"Complementi di Fisica"
Lecture 4

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Course Outline - Reminder

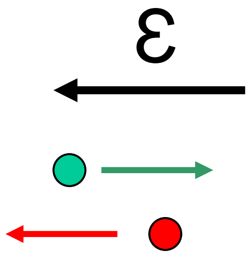
- The physics of semiconductor devices: an introduction
  - Basic properties; energy bands, density of states
  - Equilibrium carrier concentration (“intrinsic”, “extrinsic”)
  - Carrier transport phenomena
    - Drift and Diffusion
    - Generation and Recombination
    - Continuity equations
- Quantum Mechanics: an introduction
- Advanced semiconductor fundamentals

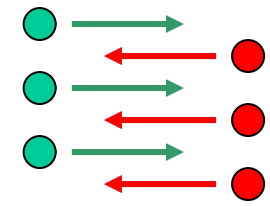
- Lecture 2: intrinsic carrier concentrations…

Lecture 4 - outline

- Carrier transport phenomena (introduction)

- Carrier drift
  - Carrier drift velocity in an external electric field:
    - Mobility
    - Scattering on vibrating lattice and on impurities
    - T- dependence of mobility
  - Electric current density in an external electric field:
    - Conductivity
    - Resistivity
  - Measurements:
    - resistivity: 4-point probe
    - Carrier type and concentration: Hall effect





Carrier transport phenomena

- Non-equilibrium conditions may arise because of:
  - External electric field ⇒ drift
  - Non-uniform doping (i.e. carrier concentration) ⇒ diffusion
  - Injection of “excess carriers” ⇒ $np \neq n_i^2$
- Return to equilibrium:
  - Dissipative phenomena (scattering)
  - Generation-recombination processes
- All above phenomena occur simultaneously: summarized in the transport equations:
  - Current density and continuity equations
- We start by studying the *drift of carriers in an external field*

Drift

Random thermal motion

Statistical mechanics:
equipartition theorem
for electrons

$$\frac{1}{2} m_n^* \left\langle v_{th}^2 \right\rangle = \frac{3}{2} kT$$

$$T = 300\ \mathrm{K} \Rightarrow \sqrt{\left\langle v_{th}^2 \right\rangle} \approx 10^7\ \mathrm{cm/s}$$

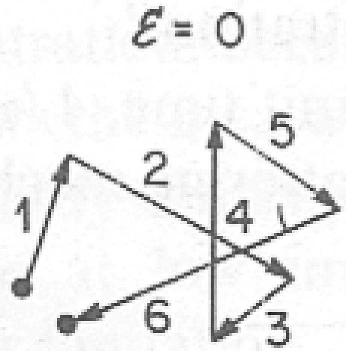
Drift combined with thermal motion

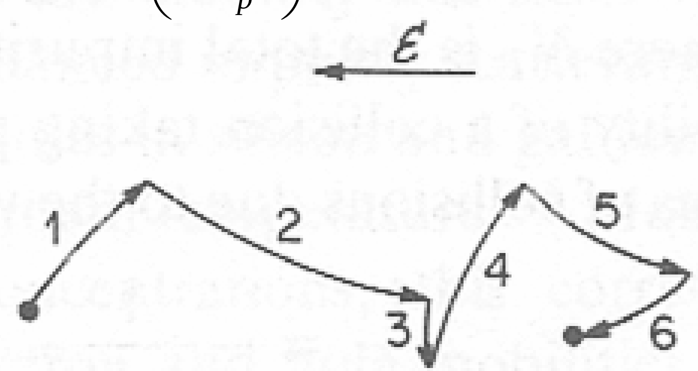
"classical electron":
- charge -|q|
- effective mass m_n^*

$$-|q| E \tau_C = m_n^* v_n \qquad E = \text{electric field}$$

$$v_n = -\left(\frac{|q|\tau_C}{m_n^*} \right) E = -\mu_n E \qquad \mu_n \equiv \frac{|q|\tau_C}{m_n^*}$$

$$v_p = \left(\frac{|q|\tau_C}{m_p^*} \right) E = \mu_p E \qquad \mu_p \equiv \frac{|q|\tau_C}{m_p^*}$$





