“Complementi di Fisica”
Lecture 5

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Course Outline - Reminder

- The physics of semiconductor devices: an introduction
  - Basic properties; energy bands, density of states
  - Equilibrium carrier concentration (“intrinsic”, “extrinsic”)
  - Carrier transport phenomena
    - Drift and Diffusion
    - Generation and Recombination
    - Continuity equations
- Quantum Mechanics: an introduction
- Advanced semiconductor fundamentals

Lecture 5 - outline

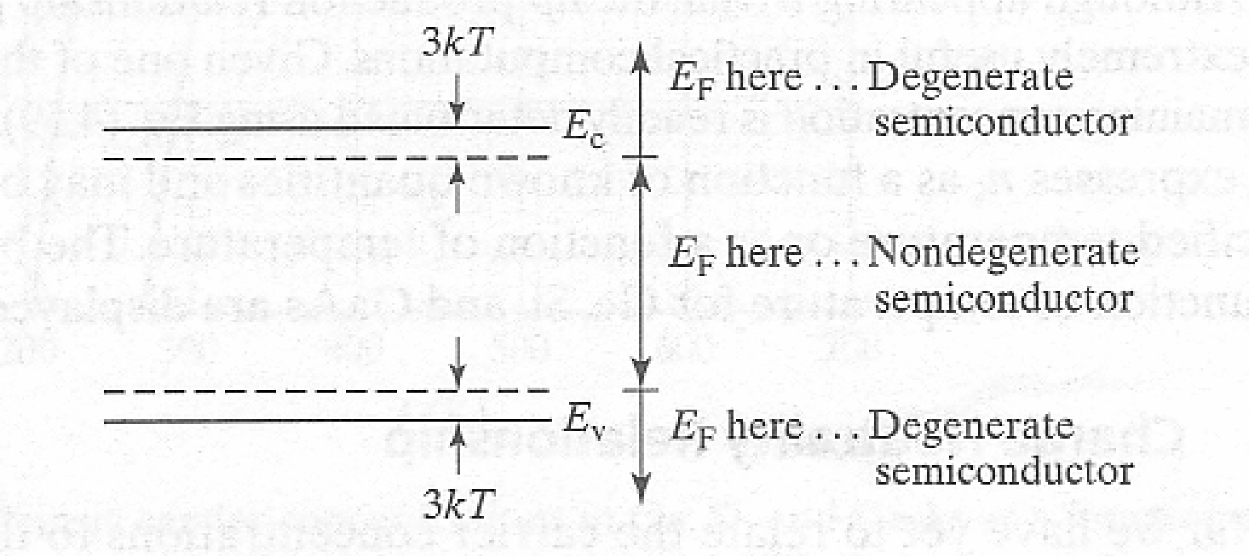
- Carrier diffusion
  - Diffusion process: current density and diffusivity
  - Einstein relation between diffusivity and mobility
  - Current density equations
- Carrier injection
  - Majority, minority and excess carriers
- Generation and recombination: individual processes
  - Direct recombination
  - Indirect recombination
  - Surface recombination
- Next lecture:
  - continuity equations: all effects together (drift, diffusion, generation, recombination); three important special cases
  - high field effects

Lecture 5 – warning / 1

- All equations will be derived and written in a simplified 1-dimensional case
  - Carrier concentrations varying only along x coordinate:
    - n(x), p(x)
  - Current density only in x direction
    - J_x
  - Derivatives with respect to x
    - d/dx
- They can be generalized to the general 3-dimensional case (see later)
  - n(x,y,z), p(x,y,z)
  - J_x , J_y , J_z
  - Partial derivatives, differential operators (div, grad)

Lecture 5 – warning / 2

- “Non-degenerate” and “degenerate” semiconductors
  - Definition:



  - Consequences:
    - For “non-degenerate semiconductors” the approximate form (Boltzmann) of the Fermi function can be used
    - Equations are simpler in this case! we will continue our discussion in this limit, to avoid obscuring the physics

Diffusion

Current density and diffusivity

Drift and diffusion together

Einstein relation

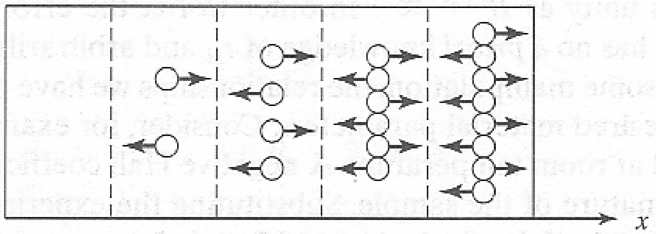
Current density equations

Diffusion process

- Qualitatively:
  - *If* there is a space variation of carrier concentration in the semiconductor material: *then* carriers tend to move predominantly from a region of high concentration to a region of low concentration

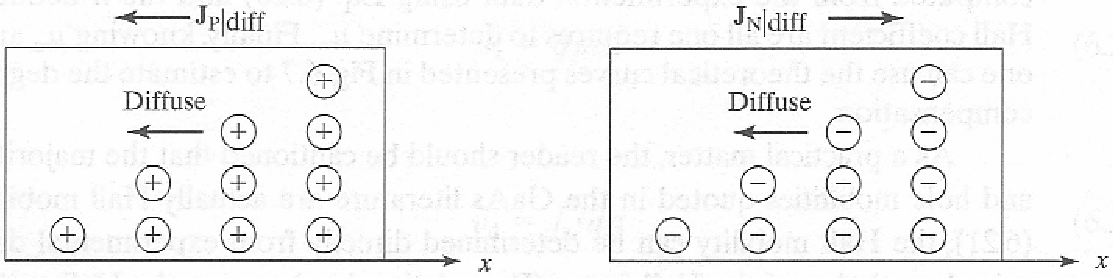
Microscopic scale: in each section,
- equal out-flow to +x and -x
- different in-flow from right and left

Net effect:
carrier concentrations tend to level out



(a)

Macroscopic scale:
current densities
$J_{n,diff}$
$J_{p,diff}$



(b)

