Electronic noise:

the first two decades

Most of the basic knowledge in the field of electronic noise was gained in the 20-year period following World War I. A considerable amount of the vacuum-tube data obtained was later translated into the semiconductor language

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You could hear a pin drop. (English saying)

You could hear the grass grow. (German version)

Such harmony is in immortal souls;
But whilst this muddy vestment of decay
Doth grossly close it in we cannot hear it.
(Merchant of Venice)

Fifty-two years ago the classical paper on noise in amplifiers was written by Dr. Walther Schottky.[31] The high-vacuum thermionic amplifier could then be called about six years old. Its development had taken place along nearly parallel lines in several countries, including Germany, mostly under rules of strict secrecy. It seems now almost incredible that out of the Germany of those years, faced with military defeat and economic collapse, could come a scientific paper of the quality and technical importance of this paper of Schottky's.

The amplifiers developed at the Siemens-Halske Works no doubt had the same kind of faults as those produced at other laboratories—poor welds, mechanical resonances, unstable cathodes, inadequate pumping, etc. These faults could distort the signals applied to the amplifiers and, since thermionic amplifiers were then being installed in commercial and military telephone systems, the faults became a technical liability. At this time I was employed in the Engineering Department of the Western Electric Company, the Engineering and Supply Division for the Bell System. I was assigned to study some of the many projects on vacuum-tube research, came early in touch with Schottky's work, and have some memories of the work that went on. With this as my background, the Editor of IEEE SPECTRUM asked me to write this article on the study of amplifier noise as I saw it develop during about the first two decades of its progress into a rather broad scientific field.

'Wärmeeffekt' and 'Schroteffekt'

In the 1918 paper, Dr. Schottky evidently assumes that the grosser current fluctuations produced by faulty tube structures such as those just enumerated have been, or can be, eliminated, and he is left with two sources of noise that are of a much more fundamental nature. One he calls the "Wärmeeffekt," in English now commonly named "thermal noise." This is a fluctuating voltage generated by electric current flowing through a resistance in the input circuit of an amplifier, not in the amplifier itself. The motion of charge is a spontaneous and random flow of the electric charge in the conductor in response to the heat motion of its molecules. The voltage between the ends of the conductor varies and is impressed upon the input to the amplifier as a fluctuating noise. This flow of energy between molecules and electric current involves not the charge of the electron but rather the rate of flow of power between charge and momentum. It involves the Boltzmann constant k times the absolute temperature T of the system, and a power flow of at least 10^{-17} watt to be audible in a telephone. Schottky believed any other noise source would be much stronger than this.

And here, for the sake of history, we may digress a bit. In estimating the total of noise that is going to be contributed by the "Schroteffekt," the integration of a certain expression is needed that has come to be called the Schottky equation. Schottky performed this integration

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and got the result $2\pi/r^2$, where r is a damping factor of the circuit, $r = R/L\omega$.

My recollection is that because of some postal delay the 1918 paper did not get to the United States until about 1920. On reading it, I became suspicious of the integration, but in the then-available tables of integration could find no solution for the Schottky equation. I asked my friend, Dr. L. A. MacColl, mathematician, for assistance. He suggested splitting the Schottky expression into four complex factors, integrating each separately and then recombining them for the final result, $2/r$. When, after much labor on my part, this was done, MacColl again looked at the equation and said this was a case for the method of poles and residues and, without putting pencil to paper, read off the correct result. This was impressive, but evidently the method had not yet penetrated down to physicists and engineers, and the more cumbersome method was left in. The method of residues was evidently used later by Fry[5] and by Hull and Williams,[11] but before them several other methods had also been suggested. We correctors, however, chose to abide by Schottky's word that the thermal-effect noise is much smaller than the shot noise, and recognition of the technical importance of the thermal noise was delayed by about a decade. This probably did not matter much, because it was a busy decade spent on other phases of the project.