Mobility

- Two main collision mechanisms for mobility
  - Scattering on lattice deformations: $\mu_L \propto \tau_L \propto T^{-3/2}$
  - Scattering on impurities: $\mu_I \propto \tau_I \propto T^{3/2}$
  - (more details later on)

$$\mu = \frac{|q|}{m^*}\tau$$

- The two mechanisms coexist:

$$\frac{1}{\tau_c} = \frac{1}{\tau_{c,\mathrm{lattice}}} + \frac{1}{\tau_{c,\,\mathrm{impurity}}} \qquad \frac{1}{\mu_c} = \frac{1}{\mu_\mathrm{L}} + \frac{1}{\mu_\mathrm{I}}$$

$\propto$ scattering probability:
in a small time interval *dt*

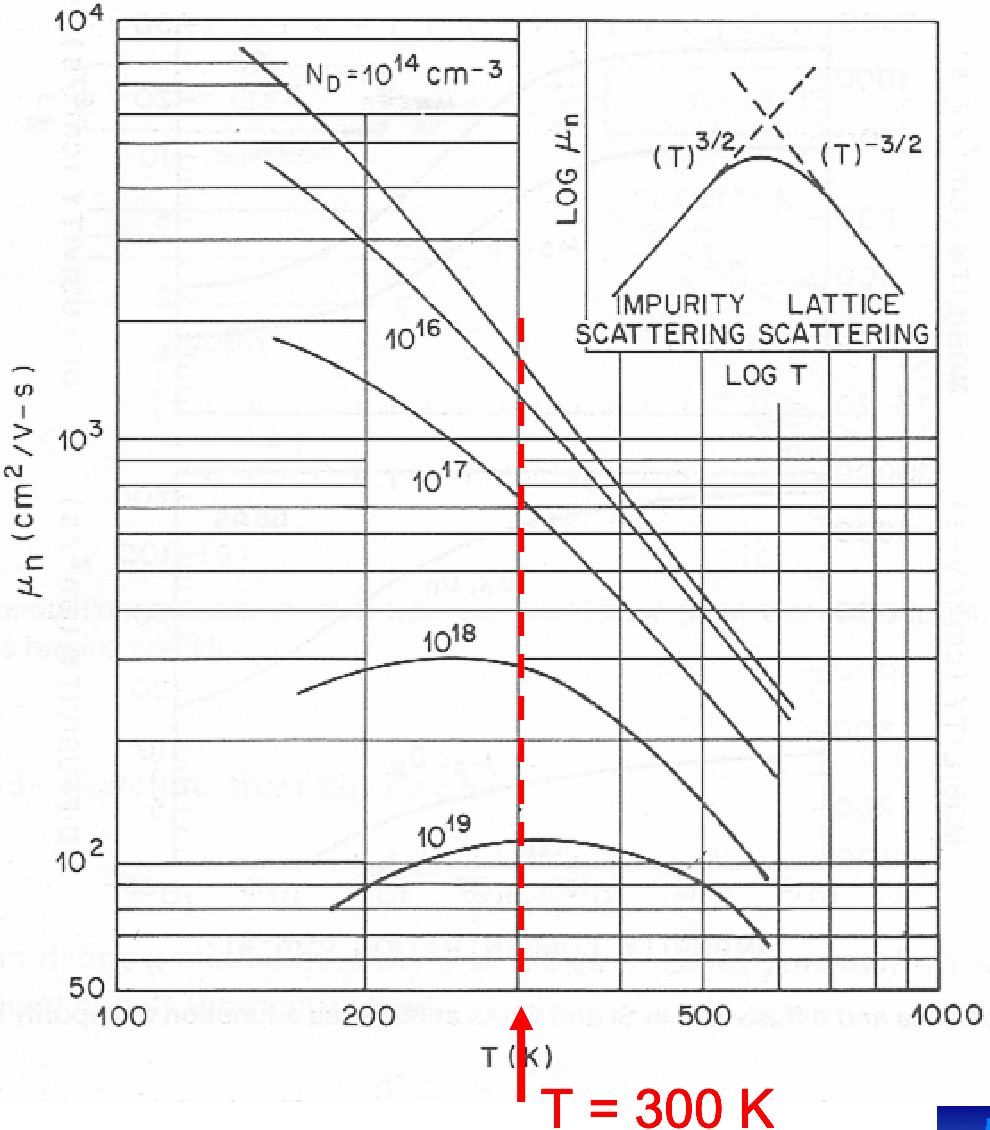
$$P = \frac{dt}{\tau_c} = P_L + P_C = \frac{dt}{\tau_L} + \frac{dt}{\tau_I}$$

