"Complementi di Fisica"
Lecture 9

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Course Outline - Reminder

- The physics of semiconductor devices: an introduction
  - Basic properties; energy bands, density of states
  - Equilibrium carrier concentration ("intrinsic", "extrinsic")
  - Carrier transport phenomena (summary)
- Quantum Mechanics: an introduction
  - Reminder on waves
  - Waves as particles and particles as waves (the crisis of classical physics); atoms and the Bohr model
  - The Schròdinger equation and its interpretation
  - (1-d) Wave packets, uncertainty relations; barriers and wells
  - (3-d) Hydrogen atom, angular momentum, spin
  - Systems with many particles
- Advanced semiconductor fundamentals (bands, etc…)

Lecture 9 - outline

- Summary: Generation, Recombination, Continuity
  - Left over: Quasi-Fermi Levels
- High field effects
  - Saturation of drift velocity, velocity for two-valley semiconductors
  - Avalanche processes and ionization rate
- (Thermionic extraction, tunnel effect etc: see later)

Generation, recombination, continuity: summary

Generation, Recombination, Continuity - 1

Net recombination rate (2)

$$U_n = R_n - G_n$$
$$U_p = R_p - G_p$$

For instance Photogener. G_L

minority excess Δn_p (Δn_p); approximations:
1-dimensional, Electric field~0,
Uniform doping $n_0 \neq n_0(x)$, $p_0 \neq p_0(x)$,
Low-injection, photogeneration GL only

Table 3.3 Carrier Action Equation Summary.

Equations of State

General

$$\frac{\partial n}{\partial t} = \frac{1}{q}\nabla \cdot \mathbf{J}_N + \left.\frac{\partial n}{\partial t}\right|_{\substack{\text{thermal}\\ \text{R-G}}} + \left.\frac{\partial n}{\partial t}\right|_{\substack{\text{other}\\ \text{processes}}}$$

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\nabla \cdot \mathbf{J}_P + \left.\frac{\partial p}{\partial t}\right|_{\substack{\text{thermal}\\ \text{R-G}}} + \left.\frac{\partial p}{\partial t}\right|_{\substack{\text{other}\\ \text{processes}}}$$

"minority diffusion" approx.

$$\frac{\partial \Delta n_p}{\partial t} = D_N \frac{\partial^2 \Delta n_p}{\partial x^2} - \frac{\Delta n_p}{\tau_n} + G_L$$

$$\frac{\partial \Delta p_n}{\partial t} = D_P \frac{\partial^2 \Delta p_n}{\partial x^2} - \frac{\Delta p_n}{\tau_p} + G_L$$

Current and R–G Relationships

$$\mathbf{J}_N = \mathbf{J}_{N|\text{drift}} + \mathbf{J}_{N|\text{diff}} = q\mu_n n\mathscr{E} + qD_N \nabla n$$

⇕ drift ⇕ diffusion

$$\mathbf{J}_P = \mathbf{J}_{P|\text{drift}} + \mathbf{J}_{P|\text{diff}} = q\mu_p p\mathscr{E} - qD_P \nabla p$$

$$\mathbf{J} = \mathbf{J}_N + \mathbf{J}_P$$

lifetime approx. (1)

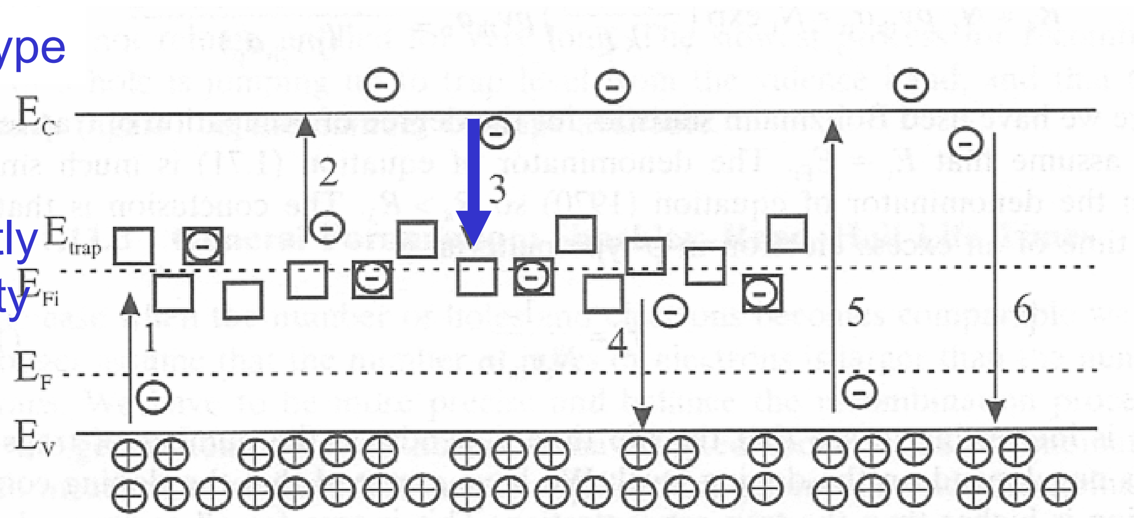
$$\left.\frac{\partial n}{\partial t}\right|_{\substack{\text{i-thermal}\\ \text{R-G}}} = -\frac{\Delta n}{\tau_n}$$

$$\left.\frac{\partial p}{\partial t}\right|_{\substack{\text{i-thermal}\\ \text{R-G}}} = -\frac{\Delta p}{\tau_p}$$

