

# Shot-noise measurements of the electron charge: An undergraduate experiment

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(Received 2 August 1994; accepted 12 October 1994)

We report the design and performance of a simple apparatus for measurement of shot-noise fluctuations in the current from a vacuum photodiode illuminated with a pilot-lamp light bulb. After calibrating the frequency-dependent gain of the measurement electronics, the charge  $e$  of the electron can be obtained by measuring the mean-square shot noise as a function of the dc photodiode current. We employ op-amp circuits and a rms-to-dc integrated circuit to amplify, filter, and detect the shot noise, so that commercial bandpass filters and rms voltmeters are unnecessary. The apparatus is therefore not expensive and can be built using readily available components. Repeated measurements employing different pilot lamps yield a value of  $e = (1.581 \pm 0.015 \pm 0.032) \times 10^{-19}$  C, where the uncertainties represent random ( $2\sigma_{\text{mean}}$ ) and systematic error, respectively. The experiment thus permits reasonably precise measurements of a fundamental constant while allowing the undergraduate student to gain a hands-on understanding of fluctuation phenomena. © 1995 American Association of Physics Teachers.

## I. INTRODUCTION

Noise is ubiquitous in laboratories and forms an intrinsic part of any experiment. Undergraduate physics students often first learn of noise in a negative context, as something which always degrades the results of an experiment, and which would disappear altogether if only the apparatus and the experimentalist were "good" enough.<sup>1-3</sup> Certainly many types of noise can (and should) be eliminated by following careful experimental and design procedures. Some types of noise however, such as Johnson and shot noise, are intrinsic properties of matter and will be present to some extent in all electronic apparatus,<sup>4-6</sup> thus setting ultimate upper limits on achievable signal-to-noise ratios.<sup>7</sup> Indeed, because Johnson fluctuations and shot noise are universal effects, these phenomena can be used to measure fundamental physical constants. Measurements of Johnson noise, arising from thermal fluctuations in the voltage across a resistor, can be used to determine Boltzmann's constant  $k$ .<sup>8,9</sup> Shot noise, the subject of the present paper, refers to current fluctuations due to quantized charges crossing a vacuum diode or  $p-n$  junction at random times, and can be used to measure the quantum of charge  $e$ .<sup>10-12</sup>

We describe here a simple apparatus that has been successfully employed for investigation of shot-noise fluctuations and determination of the electron charge in a junior-level physics laboratory course at Trinity University. Previous reports of student-laboratory shot-noise measurements include those of Portis (1964),<sup>13</sup> Mathieson (1965),<sup>14</sup> Earl (1966),<sup>15</sup> Livesey and McLeod (1973),<sup>16</sup> and Vetterling and Andelman (1979).<sup>17</sup> This type of experiment has also been carried out for a number of years in the Massachusetts Institute of Technology Junior Physics Laboratory,<sup>18</sup> one of us (D.R.S.) first learned of the instructional utility of shot-noise measurements at the MIT Workshop on Advanced Undergraduate Training in Experimental Physics<sup>19</sup> (led by Professor George Clark and Dr. Jay Kirsch). We have since constructed an apparatus using only op-amp circuits and a rms-to-dc conversion chip to amplify, filter, and quantitatively measure shot-noise fluctuations in the current obtained from an illuminated vacuum photodiode. The entire appara-

tus can be built in-house with readily available electronic components. From a pedagogical viewpoint, the experiment permits accurate and reproducible measurements of a fundamental constant, while simultaneously allowing the undergraduate student to develop an understanding of statistical noise in measurements and an appreciation for the universal importance of statistical fluctuations.

We present below a very brief and nonrigorous account of the theory of shot noise, a description of our electronics, and a report of the precision and accuracy obtained in our measurements of  $e$ .

*Shot noise: theoretical considerations.* Consider electrons emitted from a cathode and collected at an anode, as shown in Fig. 1.<sup>5,20</sup> Electron emission from the cathode material can be caused by "evaporation" (thermionic emission) or, if one illuminates the cathode with photons of sufficient energy, photoelectric emission. In the latter case, we assume that the detectable photons impact the cathode at random times; this can be achieved, for example, through the use of a quasi-blackbody source such as a tungsten-filament lamp.<sup>21,22</sup> The emitted electrons result in a current  $I(t)$ , which is measured using a current-to-voltage converter with output  $V(t) = -R_o I(t)$ . For small bias voltages  $V_b$ , a space-charge cloud will form above the cathode, and some of the emitted electrons will not reach the anode. If the bias voltage is sufficiently high, however, then the space-charge cloud disappears, and virtually all emitted electrons are collected at the anode.<sup>5,23</sup> Under these "saturated" conditions, the emission (and collection) of each individual electron is a statistically independent event. In addition, for operation in the saturated regime, the steady current  $I_{av}$  reaching the anode depends only on the temperature of the cathode (for thermionic emission) or the intensity of photon illumination (for photoelectric emission), and is independent of the bias  $V_b$ .