Diffusion process

- Quantitatively, for electrons:

Left to right through plane at x = 0

$$F_1 = \frac{1}{2}\frac{n(-l)l}{\tau_c} = \frac{1}{2}n(-l)v_{th}$$

Right to left through the same plane

$$F_2 = \frac{1}{2}n(l)v_{th}$$

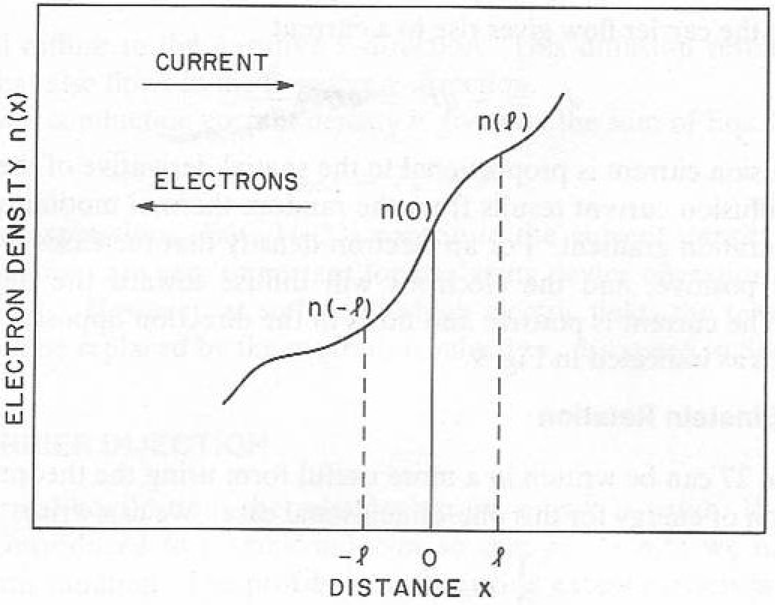
Net rate of carrier flow at x = 0

$$F = F_1 - F_2 = \frac{1}{2}v_{th}\left[n(-l) - n(l)\right] =$$

$$= -v_{th}l\frac{dn}{dx} \equiv -D_n\frac{dn}{dx}$$

$$l = v_{th}\tau_c$$

Thermal motion:
mean free path = thermal velocity × × mean free time



Taylor series expansion:

$$n(-l) = n(0) - l\left(\frac{dn}{dx}\right)_{x=0}$$

$$n(l) = n(0) + l\left(\frac{dn}{dx}\right)_{x=0}$$

Diffusion equations

- Diffusion current, electrons (1-dimensional case):

$$J_{n,x} = -|q|F = |q|D_n \frac{dn}{dx}$$

if positive gradient
dn/dx > 0
then: positive current
$J_{n,x} > 0$

- Similarly, holes:

$$J_{n,x} = |q|F = -|q|D_p \frac{dp}{dx}$$

if positive gradient
dp/dx > 0
then: negative current
$J_{p,x} < 0$

- Diffusivity:

$$D_n \equiv v_{th,n} l_n \qquad D_p \equiv v_{th,p} l_p$$

Dimensionally OK;
a more complete 3-d
analysis gives a
numerical coefficient
(1/3)

Drift and diffusion together

Drift and diffusion: Einstein relation

- Drift (mobility) and diffusion (diffusivity) coefficients are correlated!

