"Complementi di Fisica"
Lecture 6

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Course Outline - Reminder

- The physics of semiconductor devices: an introduction
  - Basic properties; energy bands, density of states
  - Equilibrium carrier concentration (“intrinsic”, “extrinsic”)
  - Carrier transport phenomena
    - Drift and Diffusion
    - Generation and Recombination
    - Continuity equations
- Quantum Mechanics: an introduction
- Advanced semiconductor fundamentals

Lecture 5 - outline

- Drift and diffusion *at equilibrium* (built-in field):
  - an example
- Continuity equations and Generation-Recombination
  - Recall continuity equation: charge conservation (and Maxwell equations…)
  - Continuity equations, separately for electrons and holes, in a semiconductor (1-d)
  - Continuity equations for minority carriers in low-injection condition
- Minority carrier lifetime: overview
  - Direct, indirect and surface recombination conditions
- Homework:
  - Derivations of net recombination and lifetimes;
  - Three important special cases
  - high field effects

Drift and diffusion

An example

Non-uniform doping: built-in field

p-type doping, thermal equilibrium
(no "external" el.field applied!):

$$J_p = |q|\mu_n p\,\mathcal{E} - |q| D_n \frac{dp}{dx} = 0$$

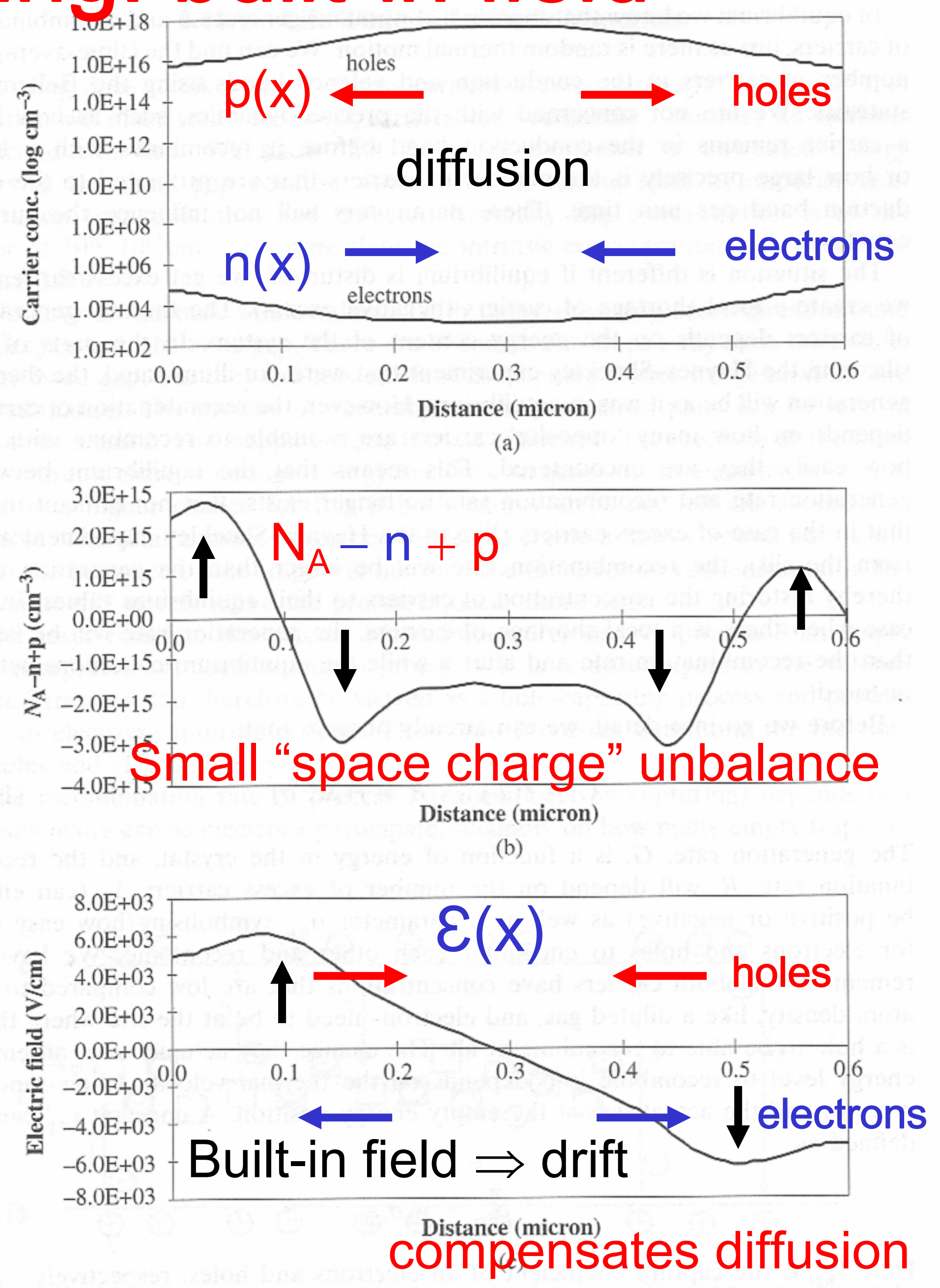
$$= |q|\mu_n p\,\mathcal{E} - |q| \frac{kT}{|q|} \mu_n \frac{dp}{dx} =$$