In 1918, Schottky<sup>10</sup> realized that, because the current reaching the anode is made up of discrete electrons emitted at random times, fluctuations in the current about its average must always occur. Schottky drew an analogy to the fluctuations in sound that one hears when listening to a steady fall of hail (or gunshot) onto a surface, and dubbed the current

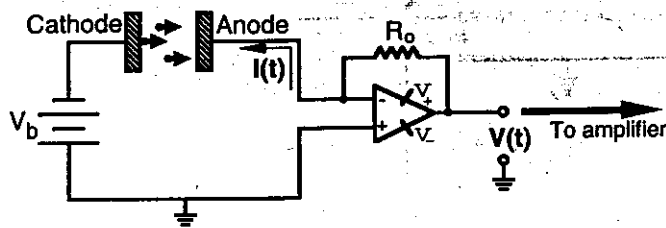


Fig. 1. Current  $I(t)$  due to electron emission from a cathode is collected at an anode and converted to a voltage  $V(t)$ . Because the current is made up of quantized charges emitted at random times, fluctuations in the current about its average value must always be present. The larger the quantum of charge, the larger the mean-square fluctuations.

fluctuations the "shot effect" (Schrotheffekt). He pointed out that, since the magnitude of the fluctuations will depend on the size of the quantum of charge  $e$ , measurement of the current fluctuations could in principle be used to determine the charge of a single electron. In 1925, Hull and Williams<sup>12</sup> used an amplifier tuned with a RLC circuit to measure shot-noise fluctuations in thermionic-emission currents and obtained, to a precision of 2%, a result for  $e$  in agreement with the currently accepted value.<sup>24</sup> Accurate values for  $e$  using similar methods were later obtained by Williams and Huxford (1929)<sup>25</sup> and Stigmark (1952),<sup>26</sup> with uncertainties of about 2% and 0.25%, respectively.

We now focus on the quantitative relation between shot-noise fluctuations and charge quantization, under the assumption (easily checked experimentally) that the effects of space charge are negligible for the apparatus under investigation. (We make no attempts here at mathematical rigor; for a more complete exposition of the theoretical foundations we refer the reader to texts by [for example] Ambrózy,<sup>27</sup> Goldman,<sup>5</sup> MacDonald,<sup>1</sup> and van der Ziel,<sup>6,7</sup> as well as the thorough reviews<sup>28,29</sup> by van der Ziel.) Suppose one repeatedly counts the number of electrons emitted from the cathode of Fig. 1 for many consecutive time intervals, each of duration  $\Delta t$ . We define the number of counts obtained in the  $i$ th interval as  $N_i$ . If  $I_{av}$  is the average (dc) current measured over a period long compared to  $\Delta t$ , then the average number of electrons emitted during each time interval is  $N_{av} = (I_{av}/e)\Delta t$ . Since the emission of each individual elec-

tron is a statistically independent event, the variance in  $N_i$  follows immediately from the theory of Poisson statistics<sup>23</sup>

$$(N^2)_{av} - (N_{av})^2 = N_{av}. \quad (1)$$

Multiplying both sides of Eq. (1) by  $e^2/\Delta t^2$ , we obtain

$$(I^2)_{av} - (I_{av})^2 = I_{av}e/\Delta t. \quad (2)$$

The left-hand side of Eq. (2) is the mean square of the fluctuations in the current about its average. Thus,

$$[(\Delta I)^2]_{av} = (\Delta I_{rms})^2 = I_{av}e/\Delta t. \quad (3)$$

Equation (3) shows that the mean-square current fluctuations can provide—at least in principle—a measure of the charge  $e$  of an individual electron. If Eq. (3) is to prove useful in practical measurements, however, we must obtain the mean-square current *per unit frequency interval*, i.e., we require the power spectrum of the current fluctuations. According to the Wiener-Khinchine theorem,<sup>29</sup> the power spectrum can be obtained from the Fourier transform of the autocorrelation of  $\Delta I(t)$ .<sup>30</sup> If the highest frequency passed by the measurement apparatus is much smaller than  $\tau^{-1}$ , where  $\tau$  is the time required for an electron to travel from the cathode to the anode, then each electron impact can be represented as a  $\delta$ -function of integrated strength  $e$ . It is then not difficult to show<sup>7,27,31</sup> using standard methods of Fourier analysis that the mean-square current fluctuation  $d[(\Delta I_{rms})^2]$  in the frequency interval  $df$  is

$$d[(\Delta I_{rms})^2] = 2I_{av}edf \quad (f \ll \tau^{-1}). \quad (4)$$

Equation (4) is Schottky's theorem. We infer that, for frequencies much smaller than the reciprocal of the transit time, the shot noise  $d[(\Delta I_{rms})^2]$  is truly "white" noise: the shot noise per frequency interval  $df$  is independent of the frequency.<sup>32</sup> Equation (4) can alternatively be written in terms of the variance and the average of the voltage  $V(t)$  measured at the output of a wide-band current-to-voltage converter with gain  $R_0$  used to measure the current:

$$d[(\Delta V_{rms})^2] = 2eR_0V_{av}df. \quad (5)$$

Now suppose these voltage fluctuations are measured using a linear amplifier with a gain of  $g(f)$ . (That is, if the input to the amplifier is the sinusoidal voltage  $v_{in}(t) = v_0 \sin[2\pi ft]$ , then the rms voltage at the amplifier