thermal velocity

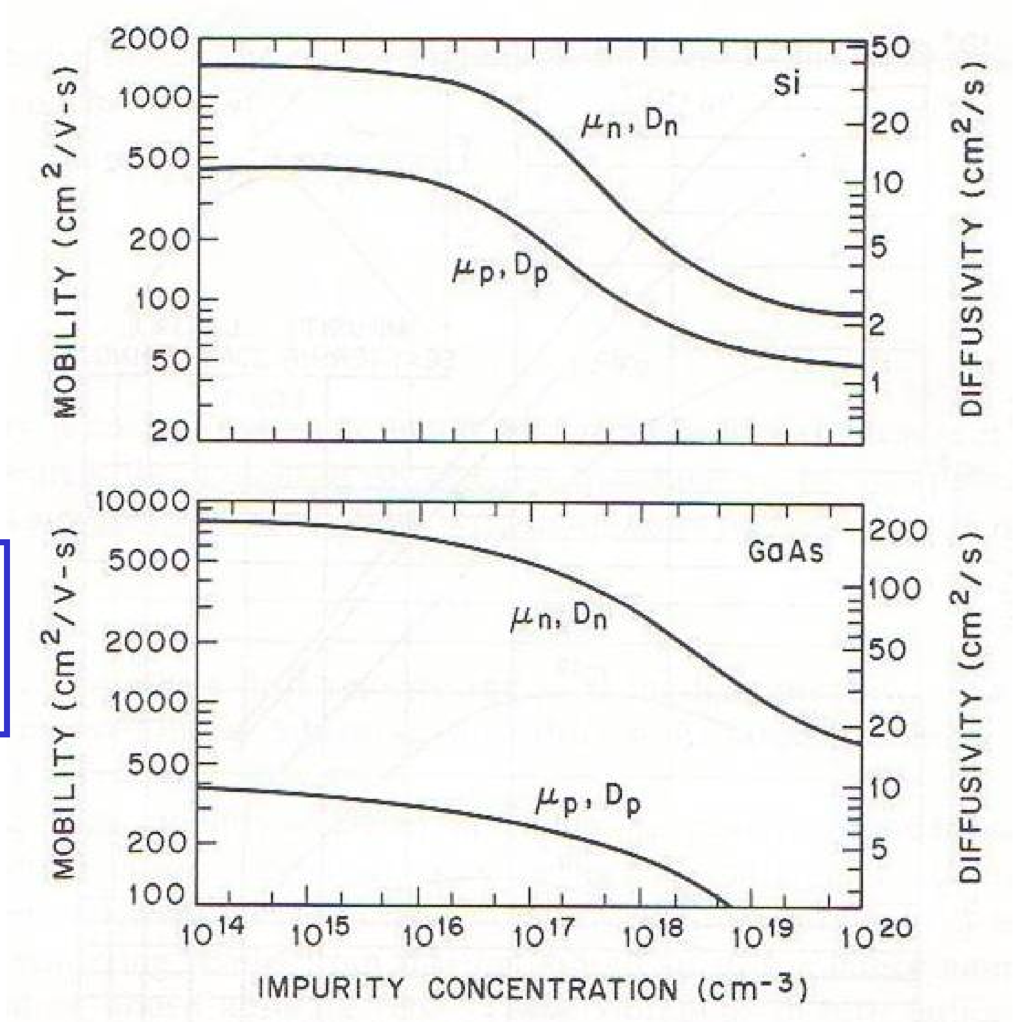
drift velocity

external field

$$D \equiv v_{th} l \qquad v_n = -\mu_n E \qquad v_p = \mu_p E$$

$$\boxed{D_n = \left(\frac{kT}{q}\right)\mu_n \qquad D_p = \left(\frac{kT}{q}\right)\mu_p}$$

- Why ?
  - Look at equilibrium conditions, no external field: a “built-in” electric field appears!



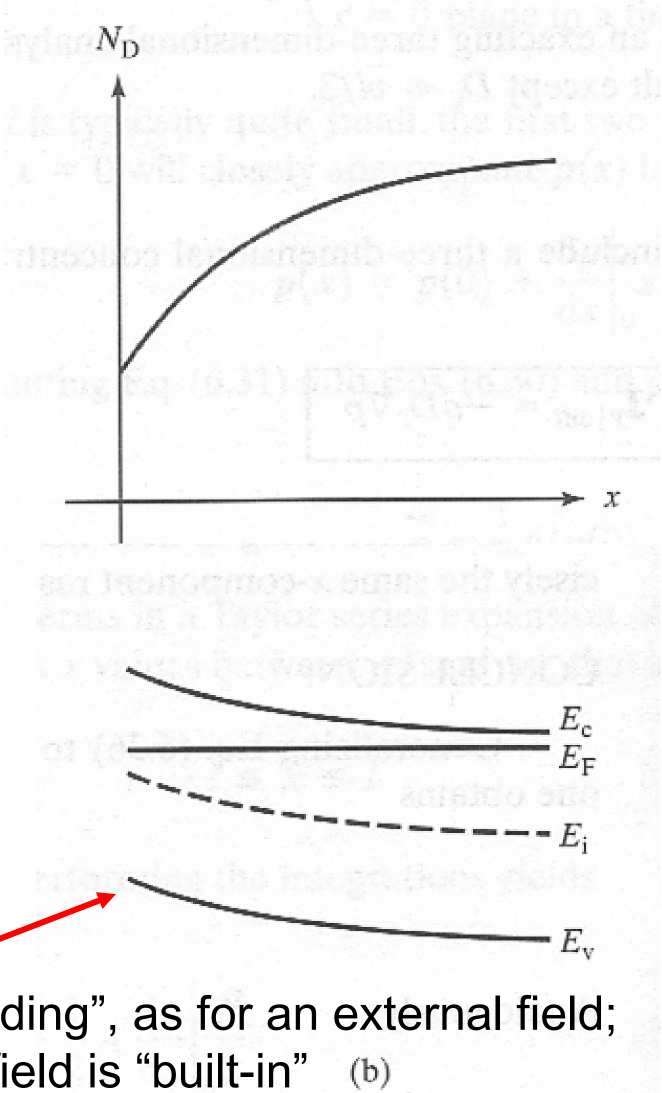
Non-uniform doping in equilibrium

- Under equilibrium conditions the Fermi level inside a material (or group of materials in intimate contact) is invariant as a function of position

$$\frac{dE_F}{dx} = \frac{dE_F}{dy} = \frac{dE_F}{dz} = 0$$

- A non-zero (“built-in”) electric field is established in nonuniformly doped semiconductors under equilibrium conditions

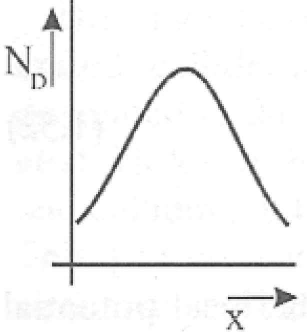
$$\mathcal{E} = (1/q)(dE_C/dx) =$$
$$= (1/q)(dE_i/dx) =$$
$$= (1/q)(dE_V/dx)$$



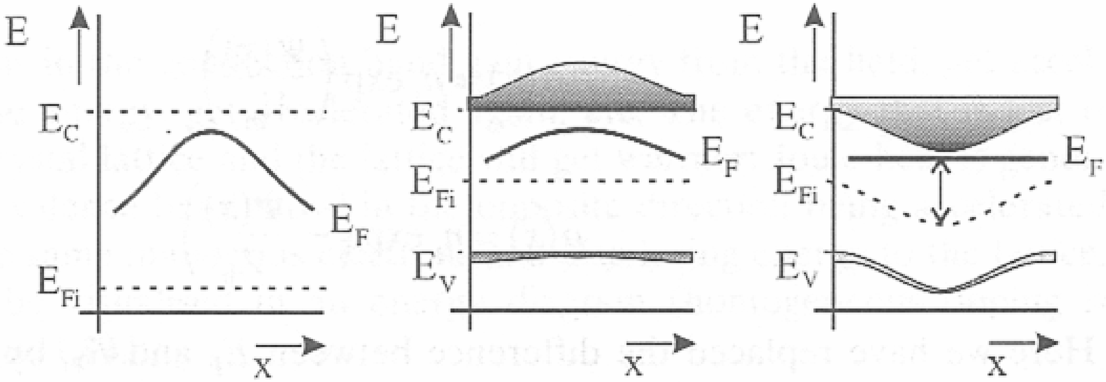
“band bending”, as for an external field; here the field is “built-in”

