# *“Complementi di Fisica”*
# *Lectures 7, 8*

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# Lectures 7, 8 - outline

- Continuity equations: three important special cases
  - Steady-state injection from one side
    - "diffusion length" $L_p$
  - Minority carriers recombination at the surface
    - diffusion length and "surface recombination velocity" $S_{lr}$
  - The Haynes-Shockley experiment
    - Evidence for simultaneous diffusion, drift and recombination

- Are we describing the behaviour of *minority* carriers alone? What about *majority* carriers?
  - Why are "minorities" important? Some examples…
  - Built-in electric field (Gauss!) and *"ambipolar" transport equations*

# System of differential equations

Continuity (transport) equations for minority carriers, 1-d case (Sze notations):

$$\frac{\partial n_p}{\partial t} = n_p \mu_n \frac{\partial E_x}{\partial x} + \mu_n E_x \frac{\partial n_p}{\partial x} + D_n \frac{\partial^2 n_p}{\partial x^2} + G_n - \frac{n_p - n_{p0}}{\tau_n}$$

$$\frac{\partial p_n}{\partial t} = p_n \mu_p \frac{\partial E_x}{\partial x} + \mu_p E_x \frac{\partial p_n}{\partial x} + D_p \frac{\partial^2 p_n}{\partial x^2} + G_p - \frac{p_n - p_{n0}}{\tau_p}$$

Gauss' law, relating the divergence of the electric field with the local charge density, 1-d case:

$$\frac{\partial E_x}{\partial x} = \frac{\rho}{\varepsilon}$$

Globally neutral, locally can be unbalanced!

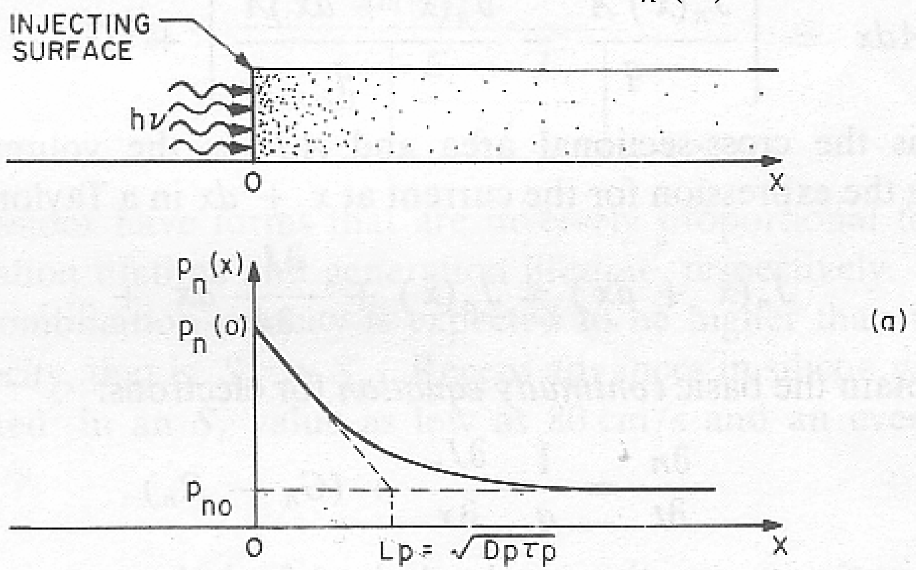
$$\rho = |q|\left(p - n + N_D^+ - N_A^-\right) \approx |q|\left(p - n + N_D - N_A\right)$$

$$N = N_D - N_A$$

To be solved with given boundary conditions!

# Steady-state injection from one side

n-type semiconductor
minority carriers: holes
concentration $p_n(x) = ?$




$$\frac{\partial p_n}{\partial t} = 0$$ steady state

$$E_x = 0$$ no applied field

$$G_p = 0$$ no generation in the bulk

$p_{n0}$ at thermal equilibrium

$p_n(0) - p_{n0}$ excess injected at x = 0 (boundary condition)

Continuity equation in this case:

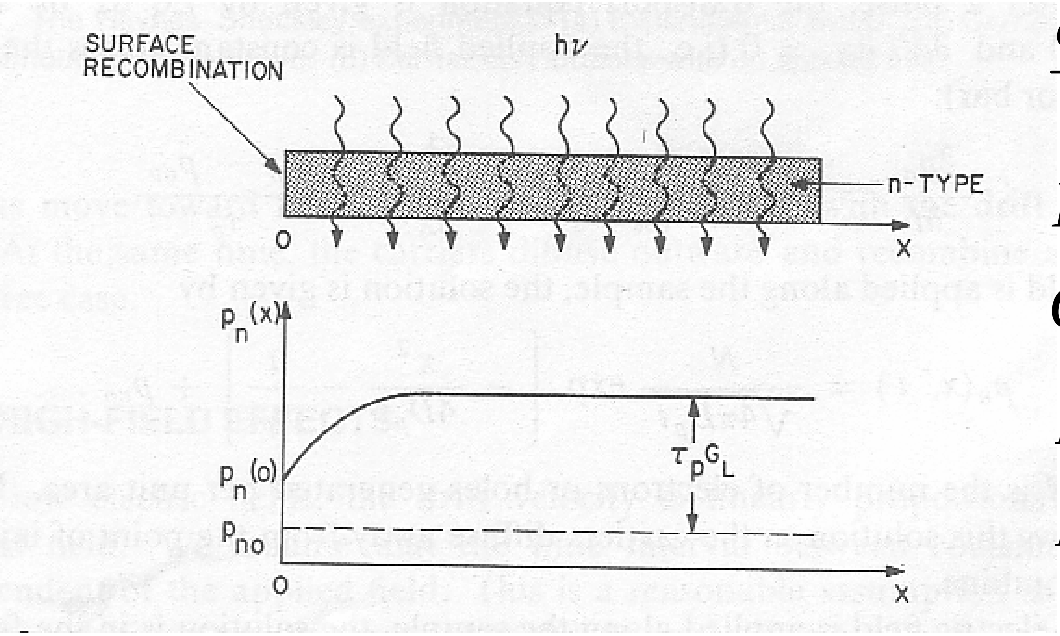
$$\frac{\partial^2 (p_n - p_{n0})}{\partial x^2} = \frac{1}{D_p \tau_p}(p_n - p_{n0})$$

"Diffusion length"

Solution: $$p_n(x) = p_{n0} + (p_n(0) - p_{n0}) e^{-x/L_p}$$

$$L_p = \sqrt{D_p \tau_p}$$

# Minority carriers at the surface




$$\frac{\partial p_n}{\partial t} = 0$$ steady state

$$E_x = 0$$ no applied field

$$G_L \neq 0$$ generation in the bulk !!!