T – dependence of mobility

Two main collision mechanisms for mobility:

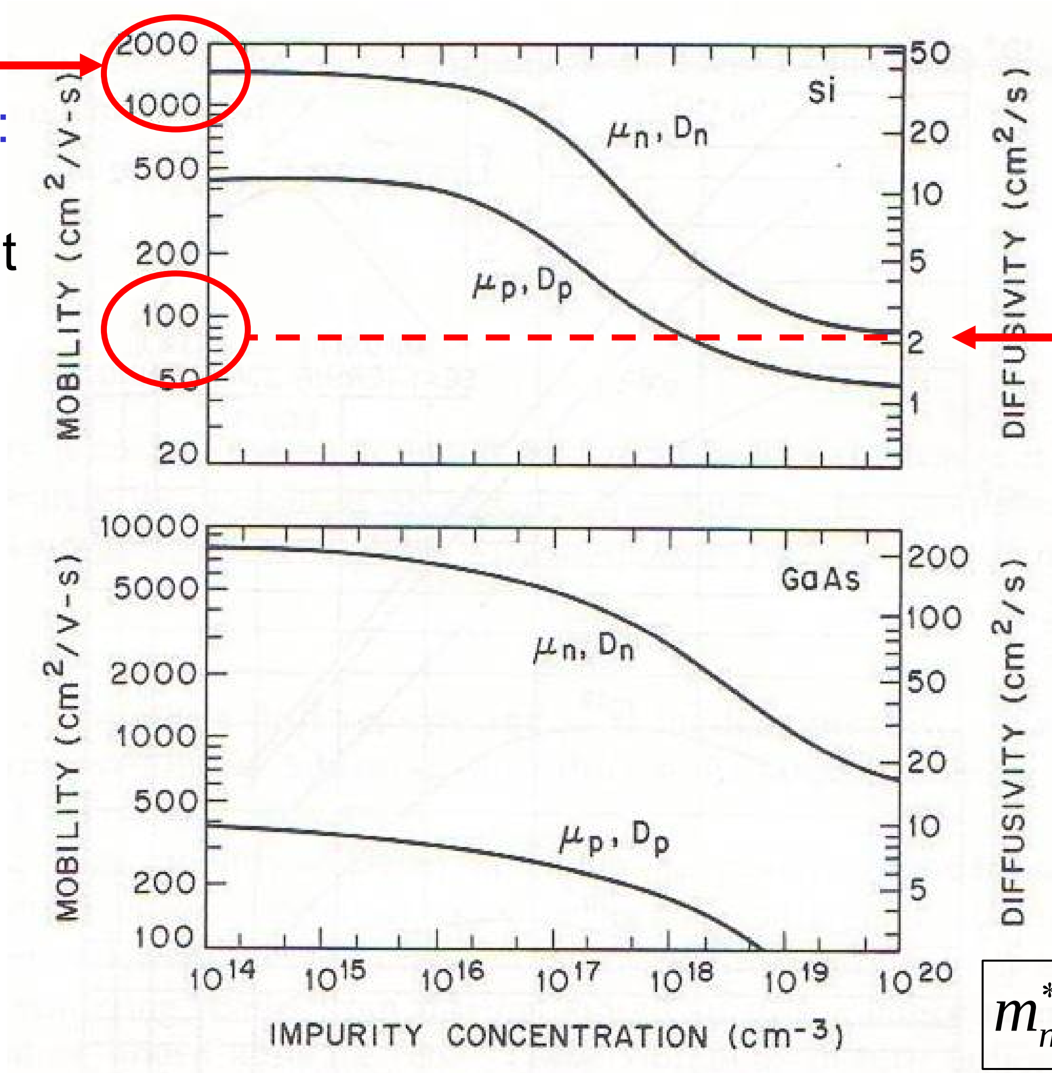
Scattering on lattice deformations:
$\mu_L \propto \tau_L \propto T^{-3/2}$