Non-uniform doping: an example

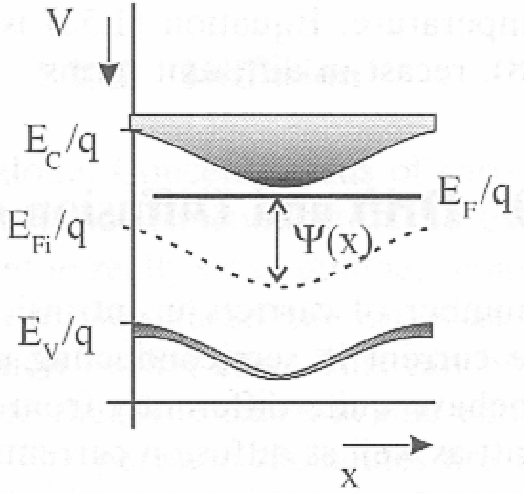
Donor concentration:



“band bending”:



⇒ Built-in electric field:



Equilibrium:

$E_F = \text{constant}$

$$\mathcal{E} = (1/q)(dE_i/dx) =$$
$$= (1/q)(d(E_F - E_i)/dx) =$$
$$= -d\Psi/dx$$

Electric potential

$$V(x) = \Psi(x) = (1/q)(E_F - E_i(x))$$

Einstein relation - justification

- Non-uniform doping at equilibrium:
  - Built-in field $\mathcal{E} \Rightarrow$ drift; the drift current compensates diffusion (*net current density is zero at equilibrium*)! For electrons:

$$\boxed{J_{n,drift}+J_{n,diff}=|q|\mu_n n\,\mathcal{E}+|q|D_n\frac{dn}{dx}=0}$$

$$\boxed{n=N_C e^{(E_F-E_C)/kT}}$$

Connection between n(x) and built-in field →

$$\frac{dn}{dx}=\frac{dn}{dE_C}\frac{dE_C}{dx}=-\frac{1}{kT}n|q|\mathcal{E}$$

$$\mathcal{E}=\frac{1}{|q|}\frac{dE_C}{dx}\Rightarrow\boxed{\frac{dE_C}{dx}=|q|\mathcal{E}}$$

$$|q|\mu_n n\,\mathcal{E}+|q|D_n\frac{dn}{dx}=|q|\,\mathcal{E}\left(\mu_n n-\frac{|q|}{kT}nD_n\right)=0$$

$$\Rightarrow\boxed{D_n=\left(\frac{kT}{|q|}\right)\mu_n}$$

Valid also for non-equilibrium conditions !

Einstein relation – numerical examples

- In the "non-degenerate" limit:

$$\frac{D_n}{\mu_n} = \frac{kT}{q} \qquad \frac{D_p}{\mu_p} = \frac{kT}{q}$$

- Typical sizes of mobility and diffusivity:

$$T = 300K \quad \Rightarrow \quad \frac{kT}{q} \approx 0.026\ \mathrm{V}$$

$$\mu_\mathrm{n} = 1000\ \mathrm{cm}^2/\mathrm{Vs} \quad \Rightarrow \quad D_n \approx 26\ \mathrm{cm}^2/\mathrm{s}$$

Current density equations

- When an *external* electric field Ɛ is present *in addition* to the concentration gradient: *no equilibrium!*
  - *Both* drift and diffusion currents will flow
  - The total current density is *different from zero* in this case!

- For electrons and holes: $J = J_n + J_p$

$$J_n = J_{n,drift} + J_{n,diff} = |q|\mu_n n \varepsilon + |q| D_n \frac{dn}{dx}$$

drift diffusion

$$J_p = J_{p,drift} + J_{p,diff} = |q|\mu_p p \varepsilon - |q| D_p \frac{dp}{dx}$$

Carrier injection

Carrier injection - introduction

- “carrier injection” = process of introducing “excess” carriers in a semiconductor, so that: $\boxed{np > n_i^2}$
  - Optical excitation:
    - shine a light on a semiconductor crystal;
    - if the energy of the photons is $h\nu > E_g$, then
    - Photons absorbed
    - “excess” electron-hole *pairs* are created: $\boxed{\Delta n = \Delta p}$
  - Other methods:
    - Forward-bias a pn junction
    - …
  - In an extrinsic semiconductor, the *relative* effect of $\Delta n = \Delta p$ is *very different for “majority” and “minority” carriers*, since $n \neq p$
  - Let us work out an example (n-type Si, n > p at equilibrium)