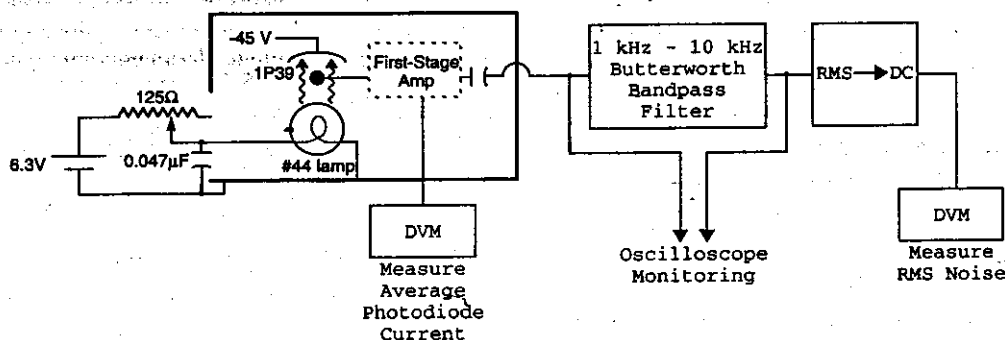


Fig. 2. A general schematic diagram of the shot-noise apparatus. A No. 44 pilot bulb illuminates a 1P39 vacuum photodiode. The dc component of the photodiode current is converted to a voltage and measured with a DVM. The ac component (shot-noise plus amplifier noise) of the current is preamplified, and then filtered using a Butterworth bandpass filter. The output of the Butterworth filter is measured using a rms-to-dc converter chip.