$$= |q|\mu_n p \left(\mathcal{E} - V_{th} \frac{1}{p} \frac{dp}{dx} \right) = 0$$

$\mathcal{E}$ is the "built-in" electric field:

$$\mathcal{E} = V_{th} \frac{1}{p} \frac{dp}{dx} \approx V_{th} \frac{1}{N_A} \frac{dN_A}{dx}$$

$$V_{th} \equiv \frac{kT}{q}$$ "thermal voltage equivalent"



Continuity equations

Summary of Classical Physics

Maxwell's equations

I. $\nabla \cdot \boldsymbol{E} = \dfrac{\rho}{\epsilon_0}$ (Flux of $\boldsymbol{E}$ through a closed surface) = (Charge inside)/ϵ_0

II. $\nabla \times \boldsymbol{E} = -\dfrac{\partial \boldsymbol{B}}{\partial t}$ (Line integral of $\boldsymbol{E}$ around a loop) = $-\dfrac{d}{dt}$ (Flux of $\boldsymbol{B}$ through the loop)

III. $\nabla \cdot \boldsymbol{B} = 0$ (Flux of $\boldsymbol{B}$ through a closed surface) = 0

IV. $c^2 \nabla \times \boldsymbol{B} = \dfrac{\boldsymbol{j}}{\epsilon_0} + \dfrac{\partial \boldsymbol{E}}{\partial t}$ c^2(Integral of $\boldsymbol{B}$ around a loop) = (Current through the loop)/ϵ_0 $+ \dfrac{\partial}{\partial t}$(Flux of $\boldsymbol{E}$ through the loop)

Conservation of charge

$\nabla \cdot \boldsymbol{j} = -\dfrac{\partial \rho}{\partial t}$ (Flux of current through a closed surface) = $-\dfrac{\partial}{\partial t}$ (Charge inside)

Force law

$$\boldsymbol{F} = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$$

Law of motion

$\dfrac{d}{dt}(\boldsymbol{p}) = \boldsymbol{F}$, where $\boldsymbol{p} = \dfrac{m\boldsymbol{v}}{\sqrt{1 - v^2/c^2}}$ (Newton's law, with Einstein's modification)

Gravitation

$$\boldsymbol{F} = -G \frac{m_1 m_2}{r^2} \boldsymbol{e}_r$$

From: The Feynman Lectures on Physics, vol.II

Conservation of charge: continuity equations

- Any net flow of charge must come from some supply!

$$\oint_S \vec{J} \bullet \hat{n}\, dS = \oint_V \vec{\nabla} \bullet \vec{J} dV = -\frac{d}{dt} \oint_V \rho\, dV = -\frac{dQ}{dt}$$

$$\vec{\nabla} \bullet \vec{J} = \frac{\partial J_x}{\partial x} + \frac{\partial J_y}{\partial y} + \frac{\partial J_z}{\partial z} = -\frac{\partial \rho}{\partial t}$$

  - The flux of a current from a closed surface is equal to the decrease of the charge inside the surface
  - ρ is the net charge density (negative and positive, algebraic sum)

- Let us consider electrons and holes, separately, in a semiconductor, in a simple one-dimensional case