Carrier injection

Example: n-type Si at 300K

thermal equilibrium

$$n_0 p_0 = n_i^2$$

$$n_i = 1.45 \times 10^{10}\ \text{cm}^{-3}$$

$$n_0 \approx N_D = 10^{15}\ \text{cm}^{-3}$$

$$p_0 \approx n_i^2 / N_D = 2.1 \times 10^5\ \text{cm}^{-3}$$

thermal equilibrium

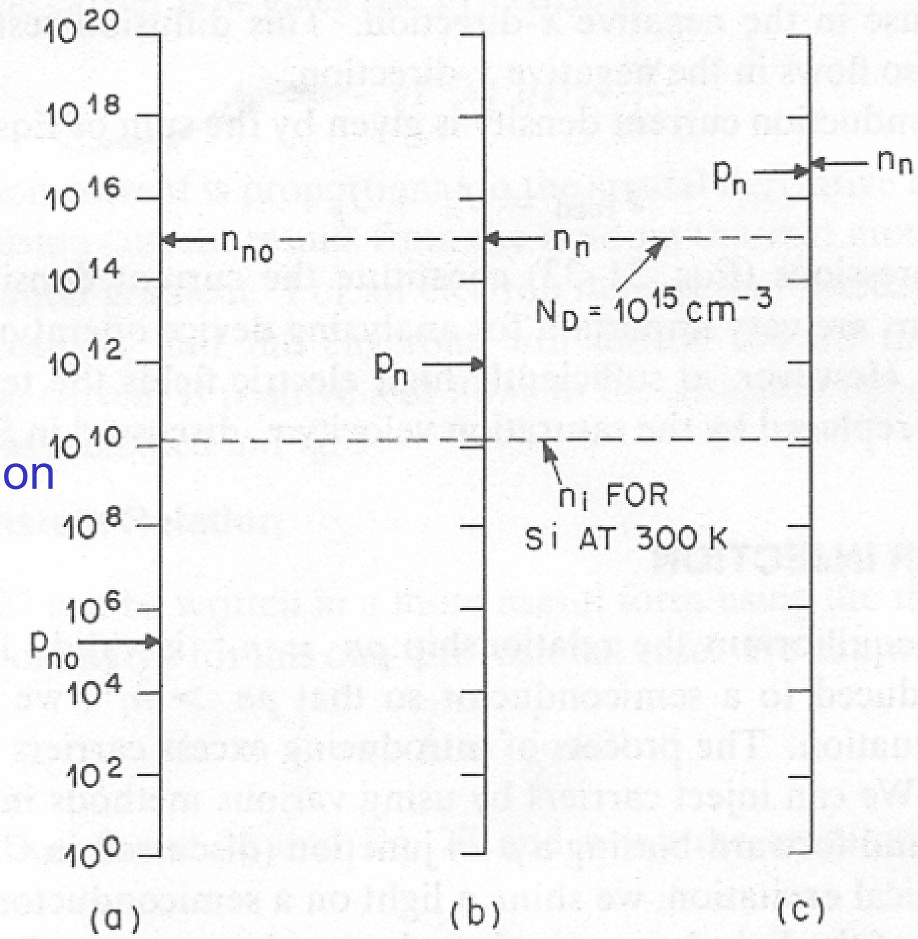
low injection

high injection

majority carriers

intrinsic concentration

minority carriers



Carrier injection

Example: n-type Si at 300K

"excess" carriers
low-injection

$$\Delta n = \Delta p = 10^{12}\ \text{cm}^{-3} << N_D$$

$$\Rightarrow np > n_i^2$$

$$n = n_0 + \Delta n \approx n_0$$

$$= 10^{15} + 10^{12} \approx 10^{15}\,\text{cm}^{-3}$$

$$p = p_0 + \Delta p \approx \Delta p$$

$$= 10^{5} + 10^{12} \approx 10^{12}\,\text{cm}^{-3}$$

thermal equilibrium

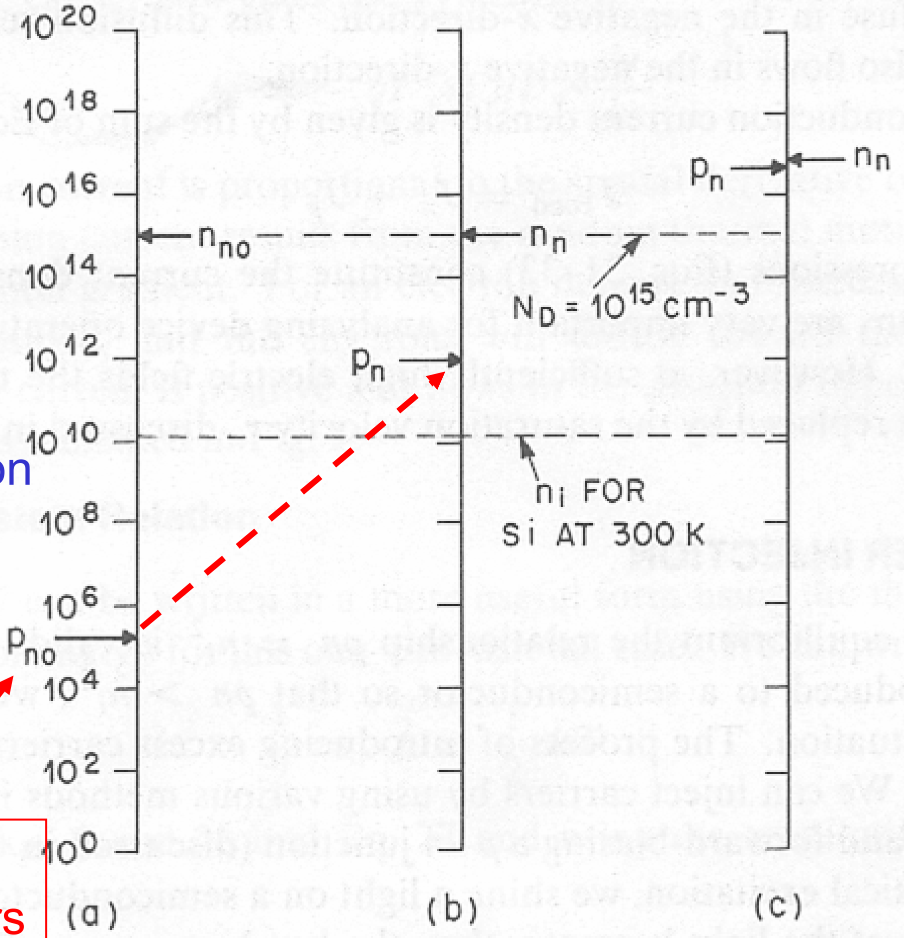
low injection

high injection

majority carriers

intrinsic concentration

minority carriers



Large increase in minority carriers

Carrier injection

Example: n-type Si at 300K

"excess" carriers
high-injection

$$\Delta n = \Delta p > N_D$$

$$\Rightarrow np > n_i^2$$

$$n = n_0 + \Delta n \approx \Delta n$$

$$p = p_0 + \Delta p \approx \Delta p$$

Large increase for both carriers
Similar concentrations

thermal equilibrium

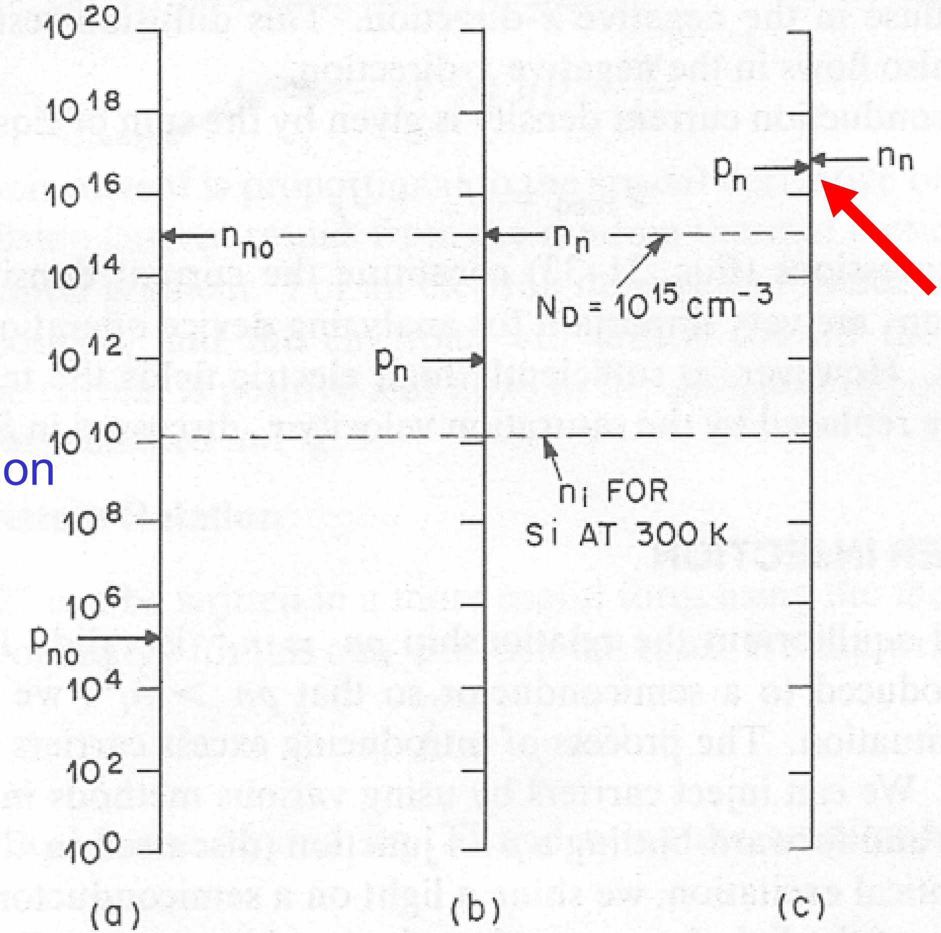
low injection

high injection

majority carriers