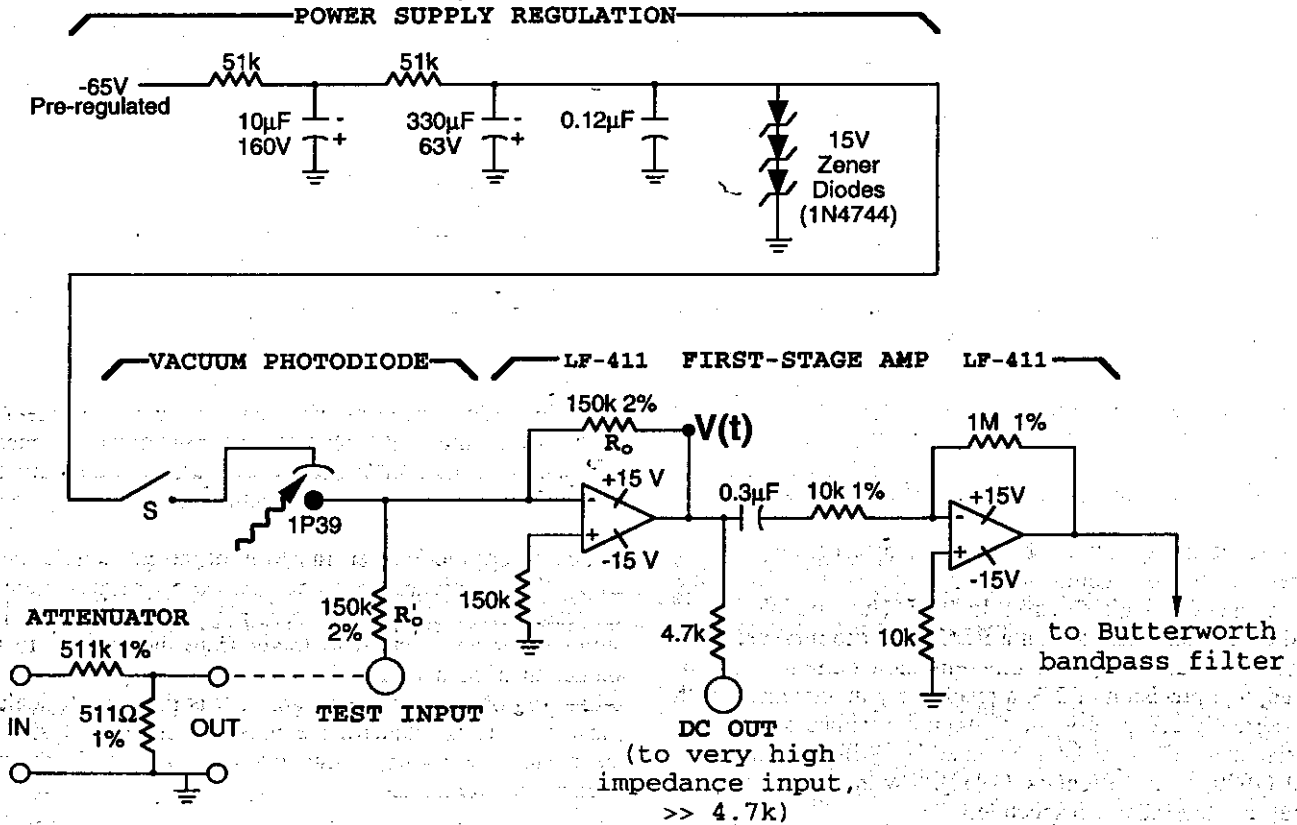


Fig. 3. Circuit schematic of the vacuum photodiode and first-stage amplifier. The photodiode bias is regulated by the Zener diode network. The photodiode current is converted to a voltage  $V(t)$  (gain  $150 \text{ k}\Omega$ ); the dc component of the voltage is measured at the point labeled "dc out." The ac component of the voltage  $V(t)$  is amplified ( $100\times$ ) and passed to the Butterworth filter. For measurement of the integrated bandwidth  $\int g^2(f) df$ , the switch  $S$  is opened, and a  $1 \text{ mV rms}$  sinusoidal voltage is connected to the point labeled "test input."

output will be  $v_{\text{out,rms}} = g(f) v_{\text{in,rms}}$ . Then, using Eq. (5), the total mean-square voltage fluctuation due to shot noise measured at the output of the amplifier is<sup>18,26</sup>

$$(\Delta V_{\text{rms}}^{\text{out}})^2 = 2eR_0 V_{\text{av}} \int_0^{\infty} g^2(f) df \quad (\text{shot noise}). \quad (6)$$

The current-to-voltage converter and the amplifier will also contribute noise, designated  $\Delta V_{\text{amp}}$ , which should be uncorrelated with the shot noise in Eq. (6). Thus the total mean-square fluctuation in the voltage measured at the output of the amplifier is

$$(\Delta V_{\text{rms}}^{\text{out}})^2 = (\Delta V_{\text{amp,rms}})^2 + 2eR_0 V_{\text{av}} \int_0^{\infty} g^2(f) df. \quad (7)$$

If the mean-square voltage fluctuation  $(\Delta V_{\text{rms}}^{\text{out}})^2$  is measured as a function of the average voltage  $V_{\text{av}}$ , and if the bandpass  $g(f)$  of the measurement apparatus can be obtained with sufficient accuracy, Eq. (7) can be applied to find the charge  $e$ .

## II. DESIGN OF THE APPARATUS

A general schematic of the apparatus used to generate and measure shot noise is displayed in Fig. 2; each of the major components shown will be described separately in detail below. A No. 44 incandescent pilot lamp, powered by a  $6.3 \text{ V}$  dc source in series with a  $125 \Omega$  rheostat, illuminates a vacuum photodiode (Centron 1P39) biased at  $-45 \text{ V}$ . The

photodiode current is fed to a first-stage amplifier, which converts the current to a voltage  $V(t)$  (as seen in Fig. 1) with a gain of  $R_0 = 150 \text{ k}\Omega$ . The remainder of the apparatus is dedicated to measurement of the average and the variance of the voltage  $V(t)$ . The dc component  $V_{\text{av}}$  can be measured directly with a digital voltmeter (DVM). The ac component of  $V(t)$  is fed to a homemade  $1 \text{ kHz} - 10 \text{ kHz}$  Butterworth bandpass filter, which defines a reproducible, easily measurable frequency window for the investigation. Finally, the rms output of the Butterworth filter is measured with a commercial rms-to-dc converter chip (Analog Devices AD637-JQ). Thus the dc output voltage of the rms-to-dc converter is the rms noise  $(\Delta V_{\text{rms}}^{\text{out}})$  appearing in Eq. (7) above.

In Fig. 3 we show the details of the vacuum photodiode and first-stage amplifier. The photodiode bias is stabilized at  $-45 \text{ V}$  using three  $15 \text{ V}$  Zener diodes powered with a dc supply. The Zener network serves to regulate the bias voltage, ensuring that the bias will not change if the dc supply suffers from small drifts. The first-stage amplifier consists of the  $150 \text{ k}\Omega$  current-to-voltage converter followed by a  $100\times$  ac amplifier.