$p_{n0}$ at thermal equilibrium

$p_n(0) - p_{n0}$ **boundary condition**

Here the boundary condition is fixed by the rate at which carriers disappear with "surface recombination velocity" depending on the "surface trap density" $N_{st}$

$$S_{lr} = v_{th} \sigma_p N_{st}$$

cm s$^{-1}$ cm s$^{-1}$ cm$^{2}$ cm$^{-2}$

Equation to be solved (x > 0, bulk):

$$\frac{\partial^2 \Delta p_n}{\partial x^2} - \frac{\Delta p_n}{D_p \tau_p} + \frac{G_L}{D_p} = 0$$

$$\Delta p_n = p_n(x) - p_{n0}$$

# Solution with boundary conditions

General solution: $\Delta p_n = A e^{x/L_p} + B e^{-x/L_p} + G_L \tau_p$

"complementary" (homegeneous) — $A e^{x/L_p} + B e^{-x/L_p}$

"particular" — $G_L \tau_p$

$$L_p = \sqrt{D_p \tau_p}$$

Boundary conditions:

$$\Delta p_n(x) \xrightarrow[x \to +\infty]{} G_L \tau_p \quad \Rightarrow \quad A = 0$$

$$\Delta p_n(x) \xrightarrow[x \to 0]{} \Delta p_n(0) \quad \Rightarrow \quad \Delta p_n(0) = B + G_L \tau_p$$

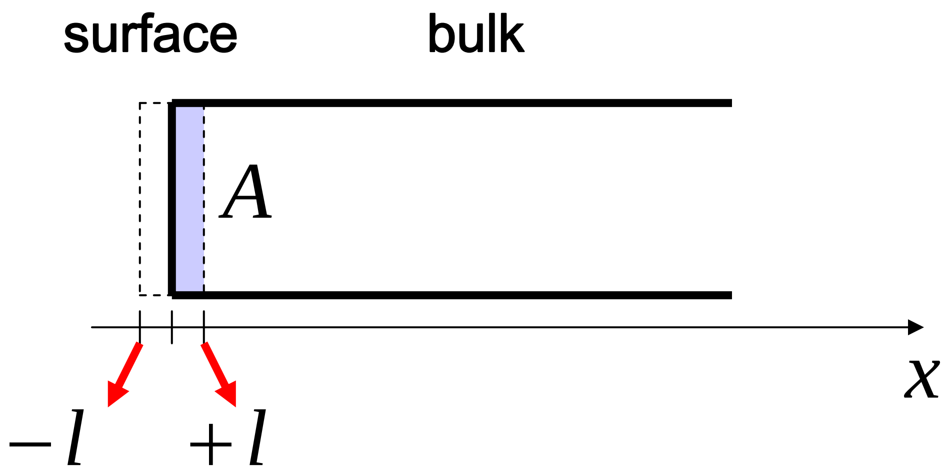
$$B = \Delta p_n(0) - G_L \tau_p$$

after some algebra, substituting A and B, our solution:

$$p_n(x) = p_{n0} + G_L \tau_p \left( 1 + \frac{\Delta p_n(0) - G_L \tau_p}{G_L \tau_p} e^{-x/L_p} \right)$$

$\Delta p_n(0) = ???$ → $S_{lr}$

# Surface boundary condition




Consider a thin volume $(A \times 2l)$ enclosing the surface:

$$-J_x(x=l)A = \left[G_L - (v_{th}\sigma_p N_t)\Delta p_n(0)\right]Al - (v_{th}\sigma_p N_{st})\Delta p_n(0)A$$

diffusion current

Gener. – recomb. (bulk)

$l \to 0$

Recombination (surface)

In the limit $l \to 0$: $\quad -J_x(0) = -(v_{th}\sigma_p N_{st})\Delta p_n(0)$

# Solution with surface recomb. velocity

$$-J_x(0) = -\left(v_{th}\sigma_p N_{st}\right)\Delta p_n(0) \quad \Rightarrow \quad D_p\left(\frac{d\Delta p_n}{dx}\right)_{x=0} = S_{lr}\ \Delta p_n(0)$$

cm$^2$ s$^{-1}$ cm$^{-4}$ cm s$^{-1}$ cm$^{-3}$

from the general solution and boundary conditions:

$$\left(\frac{d\Delta p_n}{dx}\right)_{x=0} = -\frac{B}{L_p} \quad \Rightarrow \quad D_p\left(-\frac{B}{L_p}\right) = S_{lr}\left(G_L\tau_p + B\right)$$

$$\Delta p_n(0) = G_L\tau_p + B \quad \Rightarrow \quad B = \frac{-S_{lr}G_L\tau_p}{D_p/L_p + S_{lr}}$$

Solution expressed in terms of the surface recombination velocity:

$$p_n(x) = p_{n0} + G_L\tau_p\left(1 - \frac{S_{lr}\tau_p}{L_p + S_{lr}\tau_p}e^{-x/L_p}\right)$$

# Limiting cases

Neglecting surface recombination:

$$S_{lr} \to 0 \quad \Rightarrow \quad p_n(x) = p_{n0} + G_L \tau_p$$

$$p_n(0) = p_{n0} + G_L \tau_p$$

as expected!

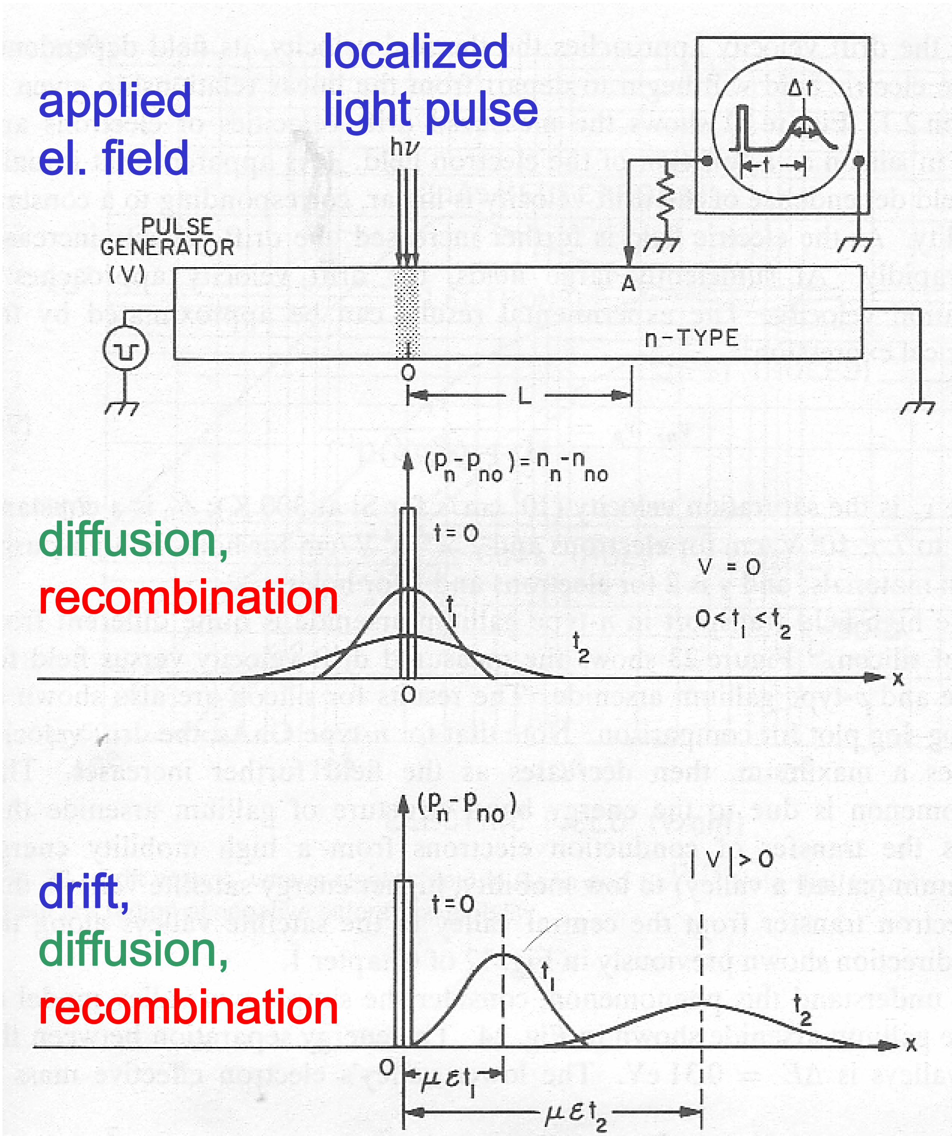
Large ("immediate") surface recombination:

$$S_{lr} \to \infty \quad \Rightarrow \quad p_n(x) = p_{n0} + G_L \tau_p \left(1 - e^{-x/L_p}\right)$$

$$p_n(0) = p_{n0}$$

as expected!