Scattering on impurities:
$\mu_I \propto \tau_I \propto T^{3/2}$



Mobility and impurity concentration

Low doping concentration:
Lattice scattering limit



T ~ 300 K

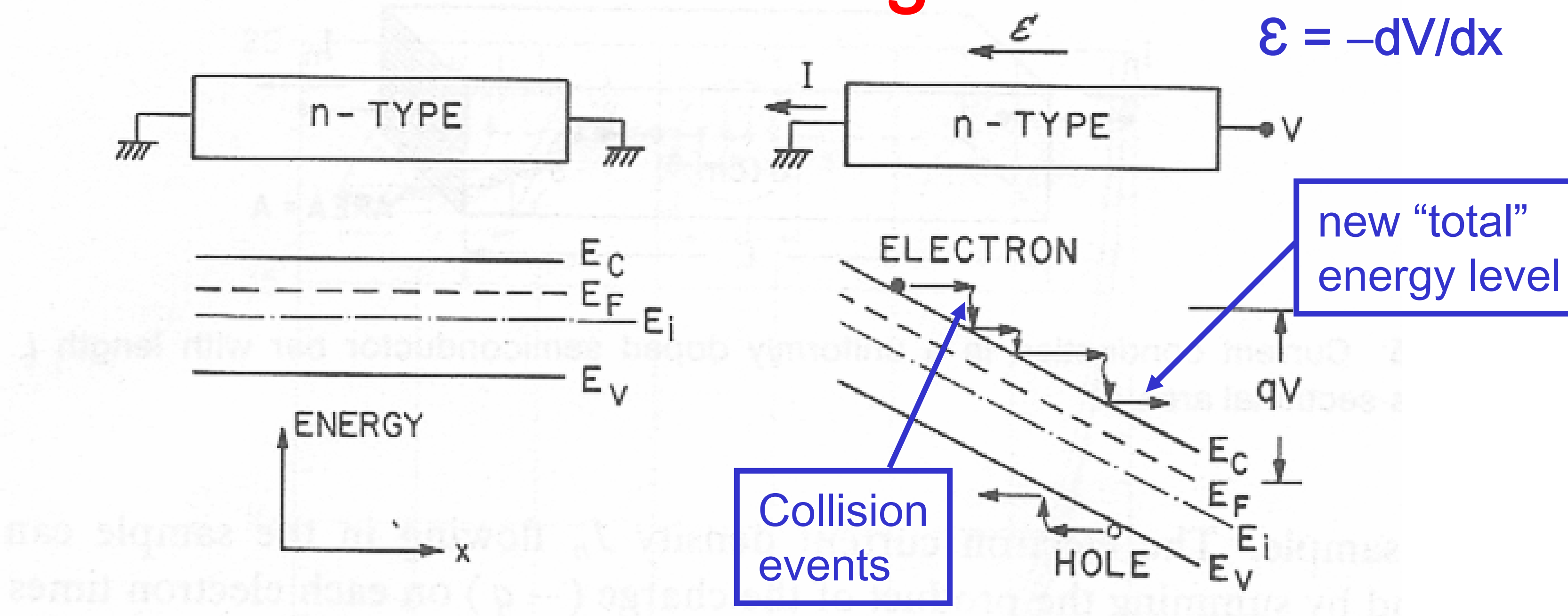
High doping concentration:
Impurity scattering limit

Electrons and holes:

$$m_n^* < m_p^* \Rightarrow \mu_n > \mu_p$$

"Band bending"

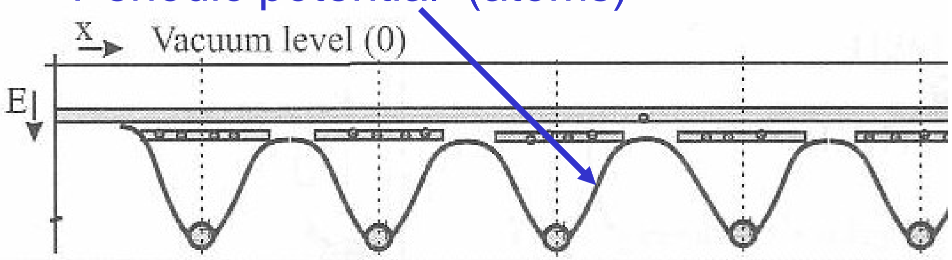
External field
$\mathcal{E} = -dV/dx$



Interpretation of levels: *total* energy

$$E = E_{potential} + E_{kinetic}$$

Periodic potential (atoms)