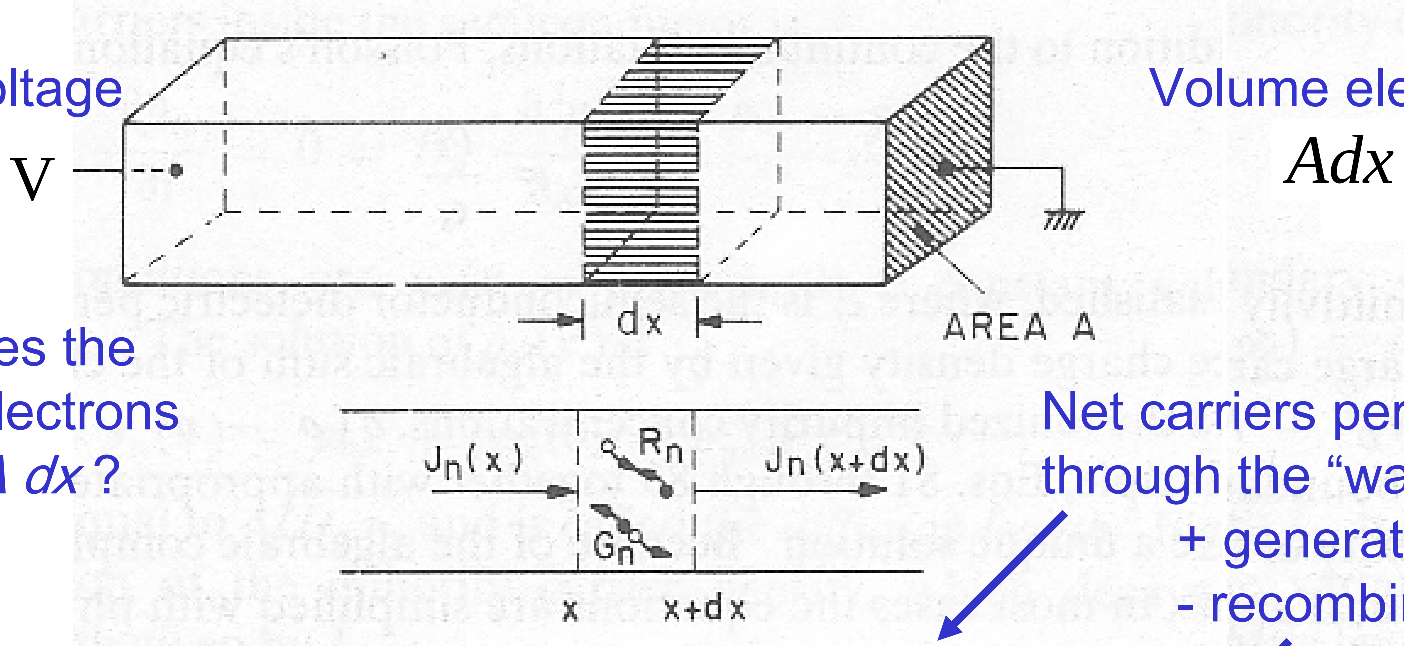
Electrons and holes

- Considering negative and positive charge densities separately, it is necessary to include also:
  - Generation rate G :
    - G = number of free carriers generated (separating electrons from holes) per second and per unit volume
    - G is usually a function of the available energy (temperature, etc.)
  - Recombination rate R:
    - R = number of free carriers “disappearing” due to recombination per second and per unit volume
    - R is usually proportional to the product of concentrations of “carriers” and “recombination centers” and to a “capture coefficient” defined as $c = v_{th}\sigma$, where v_{th} is the thermal velocity and σ is the recombination process “cross-section”
  - Net recombination effect: $U = R - G$

Continuity for electrons

External voltage

Volume element Adx



How fast does the number of electrons change in *A dx*?

$$\frac{1}{-|q|}\frac{\partial \rho}{\partial t}$$

Net carriers per second through the "walls"
+ generation
- recombination

$$\frac{\partial n}{\partial t} A dx = \left[\frac{J_n(x)A}{-|q|} - \frac{J_n(x+dx)A}{-|q|} \right] + \left(G_n - R_n\right) A dx$$

Substituting: $J_n(x+dx) = J_n(x) + \frac{\partial J_n}{\partial x} dx + \ldots$ and dividing by *A dx*

⇒ see next page…

Continuity for electrons and holes

One-dimensional

$$\frac{\partial n}{\partial t} = \frac{1}{|q|}\frac{\partial J_n}{\partial x} + (G_n - R_n)$$

$$\frac{\partial p}{\partial t} = -\frac{1}{|q|}\frac{\partial J_p}{\partial x} + (G_p - R_p)$$

Three-dimensional

$$\frac{\partial n}{\partial t} = \frac{1}{|q|}\vec{\nabla} \bullet \vec{J}_n + (G_n - R_n)$$

$$\frac{\partial p}{\partial t} = -\frac{1}{|q|}\vec{\nabla} \bullet \vec{J}_p + (G_p - R_p)$$