Figure 4 depicts the Butterworth bandpass filter. We chose the Butterworth design because of the flat frequency response over its pass band, which greatly facilitates the numerical integration required in Eq. (7). Our Butterworth device consists of an active  $1 \text{ kHz}$  high-pass filter followed by an active  $10 \text{ kHz}$  low-pass filter. We found it convenient to use one dual op-amp (Motorola LF-412) for the high pass, and a second identical dual op-amp for the low pass. The

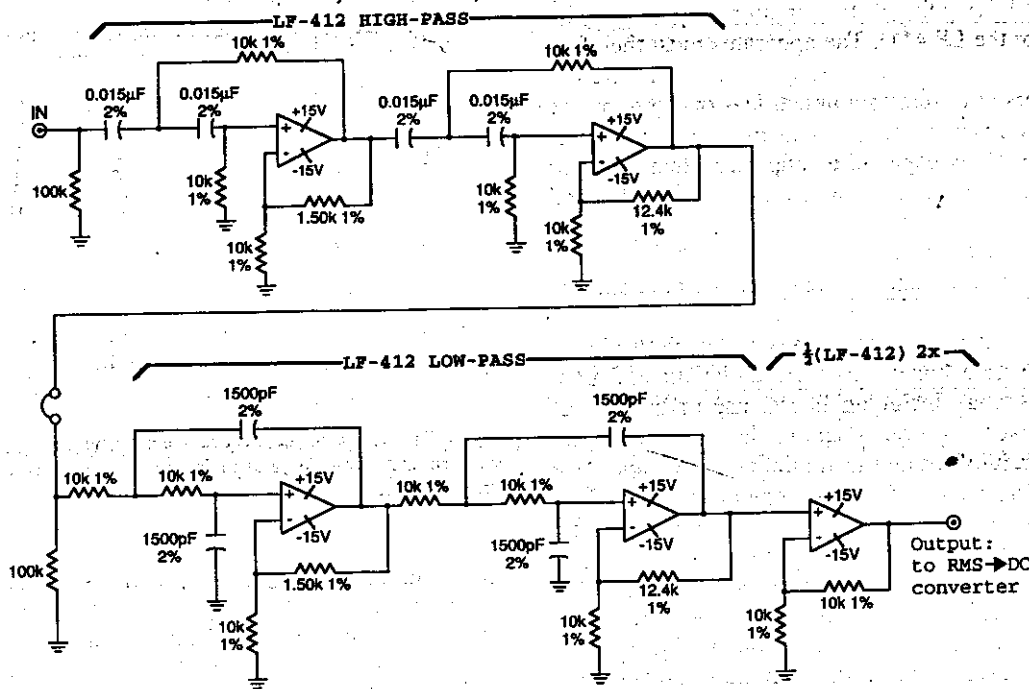


Fig. 4. Circuit schematic of the Butterworth bandpass filter designed to pass frequencies between 1 and 10 kHz. The high- and low-pass sections were constructed with dual LF-412 op-amps. The overall gain of the filter at 3 kHz, including the final 2× amplification, is about 13×.

filtered output is amplified with a final 2× noninverting amplifier. The maximum total gain of the Butterworth filter is about 13× and occurs at an input frequency of about 3 kHz.

In Fig. 5 we show the pin-out diagram for the rms-to-dc converter chip. We investigated the accuracy of the AD637 by using this chip and an oscilloscope to measure sinusoidal inputs; we were unable to detect a difference in the rms values obtained for the frequencies of interest in our measurements. Based on these observations and the specifications<sup>33</sup> of the AD637, we believe the rms-to-dc conversion in our experiments is accurate at least down to the 1% level.

To determine the electronic charge  $e$ , two separate measurements must be carried out.

(1) *Calibration of the gain: Measurement of  $\int_0^\infty g^2(f)df$ .* We first determine the frequency-dependent gain  $g(f)$  of the entire measurement apparatus (first-stage amplifier, bandpass filter, and rms-to-dc converter), using a sine wave of frequency  $f$  with a known rms voltage  $v_{in\ rms}$  input at the point labeled "test input" in Fig. 3. The switch  $S$  to the photodiode is opened for these calibration measurements. The resistances  $R_0$  and  $R'_0$  were matched to better than 0.1%, so that the voltage  $V(t)$  in Fig. 3 will be equal in magnitude (within about 0.1%) to the voltage  $v_{in}(t)$  at the test input. Thus the desired gain is just  $v_{out}/v_{in\ rms}$ , where  $v_{out}$  is the dc output of the rms-to-dc converter. By varying the frequency of the input sine wave over a wide range, enough data can be acquired to allow a satisfactory evaluation of the integral  $\int_0^\infty g^2(f)df$ .

(2) *Measurement of the shot noise.* The vacuum photodiode is now switched back into the circuit and the sinusoidal test voltage is removed. The current through the pilot lamp filament is varied using the 125 Ω rheostat shown

in Fig. 2, so that  $\Delta V_{rms}^{out}$  (as measured at the output of the rms-to-dc converter) can be recorded over a range of different values for  $V_{av}$  (as measured at the point labeled "dc out" in Fig. 3). According to Eq. (7) above, a plot of  $(\Delta V_{rms}^{out})^2$  vs  $V_{av}$  should yield a straight line with slope  $2R_0[\int_0^\infty g^2(f)df]e$ .