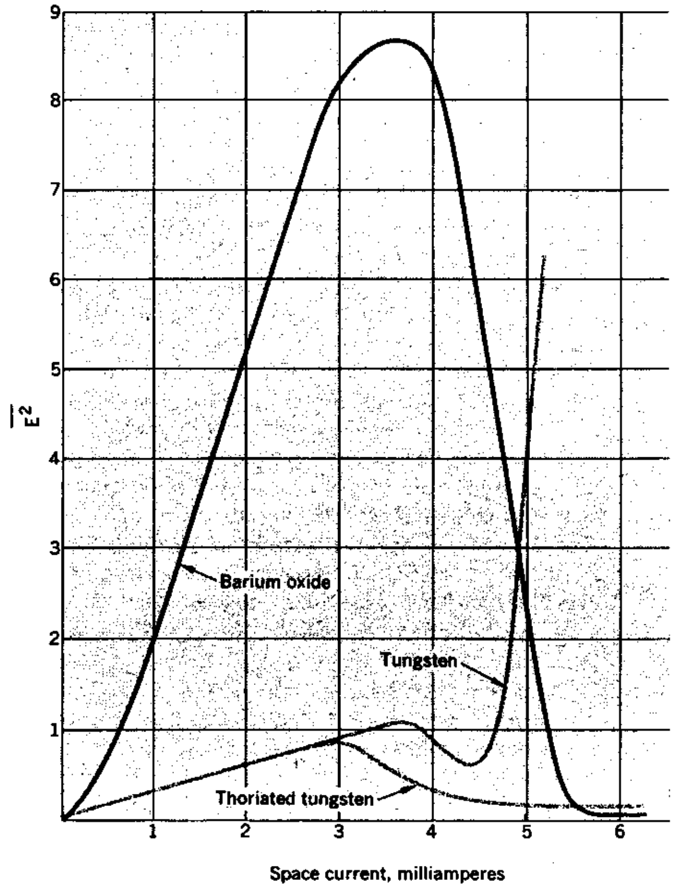
In the case of the "thermal noise," as we shall call it, the electric charge is in effect held in long bags with walls relatively impervious to electrons at low temperature. The mass transport of charge along the bag, or wires, under the influence of the heat motion, sets up the potential differences that generate the fluctuating output of the amplifier.

When now one end of the conductor, the "cathode" of the tube, is heated to incandescence, electrons can be emitted from the cathode surface to travel across the vacuum toward the anode. The electrons are emitted at random times, independent of each other, and they travel at different velocities, depending on initial velocity and voltage distribution for electron passage. In the case of a small electron emission, a small nearly steady flow of current results, with a superimposed smaller alternating current whose amplitude can be calculated from statistical theory. This small current flowing through the amplifier generates the "Schroteffekt," or shot effect, in the amplifier.

The first experimental work on identifying and measuring the shot effect was done in Schottky's laboratory and published by C. A. Hartmann in 1921. This seemed like a well-designed set of tests, but was a little ahead of its time in the new art. After corrections, it left little doubt of the existence of the shot effect.

The next step came with the publication of three papers in 1925. T. C. Fry[5] covered parts of the theory that he wanted put on a firmer mathematical basis. Through Fry, the work of Hull and Williams[11] at General Electric and Johnson[14] at Western Electric–Bell Laboratories became known to the participants, which may have given added impetus to the efforts. At GE, the first application of Hull's screen-grid tube in the amplifier increased the accuracy of the GE work to such a point that the value of the charge of the electron found by the shot effect came out close to that of the oil-drop method. The work of Johnson at lower frequencies revealed the existence of the "flicker effect," which could be many times greater than the shot effect, as well as the effect of space charge in reducing the magnitude of both shot effect and flicker effect by large factors (also recognized by Hull and Williams).