One-dimensional, under low-injection conditions, for minority carriers:

(electric field)

Electrons: n_p in p-type

$$\frac{\partial n_p}{\partial t} = n_p \mu_n \frac{\partial E_x}{\partial x} + \mu_n E_x \frac{\partial n_p}{\partial x} + D_n \frac{\partial^2 n_p}{\partial x^2} + G_n - \frac{n_p - n_{p0}}{\tau_n}$$

holes: p_n in n-type

$$\frac{\partial p_n}{\partial t} = p_n \mu_p \frac{\partial E_x}{\partial x} + \mu_p E_x \frac{\partial p_n}{\partial x} + D_p \frac{\partial^2 p_n}{\partial x^2} + G_p - \frac{p_n - p_{n0}}{\tau_p}$$

minority carrier excess

minority carrier lifetime

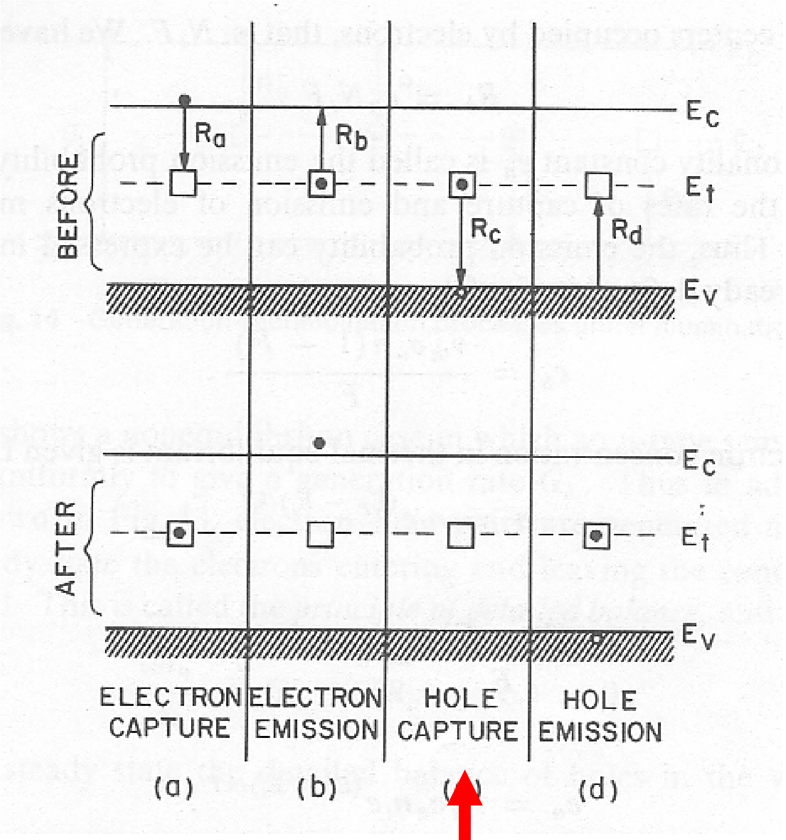
Simply substitute J=J(drift)+J(diffusion)…

Recombination: This is the tricky part!

Recombination: a hint…

- recombination is often dominated by indirect processes through "recombination centers" (direct recombination negligible for Si)

- For instance, in an n-type semiconductor, under low-injection conditions:
  - for the minority-carriers (holes !) excess-recombination, the bottleneck is "hole capture", that determines the hole "lifetime" τ_p
  - Once captured, the hole recombines quickly, since there are many electrons available