Electric field (potential): additive!
Effect of an external potential V(x):

$$E \rightarrow E + qV(x)$$

Pay attention! This notation may be misleading: "bent" levels do NOT represent the real "total" energy: just a way to represent in one picture also the SLOWLY varying external field

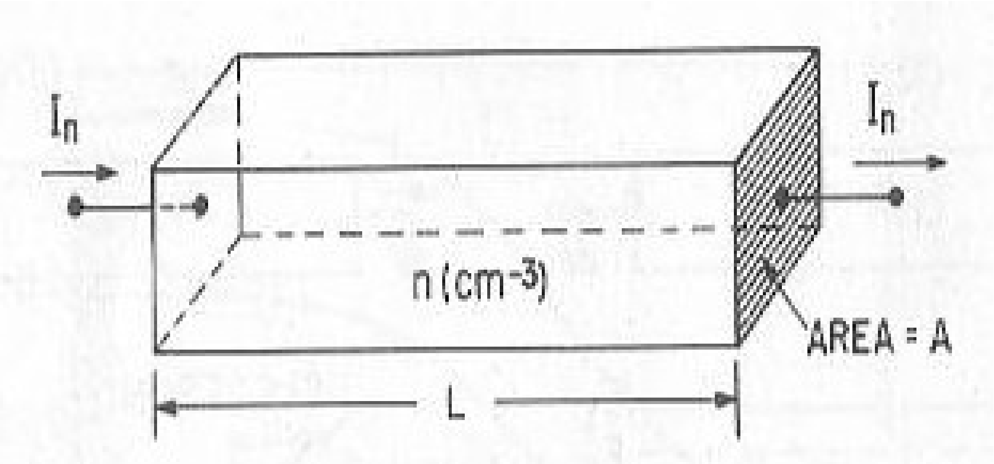
Band bending: comments

- In this representation the “external” electric field is treated separately from the inter-atomic force fields, and by definition:
  $\mathcal{E} = - dV/dx = - (1/q)dE_i/dx = \ldots$ (derivative of any “bent” level)

- Think of potential and kinetic energies in a macroscopic classical analog:
  - horizontal plane with bumps or holes, and rolling balls (some constrained by holes or springs, others free to move);
  - Tilt the plane slightly (small inclination) so that the overall change in “external” potential is of the same order of the “internal” potential difference in holes or bumps, but over a much larger distance (several orders of magnitude $\Rightarrow$ very small change on the scale of distances of holes/bumps)
  - find the analogies in representing energies…

Current density and conductivity

- Up to here, "average" behaviour of *individual* carriers in an external electrical field: *drift velocity, mobility*
- Now, *collective* behaviour: *current density*
- See blackboard for detailed calculations… result:



$$J = J_n + J_p = \left(|q| n \mu_n + |q| p \mu_p \right) \varepsilon = \sigma \varepsilon$$