FIGURE 1. The effect of space charge on fluctuation noise. Three tubes have filaments composed of tungsten, thoriated tungsten, and barium oxide. $\bar{E}^2$ is the mean-square noise voltage across the output measuring device expressed in arbitrary units. The variation in space current was obtained by changing the cathode temperature, the plate voltage remaining constant. (Copyright 1934, The American Telephone and Telegraph Co.; reprinted by permission)



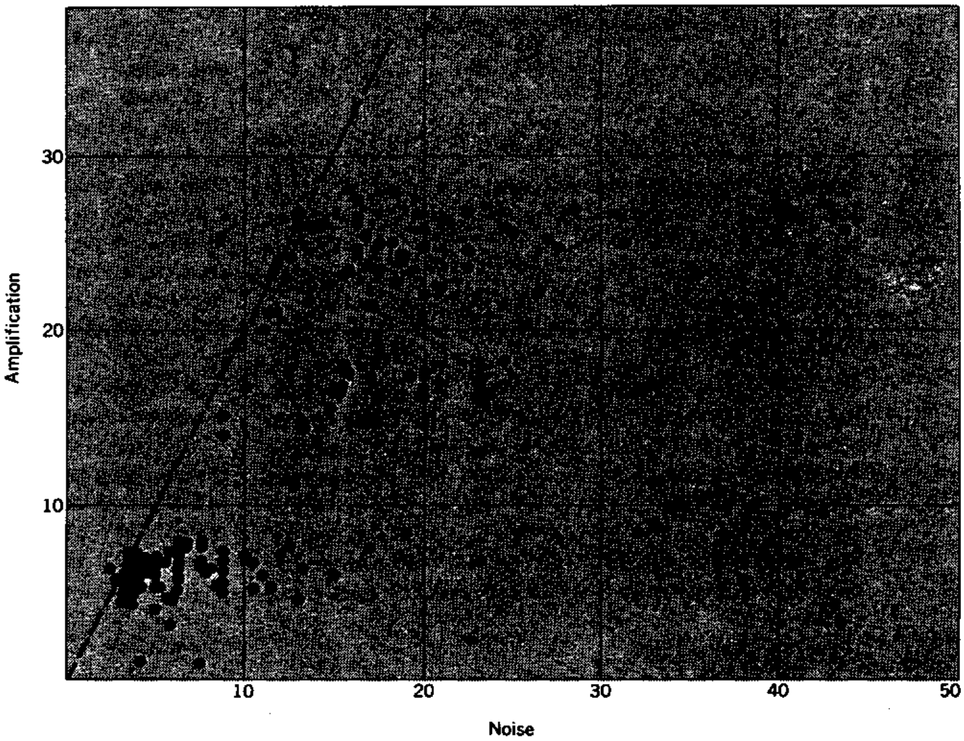


FIGURE 2. Amplification as a function of noise in three-electrode tubes; noise in arbitrary units; each point represents a tube. (Physical Review, 1925, reprinted by permission)

Each of these phenomenons will be discussed in connection with Fig. 1, which is reproduced from the 1934 paper by Pearson.[27]

By the early 1930s, the shot effect had been fairly well established for thermionic diodes, simple amplifiers, and photoelectric tubes.

A typical event that took place during the shot-effect work will be described here. We were visited by Sir J. J. Thomson, and the shot effect was demonstrated to him. Our explanation of it may not have been satisfactory, for as he left the room, the discoverer of the electron, with a forbearing smile and a gentle shake of the head, muttered, "Oh, no, no, no!"

Toward the end of the shot-noise work, a rough exploratory test was made. About 100 triode tubes of various kinds were picked out at random and tested for gain and noise in a circuit of fixed voltage, frequency range, etc. A resistance of 500 kΩ was connected across the input of the tube under test, with the output of the tube resistance–capacitance coupled to the amplifier. For each tube, the observed noise was plotted against the separately measured amplification of the tube, as in Fig. 13 of the 1925 article, here reproduced as Fig. 2. There is one point for each tube and these points are scattered over the right-hand side of the diagram. On the left, the point distribution stops abruptly along a straight sloping line. This suggests that along this line the noise pulses that the amplifier responds to have been amplified by the tube under test by its gain factor, from incoming pulses of more nearly constant value. Could this be the thermal effect predicted by Schottky?

A few simple tests, such as varying the electrical value of the input resistor, its temperature, its size, its material, soon answered the question in the affirmative. The results were discussed with Dr. H. Nyquist, who in a matter of a month or so came up with the famous formula for the effect, based essentially on the thermodynamics of a telephone line, and covering almost all one needs to know about the thermal noise.