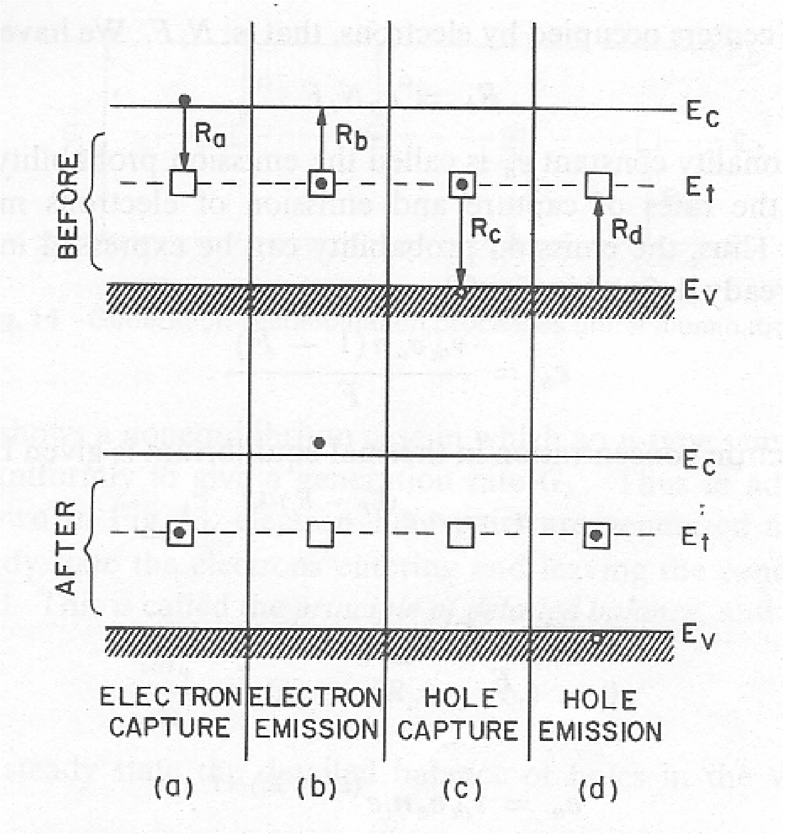
$$U \approx v_{th}\sigma_p N_t \left(p_n - p_{n0}\right)$$

$$\tau_p \equiv \frac{1}{v_{th}\sigma_p N_t}$$

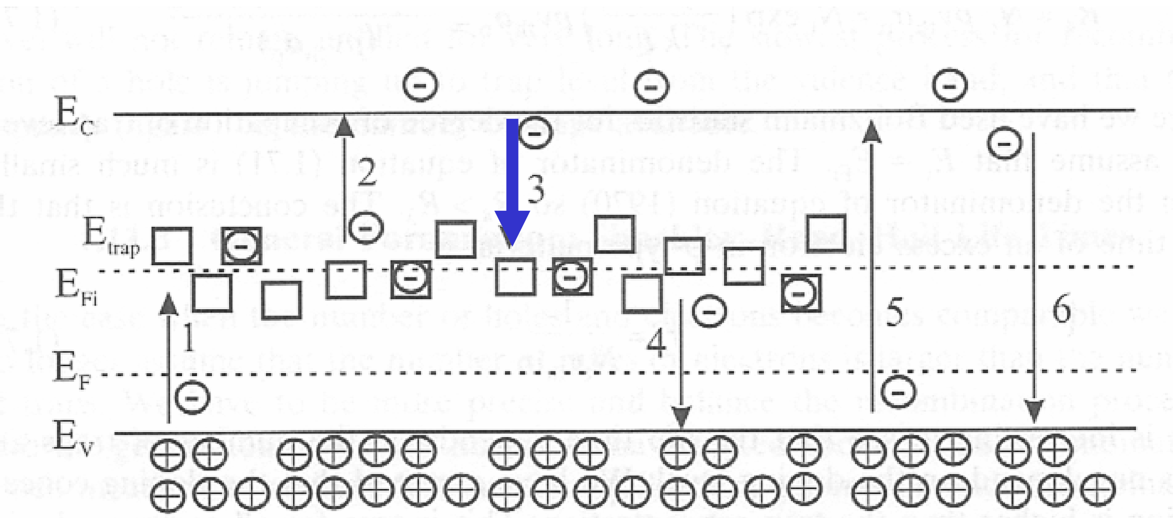
Pay attention: terminology…

- "Capture", "emission":
  - From the point of view of the trap!

- In particular (figure, next slide):
  - (a) electron capture = (3)
  - (b) electron emission = (2)
  - (c) hole capture = (4)
  - (d) hole emission = (1)