intrinsic concentration

minority carriers



Carrier injection - summary

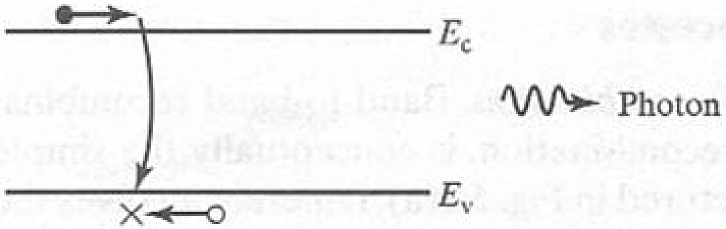
- “carrier injection” = process of introducing “excess” carriers in a semiconductor
- Several methods (optical, etc.)
- Low-level injection: relative effect on concentration
  - Negligible on majority carriers
  - Important for minority carriers (also called “minority carriers injection”)
- High-level injection
  - If very high, both concentrations become comparable
  - Sometimes encountered in device operation

Generation and recombination processes

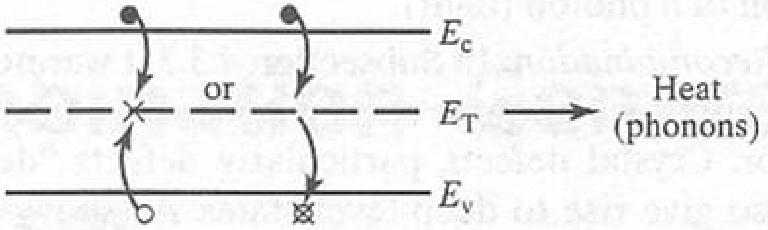
Generation and recombination: introduction

- Processes that tend to restore equilibrium ($np = n_i^2$) when it is disturbed ($np \neq n_i^2$); for instance:
  - After injection of excess carriers:
  - Recombination of injected minority carriers with majority carriers
- Recombination processes:
  - "Radiative" (with emission of a photon), "non-radiative"
  - "Direct" (dominant in direct-gap semiconductor), "indirect"
  - Surface effects, Auger recombination, … many processes
- For each, the corresponding "inverse" generation exists
  - Thermal equilibrium: "detailed balance" separately for each one
  - Off-equilibrium: overall compensation (continuity equations)
- Qualitative introduction now; quantitative next lecture