We note that the op-amp equipped with the 150 kΩ feedback resistor in Fig. 3 is used as a current-to-voltage converter for the shot-noise measurements, and as a unity-gain inverting amplifier for the calibration measurements. The frequency response of these two configurations will be the

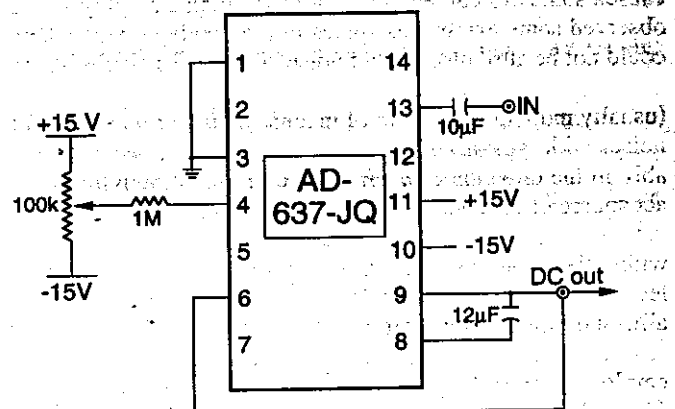


Fig. 5. Pin-out diagram of the rms-to-dc converter chip (Analog Devices AD637) used to measure the rms output of the Butterworth filter. The dc output voltage at pin 9 is equal (within better than 1%) to the rms of the input voltage at pin 13.

same, in principle, only well below the frequency  $f_T$  at which the open-loop gain of the op-amp drops to unity (about 4 MHz for the LF-411). The apparatus must therefore be designed so that the high-frequency cutoff of the Butterworth filter occurs at a frequency much less than  $f_T$ .

Since the experiment is designed to measure small fluctuations about a dc average, it is important that external noise sources be eliminated as much as possible. Several common procedures were useful in achieving this goal. (1) All components were housed in closed metal boxes. We found it convenient to mount the lamp, photodiode, and the first-stage-amplifier electronics in a single light-tight box. The Butterworth filter and the rms-to-dc converter were placed inside separate boxes. (2) Careful layout and construction was essential, including liberal use of bypass ceramic capacitors at the power inputs to the op-amps. Components were carefully located to minimize stray capacitive coupling. The performance of each independent stage of the apparatus (first-stage op-amps, filter, rms-to-dc converter) was examined separately before any experiments were carried out. An additional small capacitance ( $<10$  pF) could be placed in parallel with the feedback resistor on either first-stage op-amp, if necessary, to reduce the high-frequency components ( $>20$  kHz) at the LF-411 output. (3) All connections to external power supplies and between the different major components were made with coaxial cables or twisted-pair leads. (4) The power supplies were equipped with external decoupling capacitors. (5) Because the vacuum photodiode was very sensitive to acoustic vibrations, it proved useful to place the box containing the photodiode on a soft pad while running the experiments. (6) Continuous oscilloscope monitoring (see Fig. 2) of the voltage fluctuations was very helpful both in visualizing the shot noise, and as a diagnostic tool for identifying extraneous (nonshot) noise spikes or faulty components. Indeed, by using the "tangential" method<sup>34</sup> for oscilloscope measurements of Gaussian noise, one can obtain a fair estimate of the amplified rms shot noise without using the rms-to-dc converter.

During our first trials the operational amplifiers (LF-411 and LF-412) displayed so-called "popcorn" noise.<sup>4,35</sup> While this caused some initial problems, the popcorn noise disappeared altogether after the first few experiments, suggesting the op-amps required time to burn in. The pilot lamps also caused some difficulties: for most of the lamps employed, we observed some extraneous spikes in the anode current which could not be attributed to shot noise. The spikes, as measured at the output of the first-stage amplifier, were quite large (usually more than an order of magnitude larger than the shot noise) with typical widths of about  $50 \mu\text{s}$ , and were invariably in the direction of decreasing current. Although the exact source of the spikes is not understood at present, we have determined that these high-frequency fluctuations are caused within the pilot-lamp bulb itself: different bulbs had different levels of extraneous spikes (one bulb displayed no spikes at all), and when diffuse sunlight was instead used as a light source, none of these spikes was observed. For most bulbs employed, the spikes occurred at a low rate (on the order of 1/s) and did not affect the linearity of the data ( $[\Delta V_{\text{rms}}^{\text{out}}]^2$  vs  $V_{\text{av}}$ ) or the results for  $e$  to any measurable degree. In several extreme cases, however, the spikes were so frequent that the extraneous noise resulted in noticeable deviations from linearity and errors on the order of  $+15\%$  in the determination of  $e$ . Based on these experiences, we strongly recommend

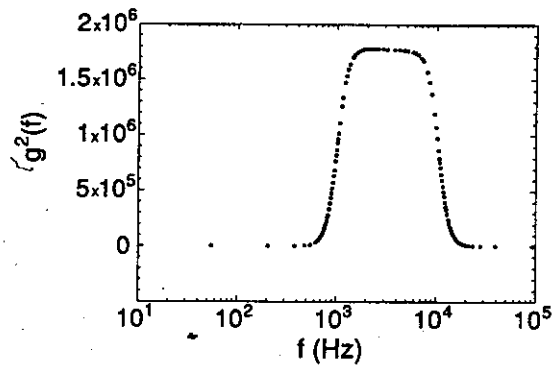


Fig. 6. The square of the frequency-dependent gain  $[g^2(f)]$  of the measurement apparatus. The area under the curve is derived from a numerical (trapezoid-rule) integration of the data.

that an initial oscilloscope study of the photodiode output be carried out using several pilot lamps to identify a bulb delivering a minimal number of extraneous spikes.