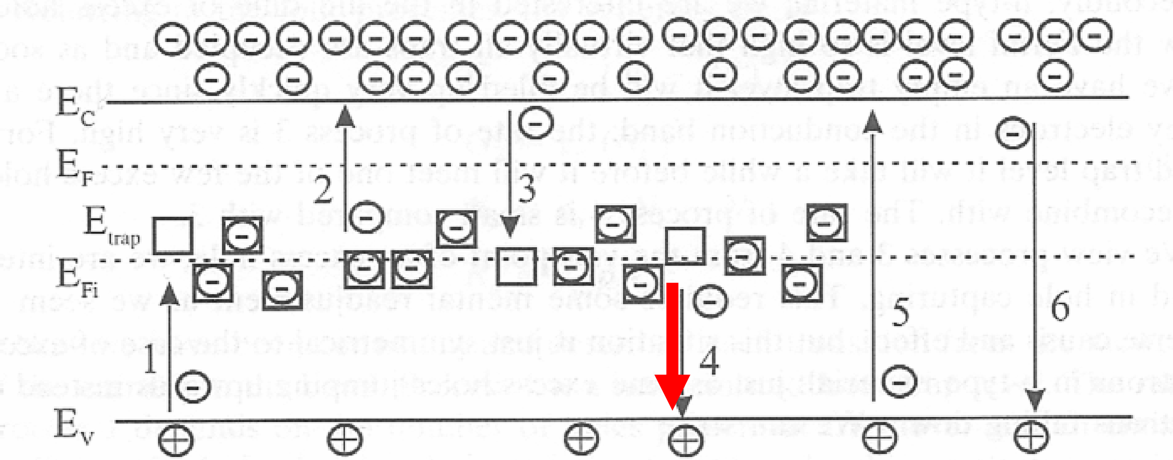
… more recombination for low-injection



p-type semiconductor: electron-lifetime dominated by “electron capture” (3) in “empty” RG centers

$$U \approx v_{th}\sigma_n N_t \left(n_p - n_{p0}\right)$$

$$\tau_n \equiv \frac{1}{v_{th}\sigma_n N_t}$$



n-type semiconductor: hole-lifetime dominated by “hole capture” (4) in “full” RG centers

$$U \approx v_{th}\sigma_p N_t \left(p_n - p_{n0}\right)$$

$$\tau_p \equiv \frac{1}{v_{th}\sigma_p N_t}$$

Lecture 6 - summary

- The continuity equations are the main tools for simulating semiconductor devices, complemented by Gauss' law $\vec{\nabla} \bullet \vec{E} = \rho / \varepsilon_0$ $\nabla^2 V = -\rho / \varepsilon_0$

- To understand the key recombination mechanisms and see some practical examples, homework for next time: reading of
  - Direct recombination (2.4.1), indirect recombination (2.4.2), surface recombination (2.4.3)
  - Applications: steady state injection from one side (2.5.1), minority carriers at the surface (2.5.2), Haynes-Shockley experiment (2.5.3)
  - High field effects (2.6), Hall effect (2.1.3)

Lecture 6 – Items to be understood…

- Some items that require more thought:
  - Continuity equations: examples, applications
  - Generation rate: orders of magnitude, different processes
  - Recombination rate: orders of magnitude, different processes
  - What happens if the low-injection approximations are not possible?
  - What about currents and continuity equation for the majority carriers? When are they relevant?

Lecture 6 - Glossary

built-in field		
continuity equations		
generation rate		
recombination rate		
low-injection		
minority carriers lifetime		

Lecture 6 - exercises

- **Exercise 6.1:** Calculate the electron and hole concentration under steady-state illumination in an n-type silicon with $G_L=10^{16}cm^{-3}s^{-1}$, $N_D=10^{15}cm^{-3}$, and $\tau_n=\tau_p=10\ \mu s$.
- **Exercise 6.2:** An n-type silicon sample has $2x10^{16}$ arsenic atoms/cm^3, $2x10^{15}$ bulk recombination centers/cm^3, and 10^{10} surface recombination centers/cm^2. (a) Find the bulk minority carrier lifetime, the diffusion length, and the surface recombination velocity under low-injection conditions. The values of σ_p and σ_s are $5x10^{-15}$ and $2x10^{-16}$ cm^2, respectively. (b) If the sample is illuminated with uniformly absorbed light that creates 10^{17} electron-hole pairs/(cm^{2}s), what is the hole concentration at the surface?
- **Exercise 6.3:** The total current in a semiconductor is constant and is composed of electron drift current and hole diffusion current. The electron concentration is constant and equal to 10^{16} cm^{-3}. The hole concentration is given by $p(x)=10^{15}\exp(-x/L)$ cm^{-3} ($x>0$), where L = 12μm. The hole diffusion coefficient is D_p=12cm2/s and the electron mobility is μ_n=1000cm^2/(Vs). The total current density is J = 4.8 A/cm^2. Calculate (a) the hole diffusion current density as a function of x, (b) the electron current density versus x, and (c) the electric field versus x.