# The Haynes-Shockley experiment




Experimental set-up

excess carrier distributions at successive times $t_1$ and $t_2$, no applied field

excess carrier distributions at successive times $t_1$ and $t_2$, with a constant applied field

# The Haynes-Shockley experiment

After a light pulse: $G_L = 0$ no bulk generation

$\frac{\partial E_x}{\partial x} = 0$ constant applied field

Transport equation for excess minority carriers (n-type semiconductor):

$$\frac{\partial \Delta p_n}{\partial t} = \mu_p E_x \frac{\partial \Delta p_n}{\partial x} + D_p \frac{\partial^2 \Delta p_n}{\partial x^2} - \frac{\Delta p_n}{\tau_p} \qquad \Delta p_n = p_n - p_{n0}$$

Solution, no applied field:

$$\Delta p_n(x,t) = \frac{N}{\sqrt{4\pi D_p t}} \exp\left( -\frac{x^2}{4 D_p t} - \frac{t}{\tau_p} \right)$$

diffusion,
recombination

Solution, with applied field:

$$\Delta p_n(x,t) = \frac{N}{\sqrt{4\pi D_p t}} \exp\left( -\frac{\left(x - \mu_p E_x t\right)^2}{4 D_p t} - \frac{t}{\tau_p} \right)$$

drift,
diffusion,
recombination

# Minority and majority carriers
# “Ambipolar” transport

If the doping is not very heavy
(conditions approaching “intrinsic”),
also the excess of majority carriers plays a role…

$\Rightarrow$ Look more carefully into the local charge distribution:
neutrality or unbalance!!!

# The role of Gauss' law

Divergence of the electric field, 1-d case:

$$\frac{\partial E_x}{\partial x} = \frac{\rho}{\varepsilon}$$

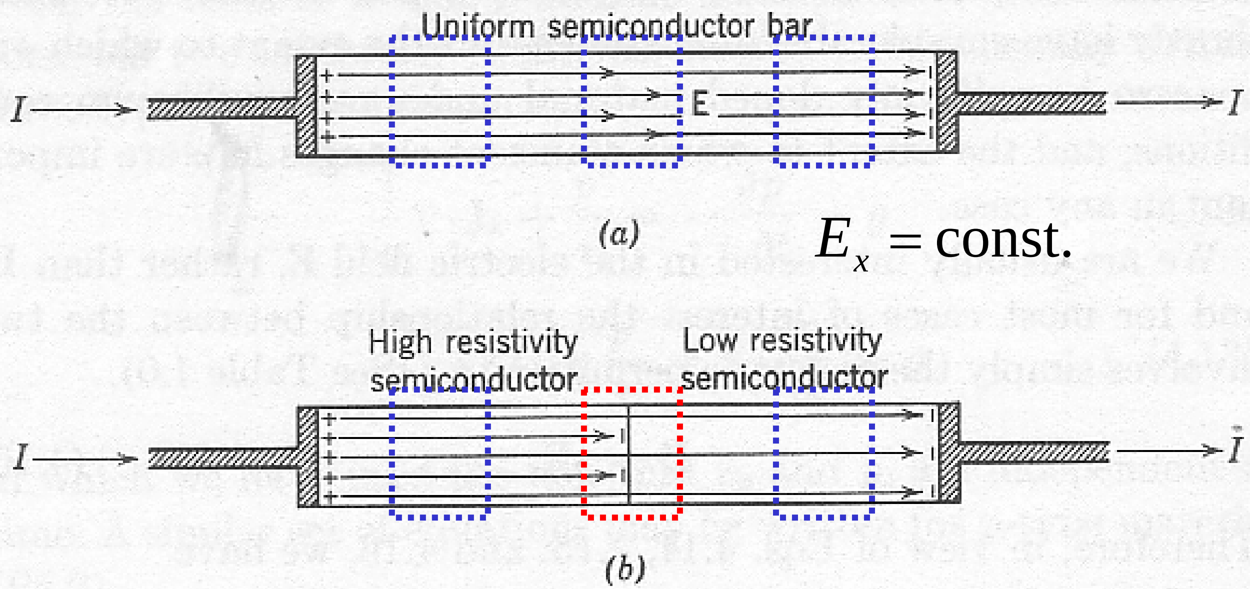
~ complete ionization

$$\rho = |q|\left(p - n + N_D^+ - N_A^-\right) \approx |q|\left(p - n + N_D - N_A\right)$$

$$N = N_D - N_A$$

divergence-less fields can be non-zero ! (due to "external" charges)

$$\frac{\partial E_x}{\partial x} = 0 \;\not\Rightarrow\; E_x = 0$$




$$E_x = \text{const.}$$

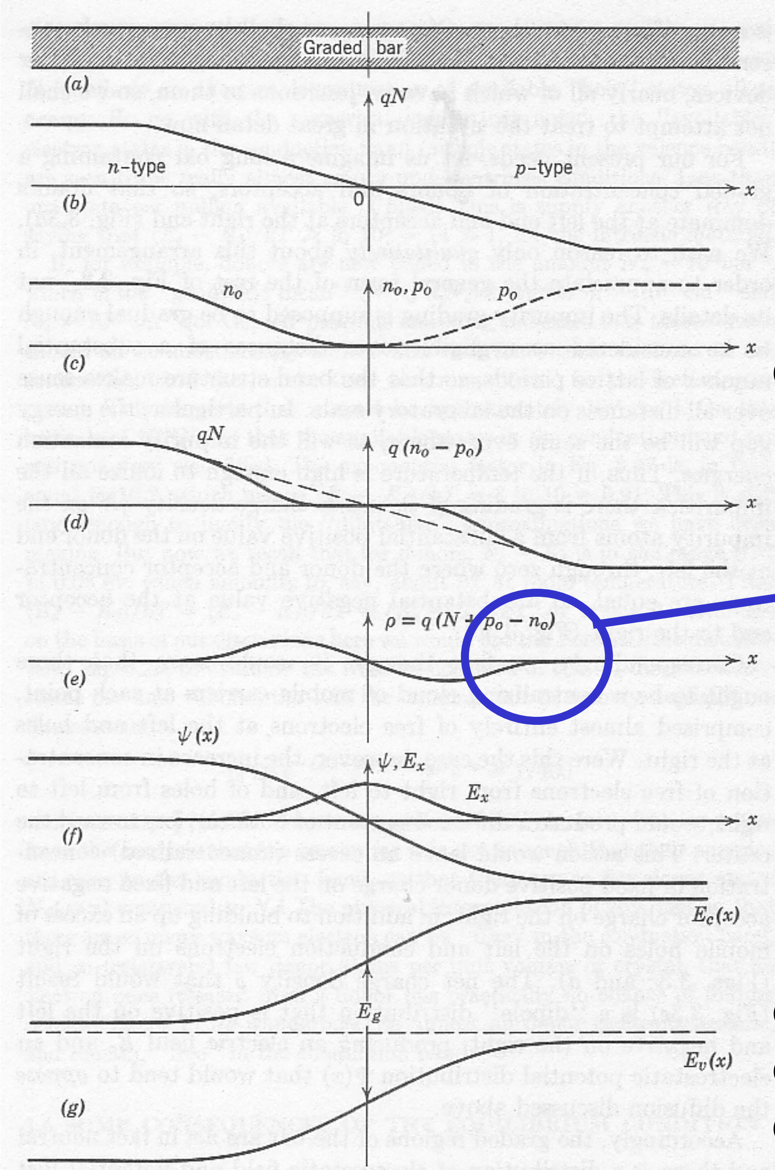
Example: current in a semiconductor

Uniform resistivity: uniform el.field, no local charge

Non-uniform resistivity: non-uniform field, local charge $\neq 0$ !

$$E_{x1}/\rho_1 = J_x = E_{x2}/\rho_2 \quad \Rightarrow \quad E_{x1} > E_{x2}$$

# Dielectric relaxation and Debye length




$N = N_D - N_A$

N: net *fixed* charge

$n_0$, $p_0$: carrier concentrations

net *mobile* charge

net *total* charge

potential $\Psi$

El.field $\mathcal{E}_x$

electron energy-band diagram

Equilibrium properties of an inhomogeneus bar

In case of a local charge unbalance:
*recovery of electrical neutrality* :
How fast?
Over what distance?