The two effects: A and B (or T and S?)

We have, then, two different sources of electrical noise obeying statistical laws. Both have the properties in common that the noise can be described as a power dissipated by the noise source at a point of the amplifier circuit, and that for frequencies above certain values the noise power is constant up to very high frequencies. For thermal noise this constant power extends also to low values, while for shot noise there are many exceptions and variations.

T: the thermal effect. By the Nyquist[23] formulation, the thermal effect may be expressed as a voltage applied by the source to the input point of the amplifier at the high-impedance grid-leak resistor:

$$\text{Thermal formulation } \overline{V_T^2} = 4kTR \qquad (1)$$

Here $\overline{V_T^2}$ is the mean-square noise fluctuation per unit bandwidth as measured by a thermocouple voltmeter; R is the resistance of the input circuit; T is temperature in degrees Kelvin; k is Boltzmann's constant, 1.38×10^{-23} joules/degree K. This can also be written

$$W_T = 4kT \text{ watts per unit resistance} \qquad (2)$$

per unit bandwidth. The total for any case is then obtained by linear integration over the resistance and bandwidth range.

There is not much more to be done with this formulation except to consider the slope resistance of the tube, which will be done later.

S: the shot effect. The Schottky formulation for the shot effect per unit bandwidth may similarly be written

$$\overline{J_S^2} = 2ei \qquad (3)$$

or

$$W_S = 2eiR_1 \qquad (4)$$

where the charge on the electron $e = 1.602 \times 10^{-19}$ coulomb; i = dc space current, in amperes, flowing in space from cathode to anode (negative); R_1 = total resistance between cathode and anode, including that internal to the tube (function of frequency); and W_S = power per cycle dissipated in R_1.

This formulation was found by the early workers to hold under some carefully controlled conditions, including choice of cathode materials, freedom from space-charge effects, and choice of frequency band. When these conditions were judiciously selected, the experiments yielded, for instance, very nearly the correct value for the charge on the electron, as was shown in the tests of the 1920s. More complicated effects were also observed; they were subjected to a rather concentrated theoretical attack in the 1930s and will briefly be described in the following paragraphs.

1. *The flicker effect.* With some cathodes there is superimposed on the pure shot effect a fluctuation in current that is much greater than the shot current itself. This is illustrated in Fig. 1. The linear portion of the curve, obtained from tubes having filaments of tungsten and thoriated tungsten, gives the values the pure shot noise should have. The noise data were recorded as the temperature of the cathode was raised, the plate voltage of the diode being supplied by a fixed battery through a constant resistance. A measure of the cathode temperature is given by the indicated total current, in milliamperes. In the barium oxide tube, the noise increased more rapidly and reached a maximum value approximately ten times that of pure shot noise. The reason for this excess noise was surmised by Johnson to be fluctuations in the work function of the cathode surface due to particle migration, and was discussed at length by Schottky, who called it "Fackelneffekt."

2. *Space-charge depression.* Still in Fig. 1, after passing through a maximum, the noise in all three of the tubes decreases toward values eventually far below the theoretical shot value, at first thought to be effectively zero. This is an important feature, for it is in this low noise range that thermionic devices can be used as amplifiers. Schottky ascribes this noise depression to the smoothing effect of a dense space-charge layer near the cathode—between cathode and grid in a triode, for instance—and he works out a plausible theory for it.

3. *Frequency and flicker effect.* With fixed operating conditions, except for the natural frequency of a narrow-band circuit that the device works into, the noise output depends on this frequency. Normally the noise varies with this frequency f as

$$\overline{J_S^2} = f^{-n} \qquad (5)$$

where n may lie in the range 1.2–0.9, depending on the material and condition of the cathode. For very pure materials, this increase in noise may be unobservable at frequencies above a few thousand hertz. Oxide cathodes, and perhaps all cathodes, show the effect down to very low frequencies, such as perhaps one cycle per month, where the noise has merged with the natural drift of the device.

