1] Una tabella si può trovare ad esempio a questo link http://www.engineeringtoolbox.com/emissivity-coefficients-d_447.html

Minimal apparatus for determination of Planck's constant

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I. BACKGROUND

Blackbody radiation is especially interesting as a macroscopic phenomenon because its complete description requires quantum mechanics. Since Planck's constant h was first invoked to ward off the "ultraviolet catastrophe"—which prevented the proper description of blackbodies before the time of Planck[1]—one would expect that information about h can be obtained via simple observations on certain common radiators. We show this last assertion to be true by describing an experiment in which a minimum of apparatus and effort results in a value for h which is within a small factor of the accepted value.

II. THEORY

Consider a hot tungsten filament as an approximate blackbody. Owing to particular scaling properties of the tungsten radiation, h can be determined with just *two* different settings of the input electrical power to the filament. Nomenclature is as follows: ν = frequency of light; h = Planck's constant; A = surface area of the filament; kT = Boltzmann constant times absolute temperature; c = speed of light. Assuming that all electrical power P goes into radiation, we have the relation known as Stefan's law (we assume unit emissivity)[2]:

$$P = A\sigma T^4, \tag{1}$$

where Stefan's constant σ is given from quantum theory by

$$\sigma = \frac{2\pi^5 k^4}{15h^3c^2}. \tag{2}$$

It is interesting that Stefan obtained Eq. (1) through fortuitous use of incorrect radiation data obtained by Tyndall from a laboratory blackbody.[3] This anecdote is relevant here because Tyndall's practical difficulties are present also in the present experiment so we do not expect really sharp quantitative results.

The detailed theory that gives Eq. (1) involves the derivation of the intensity of radiation, at frequency ν, as[4]

$$I_\nu(T) = \frac{N\nu^3}{\exp[h\nu/kT] - 1}, \tag{3}$$

where N is a normalization number whose detailed structure is not required here. It will turn out that we shall not have to determine any temperatures T explicitly. Indeed, assume that *two* powers P_1, P_2 are alternately delivered to the filament. There will be two temperatures T_1, T_2 and at a *fixed* frequency ν there will be two respective intensities of the form (3). We have the relations

$$P_j = A\sigma T_j^4 \quad j = 1, 2; \tag{4}$$

$$\frac{I_1}{I_2} = \frac{I_\nu(T_1)}{I_\nu(T_2)} = \frac{\exp[h\nu/kT_2] - 1}{\exp[h\nu/kT_1] - 1}. \tag{5}$$

It is evident that by measuring the four quantities P_1, P_2, I_1, I_2 and using values for A, ν we can solve Eqs. (4) and (5) for the fundamental constant:

$$h/c^2 = \text{function of } (P_1, P_2, I_1, I_2, A, \nu). \tag{6}$$

The function (6) cannot be given in closed form, but h/c^2 can be found by machine computation. What is more expedient, however, is to realize that $\exp[h\nu/kT] \gg 1$; an approximation good for visible colors ν even for bright orange filaments. This allows a closed form for Planck's constant:

$$h \approx \frac{15c^2}{2\pi^5 A\nu^4} \frac{P_1 P_2}{(P_1^{1/4} - P_2^{1/4})^4} \log^4 \frac{I_1}{I_2}. \tag{7}$$

This formula forms the basis for the simple experiment described below.

III. EXPERIMENT AND RESULTS

The experimental arrangement is shown in Fig. 1. A common (unfrosted) 60-W light bulb is used as an approximate blackbody radiator (B). The procedure is to first determine the surface area A of the filament. This is convincingly done by projecting the image of the energized filament onto a distant wall, taking gross dimensions of the image, then scaling down according to the optical magnification. Second, a single-color filter (such as green cellophane) is placed so that a phototransistor[5] will be irradiated by essentially just one fixed frequency ν of light. The Variac (A) is then set to deliver some arbitrary power P_1 to the filament. This power is determined with a voltmeter placed first across the bulb and then across the sensing resistor (C). This sensing resistor should be a 1-Ω, 1-W device. The resulting intensity falling on the phototransistor is monitored as a photocurrent manifest as a voltage drop across resistor (D). This load resistor should be about 1 kΩ. The sequence of voltage measurements is then repeated for a second setting of lamp power P_2. The relevant data is then the pair of arbitrary powers P_i and the associated pair of photocurrents.

An experimental run was performed with the following results:

$A = 5.2 \times 10^{-5}$ m^2,

$\nu = 5.3 \times 10^{14}$ Hz (cellophane filter);

Powers	Photocurrents
19.3 W	0.81 mA
36.5 W	7.57 mA

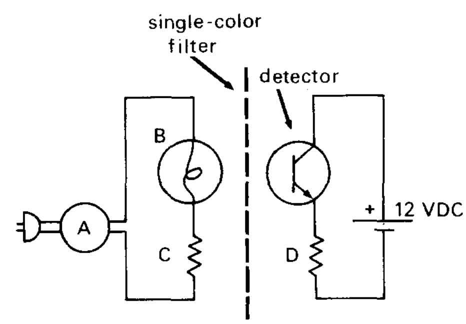

Fig. 1. Planck's constant experiment. Variac (A) delivers two different powers to filament (B). These powers and their corresponding two photocurrents through the detector[5] are determined with a roving ac/dc voltmeter, applied across B, C, and D.

with the result, from Eq. (7), $h = 4.9 \times 10^{-34}$ J s. This result is reasonably close to the accepted value $h = 6.6 \times 10^{-34}$.