NB: different from diffusion !
(diffusion of a specific carrier: driven by variations in its concentration, not by neutrality)

# Gauss' law and Debye length

$$\vec{\nabla} \bullet \vec{E} = \rho / \varepsilon \qquad \vec{E} = -\vec{\nabla}\Psi \quad \Rightarrow \quad \nabla^2 \Psi = -\frac{q}{\varepsilon}(N + p - n)$$

Gauss' law to find *departures from electrical neutrality* under *thermal equilibrium* in an *inhomogeneously doped semiconductor*

$-q \equiv -|q|$ electron charge

$$p_0 = n_i e^{-q\Psi/kT}$$
$$n_0 = n_i e^{+q\Psi/kT}$$

$(\Psi = 0 \quad \text{where} \quad p_0 = n_0 = n_i)$

$$\nabla^2 \Psi = \frac{2qn_i}{\varepsilon}\left[\sinh(q\Psi/kT) - \left(\frac{N}{2n_i}\right)\right]$$

Define:

$$u \equiv \frac{\Psi}{kT/q} = \frac{q\Psi}{kT}$$

$$\sinh u_0 \equiv \frac{N}{2n_i}$$

$$\nabla^2 u = \frac{2q^2 n_i}{kT\varepsilon}[\sinh u - \sinh u_0]$$
$$= \frac{4q^2 n_i}{kT\varepsilon}\cosh\left(\frac{u+u_0}{2}\right)\sinh\left(\frac{u-u_0}{2}\right)$$

# Gauss' law and Debye length

Special 1-d case (see fig. in slide 14, right-hand end):
uniformly *p-type*, $N < 0 \Rightarrow u \to u_0 < 0$
for $| u - u_0 | << 1$ the differential equation becomes:

$$\frac{d^2(u-u_0)}{dx^2} = \frac{q^2 N_A}{kT\varepsilon}(u-u_0) \qquad \Rightarrow \qquad (u-u_0) \approx e^{-x/L_D}$$

"Debye length" $L_D$ (~ $10^{-5}$ cm):

$$L_D \equiv \sqrt{\frac{kT\varepsilon}{q^2 N_A}} = \sqrt{\left(\frac{kT}{q}\right)\left(\frac{\varepsilon}{qN_A}\right)} \equiv \sqrt{\left(\frac{kT\mu_p}{q}\right)\left(\frac{\varepsilon}{q\mu_p N_A}\right)}$$

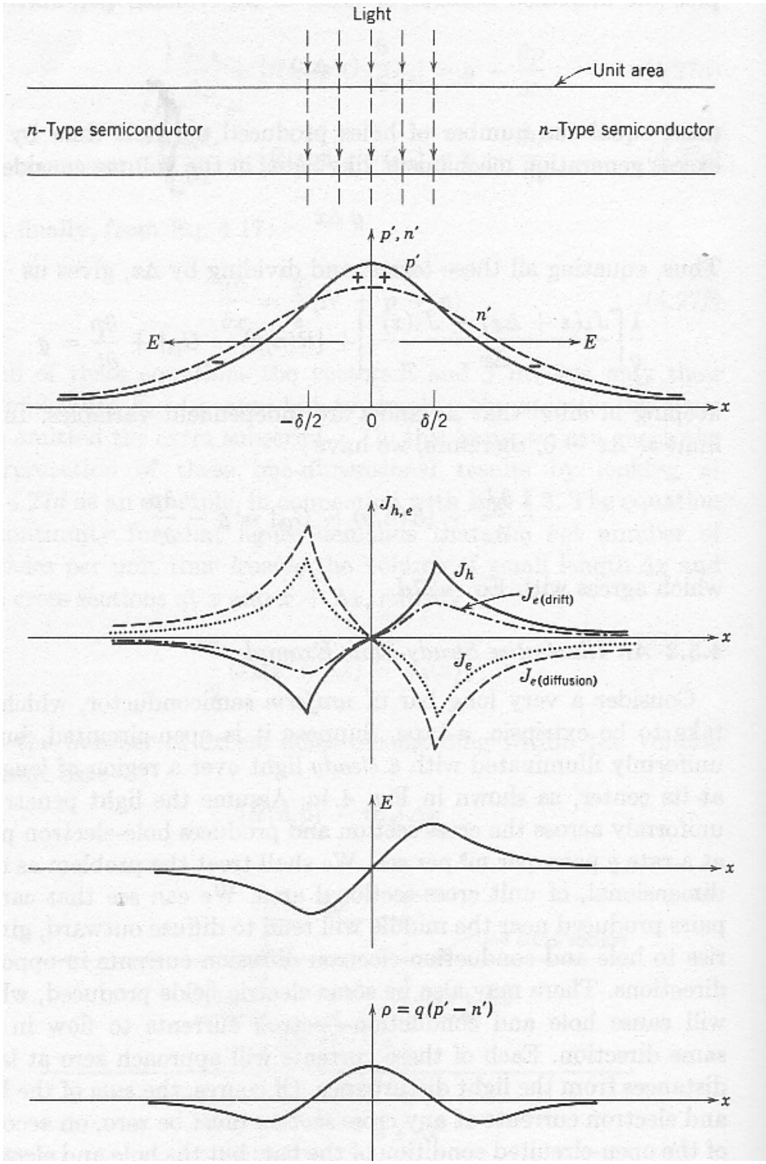
$$\approx \sqrt{D_p\left(\frac{\varepsilon}{\sigma}\right)} = \sqrt{D_p \tau_d} \qquad \tau_d \equiv \varepsilon/\sigma$$

"dielectric relaxation time" $\tau_d$ (~ $10^{-12}$ s)

Expect no significant departures from electrical neutrality, over distances greater than about 4 $L_D$ to 5 $L_D$ in uniformly doped extrinsic material, at thermal equilibrium (also true off-equilibrium!)