The f^{-n} law has been discussed theoretically by Schottky and others.

4. *Ionic effects.* Ions may be generated from gas in the device, or from the electrodes, either by photoelectric or collision processes. The ion current would normally be small and make only a small addition to the dc electron current. But if, say, a heavy positive ion becomes trapped in the negative potential well that is created by the electron space charge, then a large pulse of electrons may be released through the potential minimum to make a noise pulse. This effect was described by Johnson[14] and studied by Ballantine[1] and others. The sharp rise of the noise at high currents as depicted for the tungsten tube in Fig. 1 is a result of ions emitted from its filament.

5. *Thermal noise in plate current.* A curious situation developed in about 1930. Llewellyn[21] suggested that the internal resistance R_0 of the thermionic device is really in parallel with the external resistance R_1, the parallel combination taken as the thermal noise source of the output circuit. Llewellyn suggested that this slope or differential resistance should be considered at the cathode temperature in combining it with the external resistance at room temperature. The result seemed to give reasonable agreement with observations.

6. *The half-temperature rule.* In making more careful measurements, however, Pearson[28] concluded that the temperature of the slope resistance should be half of the absolute temperature of the cathode in order to get agreement with Eq. (1) or (2). There seems at first to be no physical basis for this peculiar situation, but further experiments seemed to agree. Some found it hard to believe that there could be such a coupling between a stream of electrons and their source (the cathode). The most careful calculation of the effect, based on certain assumptions, was made by Rack,[29] who found that over a considerable part of the mid-temperature range the value of the temperature should be taken as $0.644T$ instead of $0.500T$. The most plausible explanation of the effect is probably presented by Schottky,[33] who arrived at about 0.500 for the factor, but his presentation has to do with a certain rectification of the noise signal in the output circuit of the device and is not easy to repeat here.

Another facet of the $\frac{1}{2}T$ rule is that for very small currents the noise can be derived from either the Schottky equation ei or the Nyquist equation kT. This is in the region where the current to the anode is too small to set up appreciable space charge, because of too low a cathode temperature. This seems to have been first noticed by F. C. Williams,[42] but was also discussed by Schottky and others. The temperature must again be taken as $\frac{1}{2}T$, but tubes are probably not often used in these regions, except possibly for logarithmic response.

Rating of tubes

We have, then, two fundamental sources of noise in an electronic circuit: thermal noise, which can be calculated from the input parameters; and shot noise, which is modified by various device parameters and can, in some cases, be calculated, or can be measured for individual devices. In the device, the effects of these sources are added into a noise-power spectrum. In a diode, this is fairly simple, but in a grid-controlled tube it is more complicated since each electrode must be considered.

Up to 1940, the period of this review, the method proposed by Johnson for grid-controlled tubes was followed closely by others. This technique involved short-circuiting the input, setting the other parameters at some operating condition, and measuring the noise at the output of the device in this condition. This was then considered the noise figure introduced by the device itself, and it could be expressed in terms of a resistance at the input that would give the same amount of thermal noise. This would normally be a few hundred to a few thousand ohms.

Several measurements of this kind will be referred to but no details will be given here because methods may have changed and, moreover, because most of the tubes tested are now obsolete.

Tests on a few U.S. tube types were made by Pearson,[27] whereas Moullin and Ellis[22] reported tests on some British tubes. Spenke[38] studied some German tubes, on which he presented extended and careful discussions. Probably the most extensive and detailed discussion and measurements on U.S.-made tubes for our period were reported by Thompson, North, and Harris.[41]

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The following list of references is appended for readers who want to go a little further into the early history of our subject than is done in this brief review. It should help establish the approximate sequence of the important steps made in the first two decades. The list does not pretend to be complete, and it contains many items that are not specifically referred to in the main text.

By 1940 this basic work had been about completed, and from there on the work on noise took different directions. First, there was the highly mathematical study of how to extract a weak signal from a background of noise. Then came the transistor and the translation of the vacuum-tube data into the semiconductor language. This opened up new fields of applications, such as low-temperature work, rocketry, and space research. A few recent references may open the door to these fields.

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