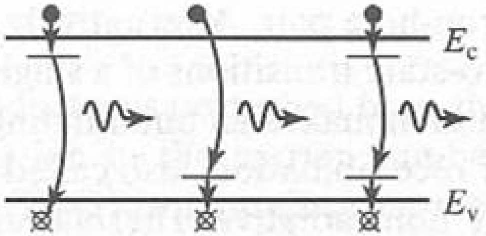
Recombination processes



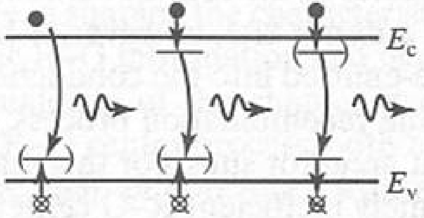
(a) Band-to-band recombination



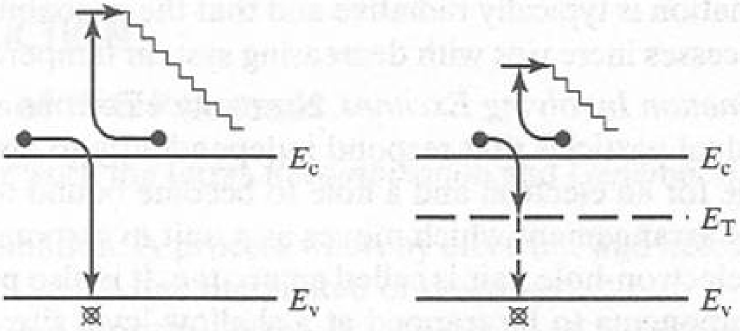
(b) R–G center recombination



(c) Recombination via shallow levels



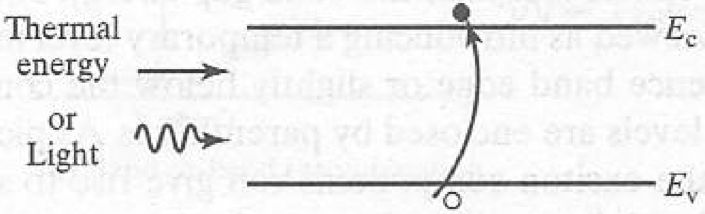
(d) Recombination involving excitons



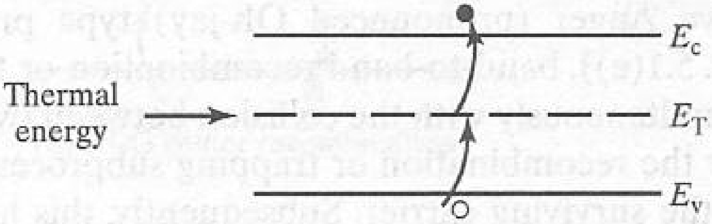
(Intrinsic) (Extrinsic)

(e) Auger recombination

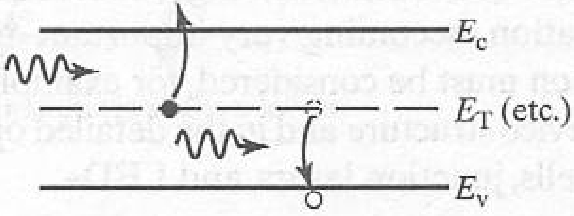
Generation processes



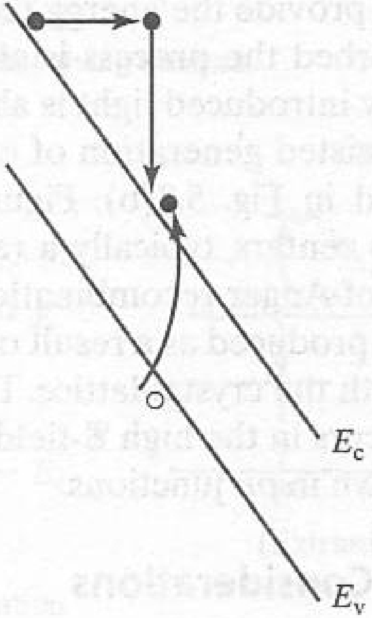
(a) Band-to-band generation



(b) R–G center generation



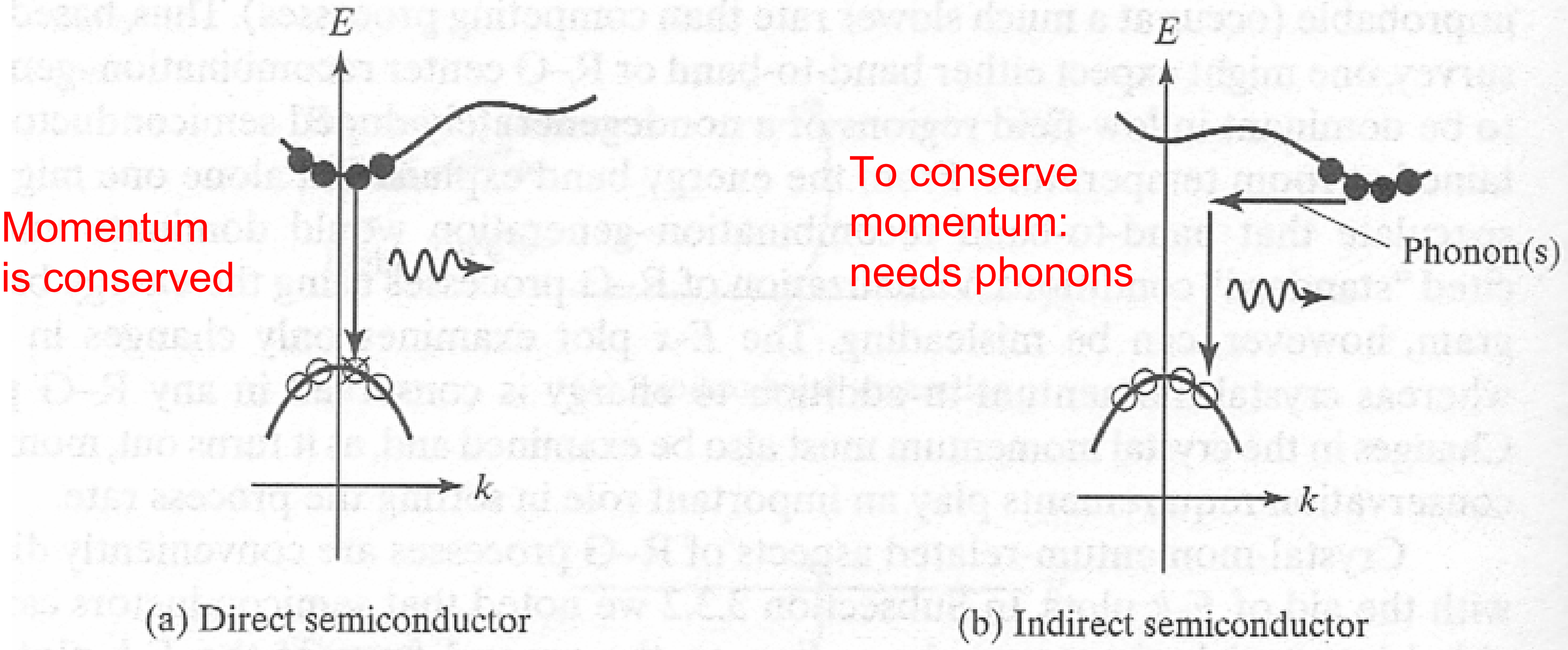
(c) Photoemission from band gap centers



(d) Carrier generation via impact ionization

Energy and momentum!