# A “steady-state” example:
## locally illuminated semiconductor bar

# Ingredients and qualitative expectations




n-type; non-equilibrium; open-circuit; Local steady illumination

Diffusion of excess carriers (*p'*, *n'*)

$$p' \equiv p - p_0 = \Delta p \qquad n' \equiv n - n_0 = \Delta n$$

Diffusion currents, but also drift currents due to the electric field $E_x$

$$J_h = q\mu_h p E_x - qD_h \frac{dp'}{dx} \qquad J_e = q\mu_e n E_x + qD_e \frac{dn'}{dx}$$

$$J = J_e + J_h = 0$$

Electric field $E_x$ (charge unbalance!)

$$\frac{dE_x}{dx} = \frac{q}{\varepsilon}\left(p' - n'\right) \neq 0$$

The local charge unbalance is small!

$$\left|\frac{p' - n'}{p'}\right| \approx \left|\frac{p' - n'}{n'}\right| << 1$$

# Ingredients and qualitative expectations

holes (h): minority
electrons (e): majority

n-type; non-equilibrium; open-circuit;
Local steady illumination

drift (e,h): n >> p

$$q\mu_e nE_x >> q\mu_h pE_x$$

Diffusion of excess carriers (*p'*, *n'*)

$$p' \equiv p - p_0 = \Delta p \qquad n' \equiv n - n_0 = \Delta n$$

$$J_h = q\mu_h pE_x - qD_h \frac{dp'}{dx}$$

$$J_e = q\mu_e nE_x + qD_e \frac{dn'}{dx}$$

Diffusion currents, but also
drift currents due to the electric field $E_x$

Diffusion (e,h): opposite currents,
comparable sizes

$$J = J_e + J_h = 0$$

$$\Rightarrow \left|q\mu_h pE_x\right| << \left|qD_h \frac{dp'}{dx}\right|$$

$$\Rightarrow J_h \approx -qD_h \frac{dp'}{dx}$$

in this case
minority carriers flow
mainly by diffusion

Electric field $E_x$ (charge unbalance!)

$$\frac{dE_x}{dx} = \frac{q}{\varepsilon}(p' - n') \neq 0$$

The local charge unbalance is small!

$$\left|\frac{p' - n'}{p'}\right| \approx \left|\frac{p' - n'}{n'}\right| << 1$$

Under conditions of:
- comparable mobilities
- small injection
in uniform extrinsic material

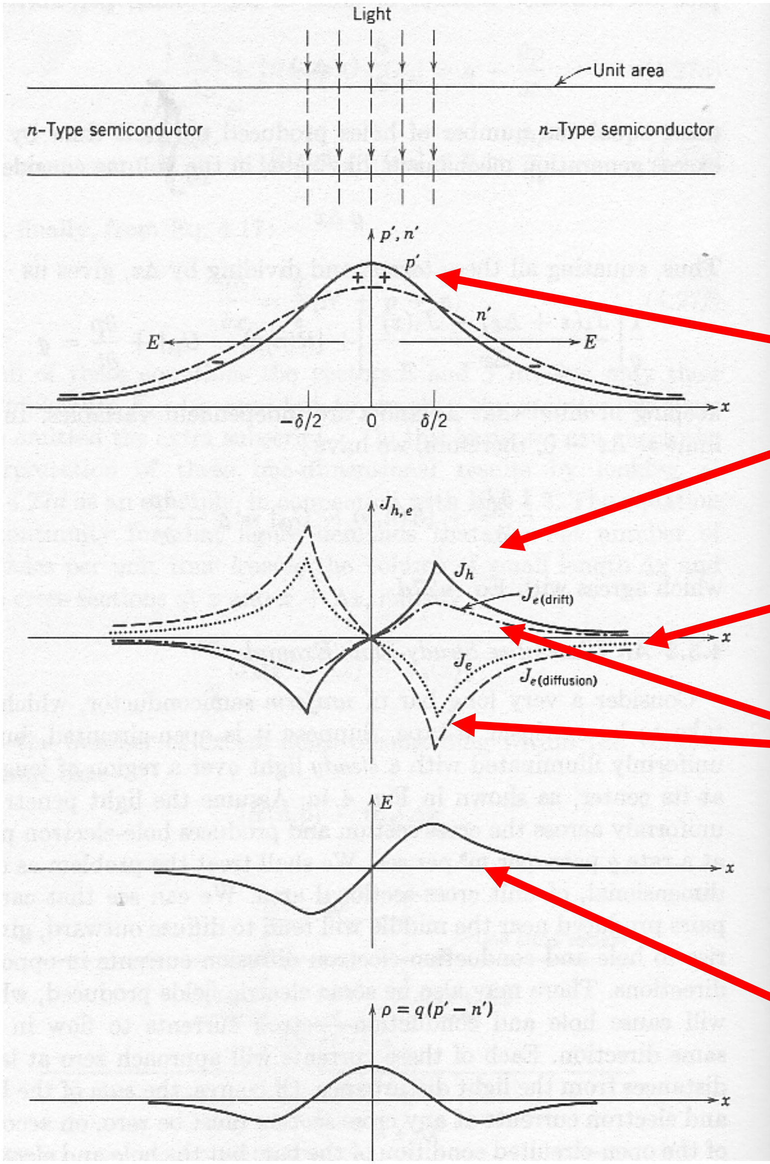
→

the minority-carrier current
will be comparable to
the majority-carrier current
only if
minority carriers
flow mainly by diffusion

# Qualitative results

$$\frac{d^2 p'}{dx^2} - \frac{p'}{D_h \tau_h} = -\frac{g_L}{D_h}\,; \qquad 0 < x < \delta/2$$

$$= 0\,; \qquad x > \delta/2$$




The continuity equations above can be solved analytically to obtain p'(x) and $J_h(x) \Rightarrow J_e(x) = -J_h(x)$

$$J_e + J_h = 0 \quad \Rightarrow \quad J_e = -J_h$$

If $\quad D_e = D_h \quad \Rightarrow \quad E_x = 0, \quad p' = n'$

If $\quad D_e > D_h \quad \Rightarrow$ Majority diffusion larger

$\Rightarrow$ Majority drift current: same direction as $J_h(x)$

Electric field $E_x$

$$J_{e(\mathrm{drift})} = q\mu_e (n_0 + n') E_x \approx q\mu_e n_0 E_x$$

Under conditions of:
- comparable mobilities
- small injection
in uniform extrinsic material

→

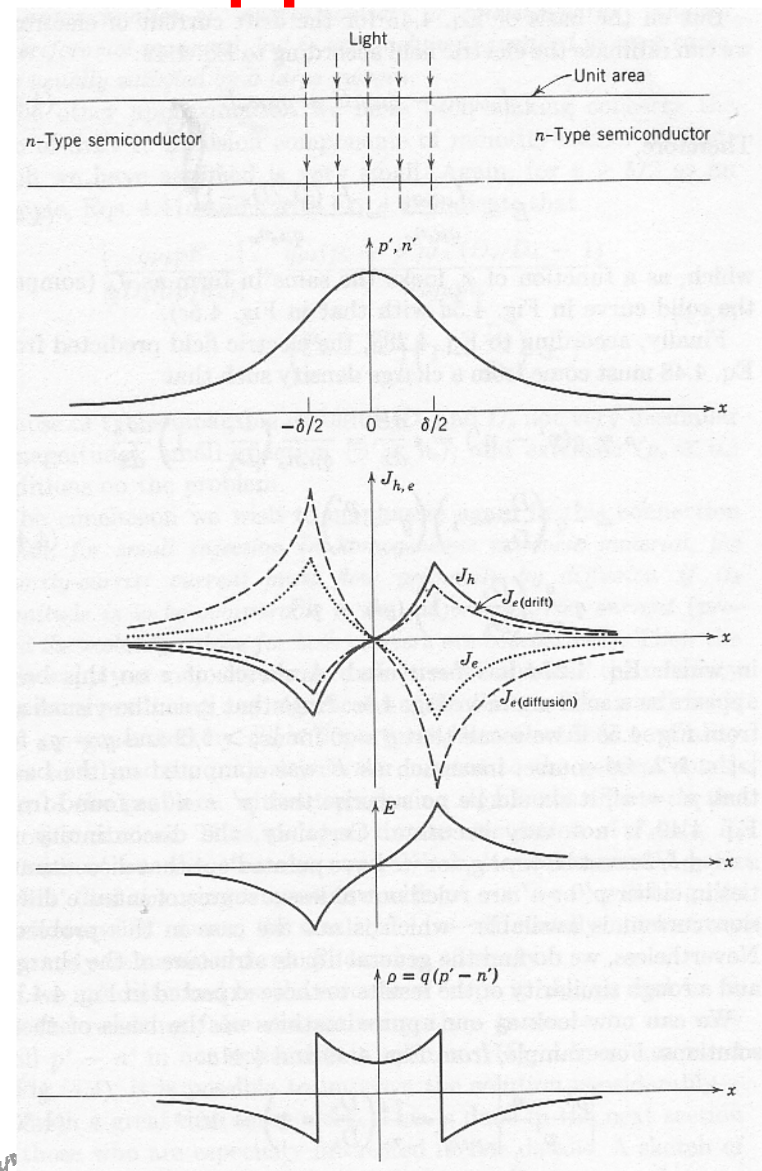
the minority-carrier current will be comparable to the majority-carrier current only if minority carriers flow mainly by diffusion

The large supply of majority carriers effectively "shields" the minority ones from producing any significant space charge.