### III. PERFORMANCE

To measure the integrated bandpass  $\int g^2(f) df$ , we used a Tenma 72-380 function generator to supply a sinusoidal test input voltage. We confirmed that the amplitude of the sine wave output of the function generator changed by less than 0.5%, and that the sinusoidal shape was preserved, when the frequency of the output was varied between 120 Hz and 20 kHz. Since the maximum gain of the amplifier-filter ensemble is on the order of  $1300\times$ , the test-input voltage  $v_{\text{in}}(t)$  should be chosen to be on the order of 1–2 mV rms. Because it is difficult to measure directly a voltage in this range with high accuracy, we attenuated a 1 V rms sinusoid output from the function generator down to 1 mV rms with a voltage divider constructed from 1% resistors. (All rms voltages were measured with the AD637 rms-to-dc converter.) This 1 mV rms signal was connected to the test input shown in Fig. 3. We found that using a commercial 60 dB attenuator in place of the simple voltage divider did not significantly improve our results.

For the bandpass measurements, the peak output voltage  $v_{\text{out}}$  of the rms-to-dc converter, occurring near a midband frequency of 3 kHz, was about  $v_{\text{out}}=1.3$  V for an input of  $v_{\text{in}}=1$  mV rms. From the limits of integration in Eq. (7) above, it is clear that the data obtained must include frequencies well outside the passband of the filter, where the gain  $g(f)$  approaches zero. For frequencies well outside the passband,  $v_{\text{out}}$  dropped to a constant baseline of  $v_{\text{out base}}=19$ –20 mV. This baseline was attributed to amplifier noise and hence was subtracted in quadrature from all data points.<sup>26</sup> The square of the gain  $g^2(f)$  was thus found from

$$g^2(f) = \frac{v_{\text{out}}^2 - v_{\text{out base}}^2}{v_{\text{in rms}}^2} \quad (8)$$

A typical result for  $g^2(f)$  is shown in Fig. 6. The area  $\int g^2(f) df$  was found using the trapezoid rule with a standard graphics package (Kaleidagraph). A fairly large number of data points (usually 100–150) was acquired to minimize errors due to the trapezoid approximation. We obtained  $\int g^2(f) df = (1.721 \pm 0.004) \times 10^{10}$  Hz, where the uncertainty

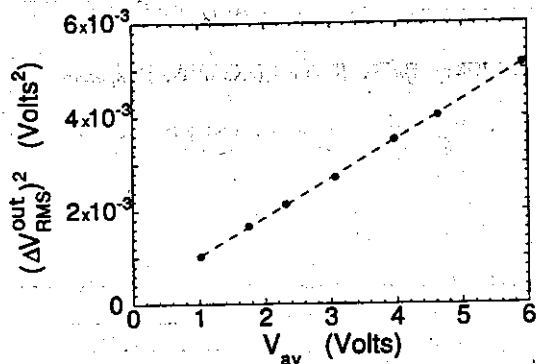


Fig. 7. The mean-square noise  $(\Delta V_{\text{rms}}^{\text{out}})^2$  plotted as a function of the average voltage  $V_{\text{av}}$ . The slope of the best-fit line is the product  $2R_0[\int g^2(f)df]e$ .

(about 0.2%) is the spread ( $2\sigma_{\text{mean}}$ ) obtained from three independent measurements taken over a three-week period. To check the validity of the trapezoid integration procedure for our data, we repeated the numerical integration for one data set with every other point deleted, and found that the integral changed by only about 0.1%.

The actual shot-noise data were acquired by varying the voltage across the pilot lamp filament over the range 0–6 V. The same DVM was used to measure the output of the rms-to-dc converter in the calibration and shot-noise measurements. With a typical bulb almost in contact with the photodiode, this procedure resulted in a maximum photodiode current of about  $I_{\text{av,max}} = 40 \mu\text{A}$ , which in turn resulted in maximum dc-average and shot-noise voltages of about  $V_{\text{av,max}} = 6 \text{ V}$  and  $[\Delta V_{\text{rms}}^{\text{out}}]_{\text{max}} = 70 \text{ mV}$ , respectively. To investigate the possible effects of space charge, we temporarily removed the Zener diodes shown in Fig. 3 and repeated the shot-noise measurements with the photodiode bias voltage set to  $-90 \text{ V}$ . Within our uncertainty, the results at  $-90$  and  $-45 \text{ V}$  were not different, implying space charge has no measurable effects on our experiments.