Electric field

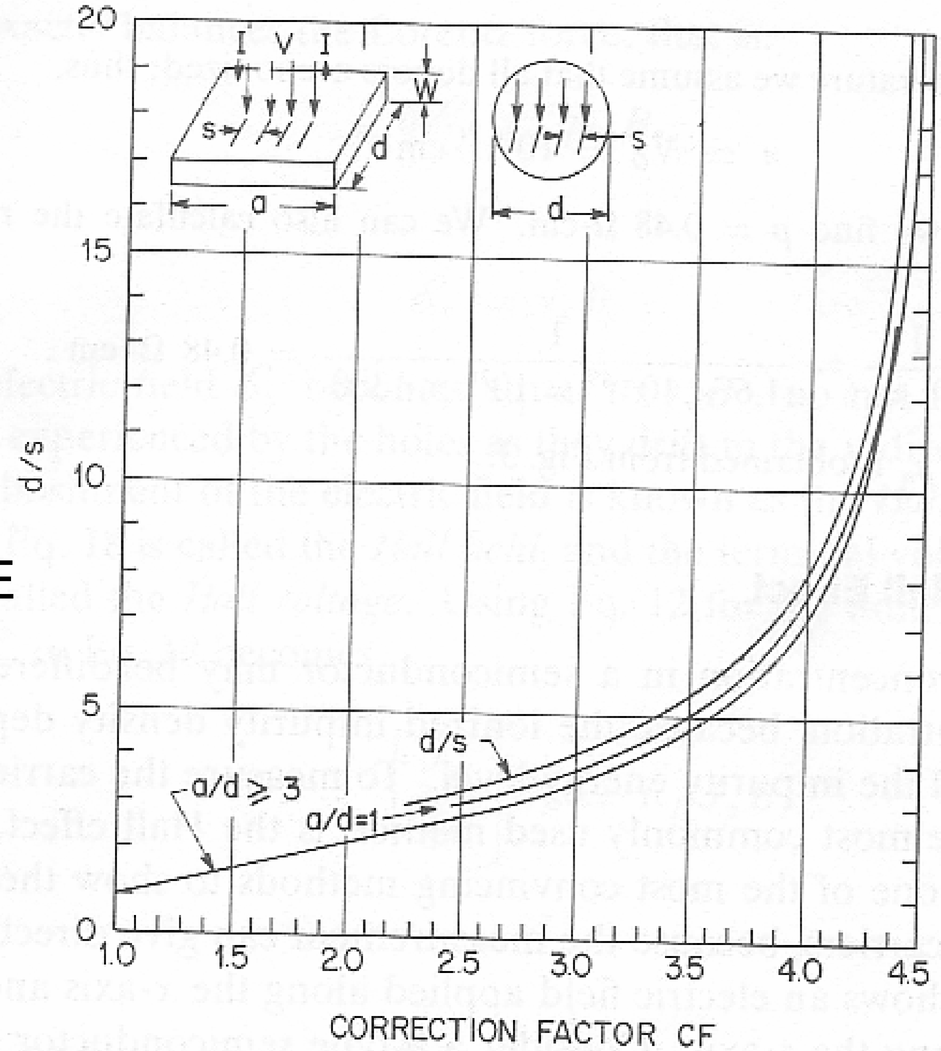
Resistivity

$$\rho \equiv \frac{1}{\sigma} = \frac{1}{qn\mu_n + qp\mu_p} = \frac{1}{q\left(n\mu_n + p\mu_p\right)}$$