- Optical transitions in “direct” and “indirect” semiconductors



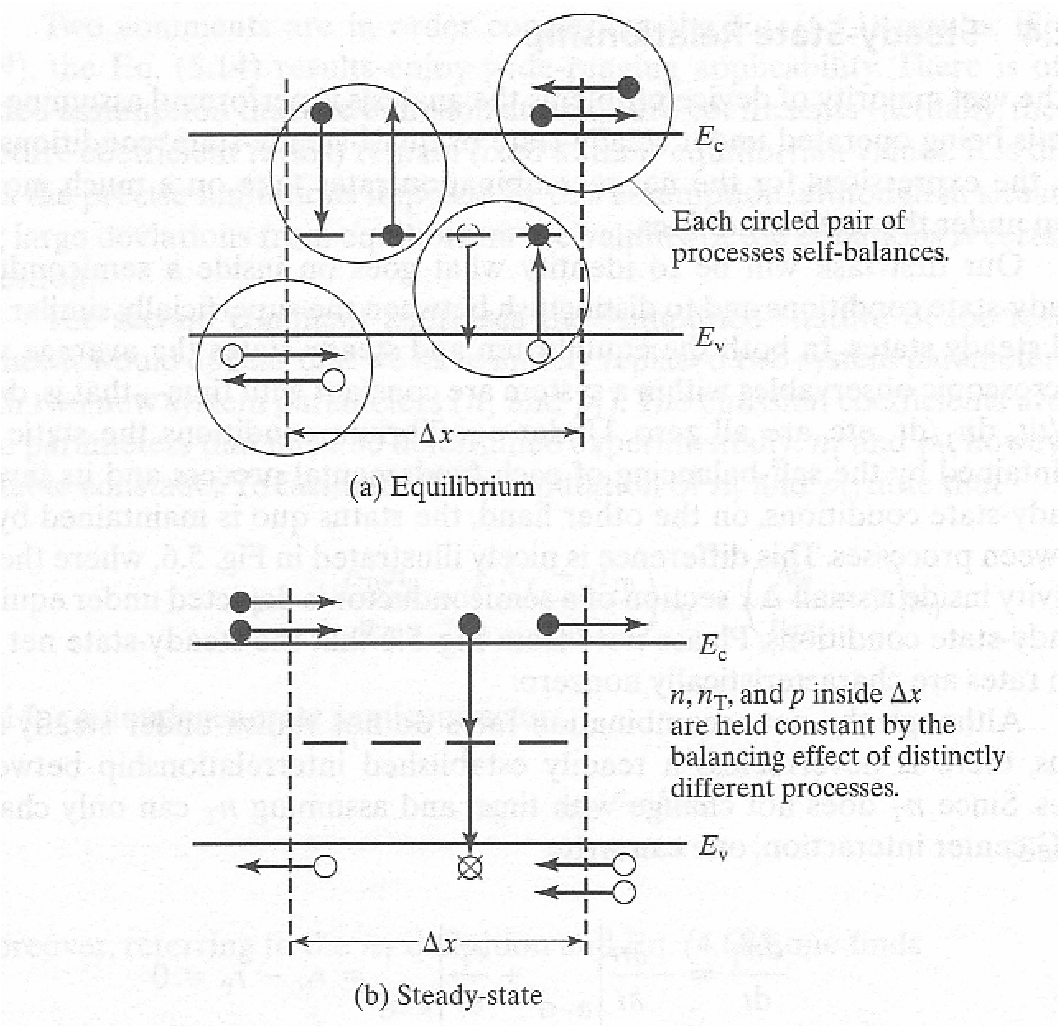
Momentum is conserved

To conserve momentum: needs phonons

(a) Direct semiconductor

(b) Indirect semiconductor

“Equilibrium” vs “steady state”



Each circled pair of processes self-balances.

(a) Equilibrium

n, n_T, and p inside Δx are held constant by the balancing effect of distinctly different processes.

(b) Steady-state

“Equilibrium”: detailed balance, for *each* process

“steady state”: overall balance

Recombination and generation: summary

- Quick and qualitative survey of generation and recombination processes…

- Quantitative description of their rates and equations combining them together with drift and diffusion?
  - Next lecture!

Lecture 5 - summary

Lecture 5 – Items to be understood…

- Some items that require a deeper explanation:
  - photon
  - photon energy
  - Phonons
  - Excitons ?
  - Quantitative description of generation and recombination processes (theoretical, experimental)
  - Diffusivity, mobility: how can they be measured? How do they vary with doping, temperature, etc.?
  - What about degenerate semiconductors? What equations need to be modified?

Lecture 5 - Glossary

degenerate semiconductor		
non-degenerate semiconductor		
diffusion		
current density		
diffusivity		
mobility		
thermal voltage		
band bending		
carrier injection		
thermal equilibrium		
steady state		
detailed balance		
low injection		
high injection		
generation		
recombination		
radiative recombination		
non-radiative recombination		
direct R-G		
indirect R-G		
R-G centers		
Auger recombination		
surface recombination		

Lecture 5 - exercises

- **Exercise 5.1:** An intrinsic Si sample is doped with donors from one side such that $N_D=N_0\exp(-ax)$. (a) Find an expression for the built-in electric field E(x) at equilibrium over the range for which $N_D \gg n_i$. (b) Evaluate E(x) when a = 1μm^{-1}.
- **Exercise 5.2:** An n-type Si slice of thickness L is inhomogeneusly doped with phosphorous donor whose concentration profile is given by $N_D(x) = N_0 + (N_L - N_0)(x/L)$. What is the formula for the electric potential difference between the front and the back surfaces when the sample is at thermal and electric equilibria regardless of how the mobility and diffusivity vary with position? What is the formula for the equilibrium electric field at a plane x from the front surface for a constant diffusivity and mobility?

Backup slides