A typical result for  $(\Delta V_{\text{rms}}^{\text{out}})^2$  vs  $V_{\text{av}}$  is shown in Fig. 7. To evaluate the accuracy and precision of the apparatus, this measurement was repeated with four different No. 44 pilot lamps. The average slope was  $(8.17 \pm 0.07 \pm 0.16) \times 10^{-4} \text{ V}$ , where the first error (random) is the  $2\sigma_{\text{mean}}$  spread in the four slopes obtained, and the second error (systematic) is a cautious estimate of the uncertainty in the rms-to-dc conversion (1% for the conversion, hence a 2% uncertainty for the mean square). Using this slope and the measured value for  $\int g^2(f)df$  in Eq. (7), we obtain finally  $e = (1.581 \pm 0.015 \pm 0.032) \times 10^{-19} \text{ C}$ , which can be compared to the accepted value of  $1.6022 \times 10^{-19} \text{ C}$ .<sup>24</sup> We note the final uncertainty in our result for  $e$  (about 3%) is dominated by the uncertainty in the slope of  $(\Delta V_{\text{rms}}^{\text{out}})^2$  vs  $V_{\text{av}}$ .

From the  $V_{\text{av}} \rightarrow 0$  intercept of the  $(\Delta V_{\text{rms}}^{\text{out}})^2$  vs  $V_{\text{av}}$  plots, we obtain the amplifier noise  $\Delta V_{\text{amp,rms}} = 15 \pm 1 \text{ mV}$  ( $2\sigma_{\text{mean}}$  uncertainty). Consideration of the known input noise sources (Johnson and op-amp noise in the first-stage amplifier) yields a rms noise figure of  $\sim 10 \text{ mV}$  which, given the typical uncertainties involved in assigning values to equivalent noise generators, agrees reasonably well with our intercept. The shot-noise intercept is somewhat smaller than the baseline observed in the bandwidth calibration, due mainly to the

Johnson noise introduced by the additional resistors used, in the latter measurement, at the input to the first-stage amplifier.

#### IV. CONCLUDING REMARKS

Given the widespread significance of statistical fluctuations in physics, we feel it is important that students gain direct hands-on experience with universal fluctuation phenomena, and we hope this work encourages increased study of such phenomena in undergraduate laboratories. We have attempted to use carefully constructed homebuilt circuits in lieu of commercial equipment whenever possible. The expense of the apparatus is minimal, and the circuits required for its construction are not overly complicated. Our measurements indicate accurate values for  $e$  and good reproducibility. It should nonetheless be pointed out that further improvements in the apparatus are undoubtedly feasible; for example, we are currently investigating alternative light sources (such as halogen-filled lamps) that may display better high-frequency stability than the ordinary pilot lamps employed to date.

#### ACKNOWLEDGMENTS

We are very grateful to George Clark for instruction on shot-noise measurements and for helpful advice. We thank Farzan Abdolsalami, Richard Bartels, Craig Bohren, David Cannell, and Wolfgang Rueckner for useful suggestions and/or critiques of the manuscript, and we are grateful to E. J. Krick for technical assistance. This work was supported in part by a grant from the National Science Foundation (Instrumentation and Laboratory Improvement Program, Grant No. DUE-935 1838).

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## Coherent backscattering of light

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(Received 2 August 1994; accepted 20 October 1994)

Coherent backscattering of light (CBS) is a phenomenon in which partial waves traversing time-reversed (momentum-reversed) scattering paths interfere constructively in the backscattering direction leading to the appearance of an intensity cone. This increase in reflectivity reduces the amount of light transported through the colloidal media. As an advanced undergraduate laboratory CBS introduces the student to low-level-light detection, precision angular resolved measurements, and sophisticated data analysis techniques. Additionally, the concept of photon self-interference, usually discussed in Modern Physics class in the context of Young's double slit experiment, is beautifully illustrated by CBS. © 1995 American Association of Physics Teachers.

### I. INTRODUCTION

Light scattering experiments have a long and time honored history in physics. Early experimental investigations into the nature and behavior of light were conducted by Newton, Huygens, Young, Fresnel, and Rayleigh to name a few. Our understanding has progressed from a belief in ether-borne vibrations to a self-consistent electromagnetic theory that incorporates all known light scattering phenomena. Light scattering experiments have enjoyed a renaissance since the invention of the laser and have gained further prominence as potential applications of quantum optics, nonlinear optical

materials, and photon localization are conceived. It is the possibility of achieving photon localization<sup>1</sup> that has spurred interest in coherent backscattering of light (CBS) as a current research topic. This experiment also has much to offer as an advanced undergraduate laboratory.

CBS is a photon self-interference effect which leads to an enhanced intensity cone in the backscattering direction. A schematic representation of the now classic CBS experiment is illustrated in Fig. 1. Laser light is incident on a dense colloidal suspension containing polystyrene spheres,<sup>2</sup> in water, undergoing Brownian motion. The diffuse intensity pro-