IV. ALTERNATIVES

The experiment can be simplified even further by using a low-voltage lamp and a power supply to run both the lamp and the transistor circuit. This approach has the advantage of human safety and requires only dc measurements. We found, however, that results for h using these low-voltage radiators tend to be less accurate, usually on the order of 2×10^{-34} for the procedure outlined above.

Results for Planck's constant should improve through use of more precise filters[6] and more ideal blackbody radiators. In particular, for truly accurate results some attention must be paid to the emissivity number for the radiator.

[1]P. Tipler, *Modern Physics* (Worth, New York, 1978).
[2]Powell and Crasemann, *Quantum Mechanics* (Addison-Wesley, Reading, MA, 1961).
[3]M. Jammer, *The Conceptual Development of Quantum Mechanics* (McGraw-Hill, Hightstown, NJ, 1968).
[4]E. Cohen, K. Crowe, and J. Dumond, *Fundamental Constants of Physics* (Interscience, New York, 1957).
[5]Phototransistor Motorola MRD-3052 or equivalent. It is interesting and advantageous that the spectral response of this device is irrelevant in the present setting: the color is never changed throughout the experiment.
[6]Extremely sharp but relatively inexpensive stock filters can be obtained from Corion, Inc., Holliston, MA 01746.

Circularly polarized waves: A mechanical demonstration

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The existence of circularly of elliptically polarized waves can easily be demonstrated by using electromagnetic waves. When a linearly polarized wave falls on a thin plate of a uniaxial material, cut with the faces parallel to the optical axis, the wave will in general be elliptically polarized after traversing the anisotropic plate. Under certain conditions the wave is circularly polarized.

In a general physics course, when these kind of waves are discussed, undergraduates have no problem understanding what happens at a well-defined point. However, they sometimes have problems, concerning the spatial extension of the rotating vector, and understanding what happens at different points in space.

Therefore we designed a mechanical apparatus, as shown in Fig. 1, to demonstrate the propagation of circularly polarized waves. This apparatus allows one to study the relative orientation of the rotating vectors at different points in space and to study the propagation of a particular state of the rotating vector between two points. The construction of this apparatus is such that a vector, with fixed orientation in space, moves between two points while the other vectors rotate.

The construction of the apparatus is shown in Fig. 2. Five rotating vectors (1–5), constructed of reinforced phenolic and provided with a slit, are mounted on a polymethylmetacrylate tube A. The five rotating vectors are spread out over a distance of 80 cm, representing one wavelength. The distance between two adjacent vectors is 20 cm (corresponding to $\lambda/4$) illustrating that the angle between them is $\pi/2$ rad. The lengh of tube A is 100 cm and its diameter is 30 cm. Turning hand wheel B clockwise simultaneously moves the chain F and rotates tube A through transmission C and gear D (see Table I). When tube A rotates, vectors 1–5 also rotate, while at the same time vector E moves from 1 to 5 through a slit in cylinder I as it is carried by the chain F. During its translation motion, E will pass through the slits of the different rotating vectors when they are oriented upwards. A mechanism on axis H is provided to prevent the system from being rotated counterclockwise. The vector E is returned to its starting position by continuing the clockwise rotation of wheel B. Each end is supported by, and pivots about, a 5-cm diameter tube (I) which also prevents the chain from sagging.

Table I. Gear characteristics.

	tooth	metric module, pitch (mm)	gear ratio
Gear transmission HC (Bevel gears)			
gear	30	2	2:1
pinion	15	2	
Gear transmission CD			
pinion	60	2	1:2.67
gear	160	2	
Sprocket wheel G	12	50.8	

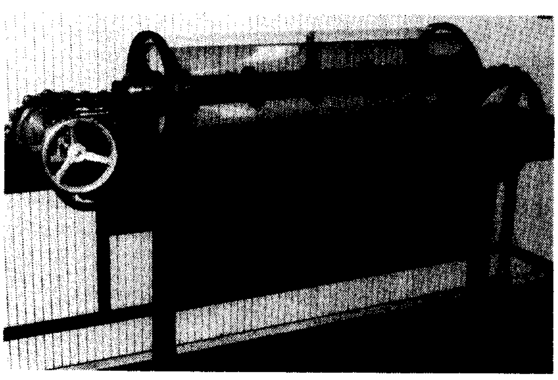

Fig. 1. Photograph of the apparatus.

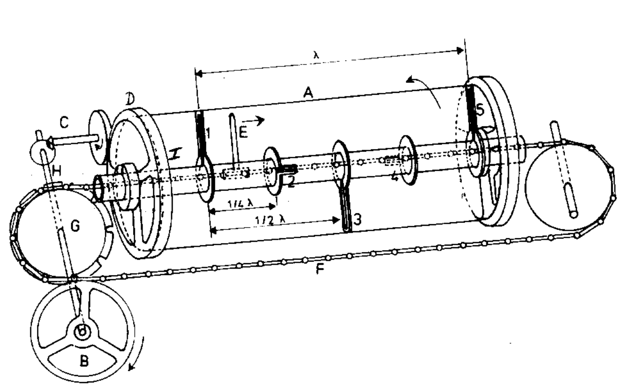

Fig. 2. Sketch of the apparatus.