Measurements: resistivity

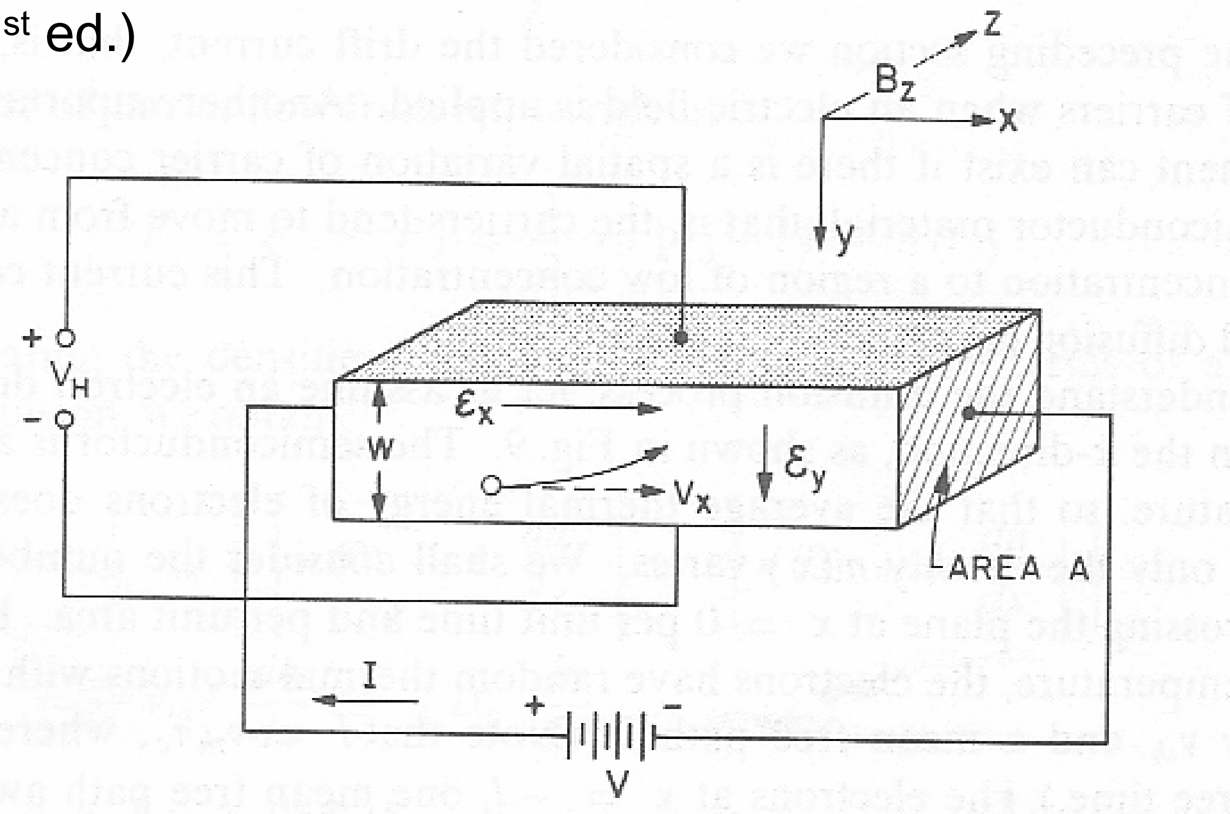
$$\rho = \frac{V}{I} \times W \times CF \quad [\Omega\,\mathrm{cm}]$$

More details of measurement methods: see for instance SZE 2.1.2 (1st ed.)



Carrier type and concentration: Hall effect

Hall effect: see for instance SZE 2.1.3 (1st ed.)



Lecture 4 - summary

- We discussed several aspects of carrier drift in a semiconductor when an external electric field is applied:
  - Qualitative microscopic mechanism, proportionality of drift velocity to the electric field, mobility coefficients
  - "band bending": representation of the external field as a potential energy, depending on position, added to the energy levels appearing in band diagrams
    - Variation is small on the scale of atomic distances; energy levels retain their meaning on that scale (~constant total energy);
    - On a larger scale, the band edges, donor and acceptor levels etc are no longer "constant total energy levels" in this representation!
  - Current densities, resistivity etc. resulting from drift motion of carriers

Lecture 4 – Items to be understood…

- Some items that require more thought:
  - Orders of magnitude for mobility, dependence on concentration, temperature, …
  - Measurement of mobility,conductivity, resistivity?
  - Theoretical predictions ? Underlying scattering processes?
  - Hall effect

Lecture 4 - Glossary

drift		
mobility		
scattering		
band bending		
conductivity		
resistivity		
Hall effect		

Lecture 4 - exercises

- **Exercise 4.1:** Find the electron and hole concentrations, mobilities and resistivities of silicon samples at 300K, for each of the following impurity concentrations: (a) $5x10^{15}$ boron atoms/cm^3; (b) $2x10^{16}$ boron atoms/cm^3 together with $1.5x10^{16}$ arsenic atoms/cm^3; and (c) $5x10^{15}$ boron atoms/cm^3, together with 10^{17} arsenic atoms/cm^3, and 10^{17} gallium atoms/cm^3.
- **Exercise 4.2:** For a semiconductor with a constant mobility ratio $b \equiv \mu_n \mu_p > 1$ independent of impurity concentration, find the maximum resistivity ρ_m in terms of the intrinsic resistivity ρ_i and of the mobility ratio.
- **Exercise 4.3:** A semiconductor is doped with N_D (N_D>>n_i) and has a resistance R_1. The same semiconductor is then doped with an unknown amount of acceptors $_{NA}$ (N_A>>N_D), yielding a resistance of $0.5R_1$. Find N_A in terms of N_D if the ratio of diffusivities for electrons and holes is D_n/D_p=50.

Backup slides