The small fields that are generated by slight departures from neutrality serve to adjust the majority-carrier current to the general conditions of the problem, without producing significant effects on minority carriers.

An approximate calculation of $J_h$, $J_e$, $E_x$, p', n' in the "quasi-neutral" $n' \approx p'$ approximation (*without* enforcing Gauss' law with $p' - n' = 0$) will be quite satisfactory; of course, the small $p' - n'$ will not be very accurately determined from $E_x$ found in this way

# Approximate quantitative solution




Assuming "quasi-neutral" behaviour:

$$p' \approx n' \qquad \frac{dp'}{dx} \approx \frac{dn'}{dx}$$

(well justified in most cases)

$$J_e = J_{e(\text{drift})} + J_{e(\text{diffusion})} = -J_h$$

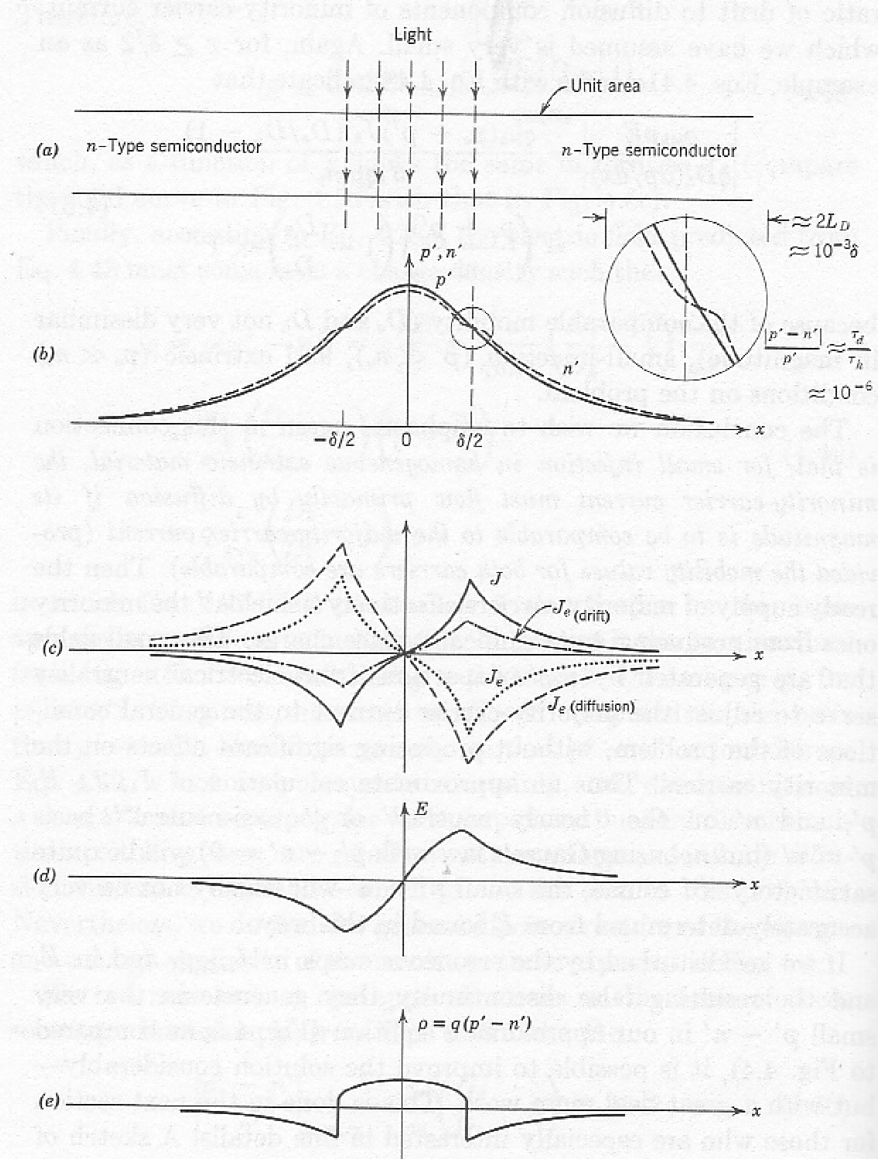
$$J_{e(\text{diffusion})} = qD_e \frac{dn'}{dx} \approx qD_e \frac{dp'}{dx} = -\frac{D_e}{D_h} J_h$$

$$J_{e(\text{drift})} = -J_{e(\text{diffusion})} - J_h \approx J_h \left( \frac{D_e}{D_h} - 1 \right)$$

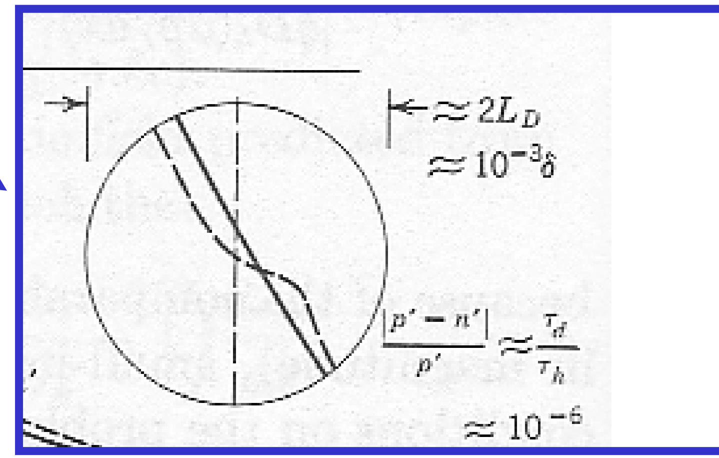
$$E_x = \frac{J_{e(\text{drift})}}{q\mu_e n} \approx \frac{J_{e(\text{drift})}}{q\mu_e n_0} \approx \frac{J_h \left( D_e / D_h - 1 \right)}{q\mu_e n_0}$$

Approximate charge unbalance ($dE_x/dx$)

# Nearly exact solution




A more accurate solution, not using the $p' \approx n'$ approx. to evaluate $J_{e\ (diffusion)}$




In this example:

$\delta \approx L_h >> L_D$

light beam width $\delta$

hole diffusion length $L_h$

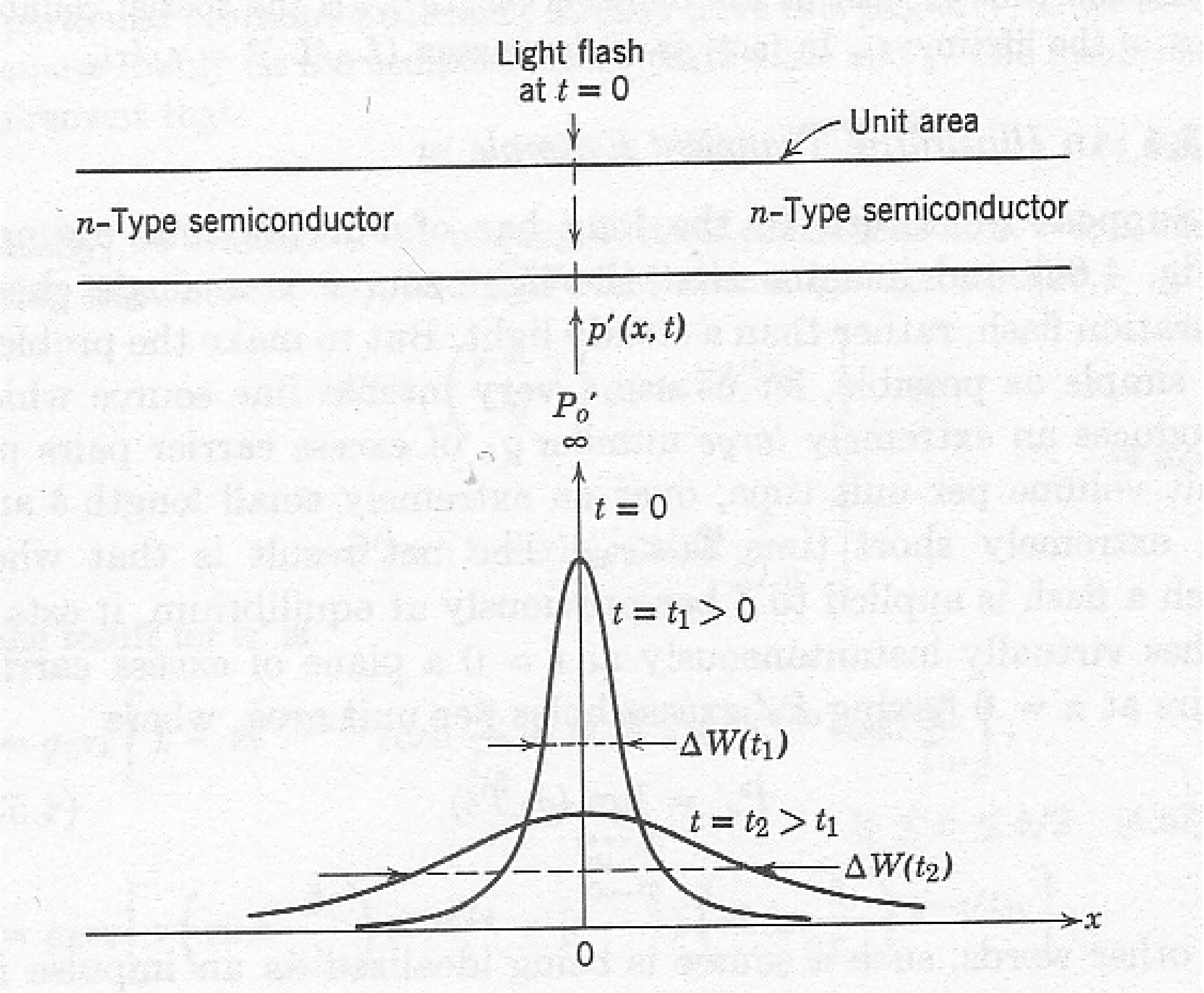
Debye length $L_D$

# “transient” examples:

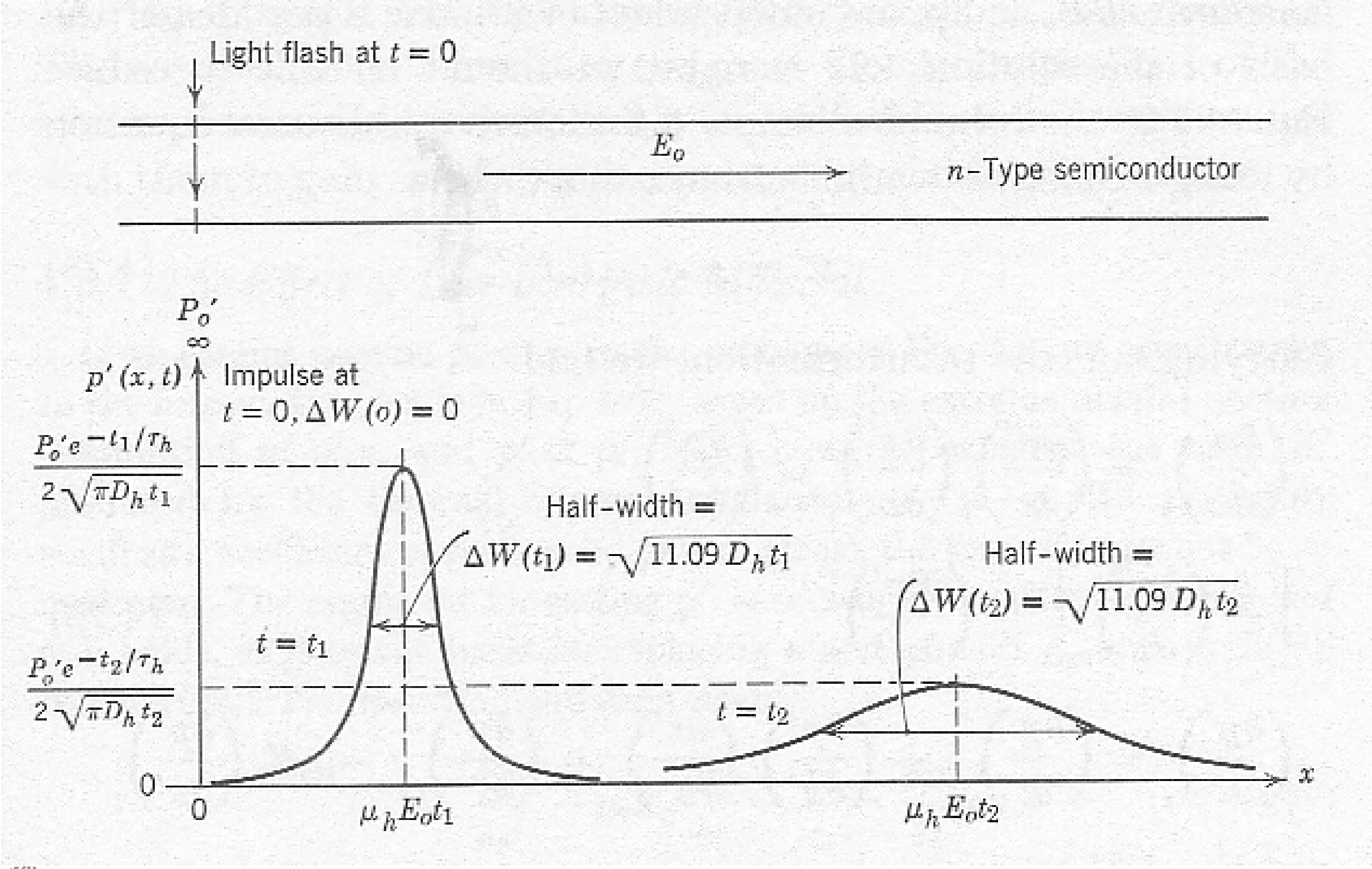
response to impulse light source
Haynes-Shockley experiment

$\Rightarrow$ “ambipolar” transport

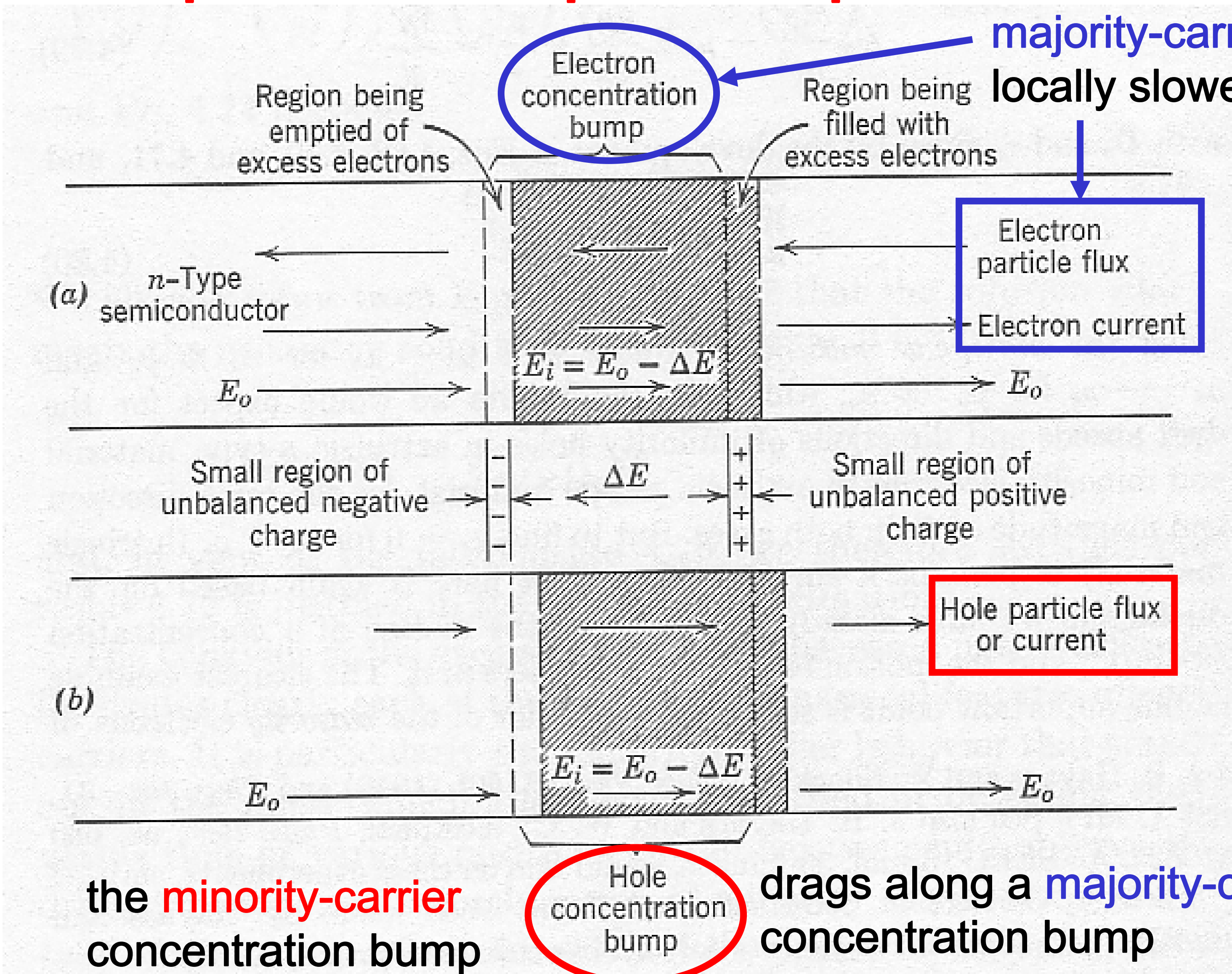
# Light flash at t=0, x=0



# Heynes-Shockley experiment



# "Ambipolar transport" - qualitative




majority-carriers, locally slowed down

the minority-carrier concentration bump drags along a majority-carrier concentration bump

# “Ambipolar transport” - equations

Special case: homogeneous semiconductor $\Rightarrow$
Thermal equilibrium concentrations $n_0$, $p_0$ constant (time and space)

$$D_p \frac{\partial^2 p'}{\partial x^2} - \mu_p \left( E_x \frac{\partial p'}{\partial x} + p \frac{\partial E_x}{\partial x} \right) + g_p - \frac{p}{\tau_p} = \frac{\partial n'}{\partial x}$$

$$D_n \frac{\partial^2 n'}{\partial x^2} - \mu_n \left( E_x \frac{\partial n'}{\partial x} + n \frac{\partial E_x}{\partial x} \right) + g_n - \frac{n}{\tau_n} = \frac{\partial n'}{\partial x}$$

$$\vec{\nabla} \bullet \vec{E} = \frac{\partial E_x}{\partial x} = \frac{q}{\varepsilon}(p' - n') \qquad p' \equiv p - p_0 \quad n' \equiv n - n_0$$

Assume:

- Small internal electric field, with respect to the applied field $\quad |E_{\text{int}}| << |E_{\text{app}}|$
- Almost complete balance of electron and hole concentrations $\quad n' \approx p'$
- Generation, recombination $\quad g_n = g_p \equiv g \qquad \frac{n}{\tau_{nt}} = \frac{p}{\tau_{pt}} \equiv R$

# "Ambipolar transport" - equations

We get then :

$$D_p \frac{\partial^2 n'}{\partial x^2} - \mu_p \left( E_x \frac{\partial n'}{\partial x} + p \frac{\partial E_x}{\partial x} \right) + g - R = \frac{\partial n'}{\partial t} \qquad \times \mu_p p$$

$$D_n \frac{\partial^2 n'}{\partial x^2} - \mu_n \left( E_x \frac{\partial n'}{\partial x} + n \frac{\partial E_x}{\partial x} \right) + g - R = \frac{\partial n'}{\partial t} \qquad \times \mu_n n$$

Multiply (see above), add and divide by $\mu_n n + \mu_p p$

$$\boxed{D' \frac{\partial^2 n'}{\partial x^2} + \mu' E_x \frac{\partial n'}{\partial x} + g - R = \frac{\partial n'}{\partial t}}$$

"ambipolar transport equation"
Non-linear!

With "ambipolar diffusion coefficient" and "ambipolar mobility":

$$D' = \frac{\mu_n n D_p + \mu_p p D_n}{\mu_n n + \mu_p p} \qquad \mu' = \frac{\mu_n \mu_p (p - n)}{\mu_n n + \mu_p p}$$

# "Ambipolar transport"

In an extrinsic semiconductor under low injection, the ambipolar mobility coefficients reduce to the minority-carrier parameter values, that are constant

p-type
minority: electrons

$$D_n \frac{\partial^2 n'}{\partial x^2} + \mu_n E_x \frac{\partial n'}{\partial x} + g' - \frac{n'}{\tau_{n0}} = \frac{\partial n'}{\partial t}$$

n-type
minority: holes

$$D_p \frac{\partial^2 p'}{\partial x^2} - \mu_p E_x \frac{\partial p'}{\partial x} + g' - \frac{p'}{\tau_{p0}} = \frac{\partial p'}{\partial t}$$

The behaviour of excess majority carriers follows that of minority!!!

# Lecture 7 - exercises

- **Exercise 7.1:** Excess electrons have been generated in a semiconductor so that at t = 0 the excess concentration is $\Delta n(0) = 10^{15}\text{cm}^{-3}$. Assuming an excess-carrier lifetime $\tau_n = 10^{-6}$ s, calculate the excess electron concentration and the recombination rate for t = 4$\mu$s.
- **Exercise 7.2:** Excess electrons and holes are generated at the end of a silicon bar (at x = 0); the silicon bar is doped pith phosphorus atoms to a concentration $N_D = 10^{17}\text{ cm}^{-3}$. The minority lifetime is $10^{-6}$ s, the electron diffusion coefficient is $D_n$ = 25 cm$^2$/s, and the hole diffusion current is $D^p$ = 10 cm$^2$/s. Determine the steady-stae electron and hole concentrations as a function of x (for x >0) and their diffusion currents at x = 10$\mu$m.