(1) "Low-injection minority lifetime" approximation

p-type

mostly empty



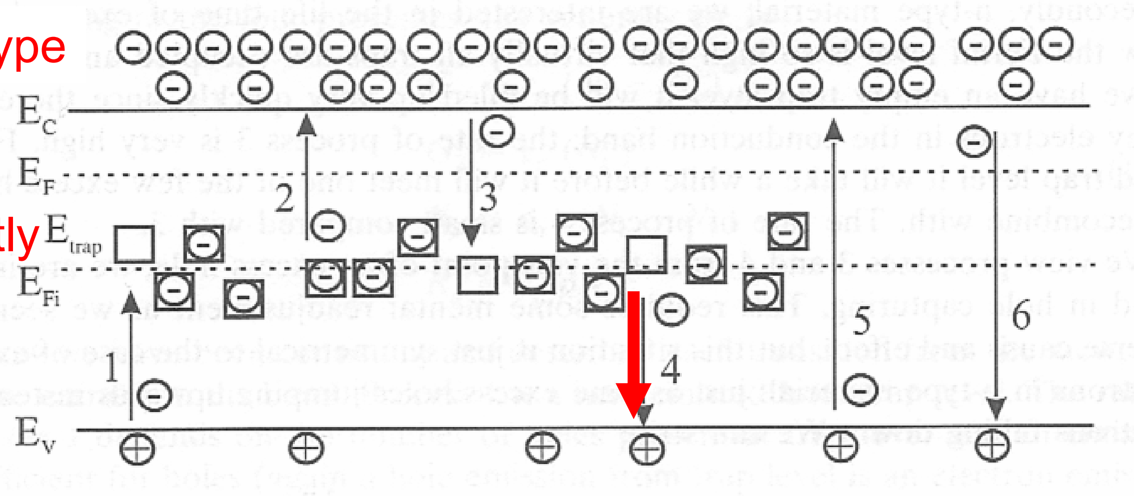
p-type semiconductor: electron lifetime dominated by "electron capture" (3) in "empty" RG centers

$$U \approx v_{th}\sigma_n N_t \left(n_p - n_{p0}\right)$$

$$\tau_n \equiv \frac{1}{v_{th}\sigma_n N_t} \approx 1.0\,\mu\text{s (Si)}$$

n-type

mostly full



n-type semiconductor: hole lifetime dominated by "hole capture" (4) in "full" RG centers

$$U \approx v_{th}\sigma_p N_t \left(p_n - p_{n0}\right)$$

$$\tau_p \equiv \frac{1}{v_{th}\sigma_p N_t} \approx 0.3\,\mu\text{s (Si)}$$

(2) General: "Shockley-Read-Hall" lifetimes

- What happens if these approximations are not valid?
  - n, p may be comparable (no longer true that n >> p or p >> n)
  - carrier lifetime no longer dominated by availability of:

    p-type: "empty" traps for "electron capture" ($N_t^0 = N_t(1-F) \approx N_t$)

    n-type: "full" or "ionized" traps for "hole capture" ($N_t^- = N_t F \approx N_t$)
- All four "indirect" processes must be taken into account (see also SZE 2.4.2, "indirect recombination")
  - 4 processes in the previous figure, notation compared to SZE 2.4.2:

    1=d "hole emission" (from a trap) $G_1 = e_p\left(N_t - N_t^-\right)$ $R_d = e_p N_t (1-F)$

    2=b "electron emission" (from a trap) $G_2 = e_n N_t^-$ $R_b = e_n N_t F$

    3=a "electron capture" (in a trap) $R_3 = c_n n\left(N_t - N_t^-\right)$ $R_a = c_n N_t (1-F)$

    4=c "hole capture" (in a trap) $R_4 = c_p p N_t^-$ $R_c = c_p N_t F$
  - Net recombination rates:

$$U_n = R_n - G_n = R_3 - G_2$$

$$U_p = R_p - G_p = R_4 - G_1$$

(2) General: "Shockley-Read-Hall" lifetimes

- From equilibrium conditions ($U_n = U_p = 0$)
  - emission coefficients (e_n, e_p) in terms of:
  - capture coeff. ($c_n = v_{th}\sigma_n$, $c_p = v_{th}\sigma_p$)

$$e_n = c_n n_1 \qquad n_1 = n_i e^{(E_i - E_t)/kT}$$

$$e_p = c_p p_1 \qquad p_1 = n_i e^{(E_t - E_i)/kT}$$

- In non-equilibrium steady-state ($U_n = U_p \neq 0$): see SZE eq. (63)

$$U = U_n = U_p = \frac{np - n_i^2}{\frac{1}{c_n N_t}(n + n_1) + \frac{1}{c_p N_t}(p + p_1)} = \frac{np - n_i^2}{\tau_n (n + n_1) + \tau_p (p + p_1)}$$

- This is a general result, usually implemented in device simulations
  - A special case: the previous "Low-injection minority lifetime" result
  - For very high doping concentrations, direct transitions become likely: this can be modeled by making τ_n and τ_p concentration-dependent

Generation, Recombination, Continuity - 2

Einstein relationships

Key Parametric Relationships

minority carriers diffusion lengths

(el.) $L_N \equiv \sqrt{D_N \tau_n}$

(h.) $L_P \equiv \sqrt{D_P \tau_p}$

$$\frac{D_N}{\mu_n} = \frac{kT}{q}$$

$$\frac{D_P}{\mu_p} = \frac{kT}{q}$$

$$\tau_n = \frac{1}{c_n N_T}$$

$$\tau_p = \frac{1}{c_p N_T}$$

minority carrier lifetimes

Resistivity and Electrostatic Relationships

$$\rho = \frac{1}{q(\mu_n n + \mu_p p)}$$

$$\rho = \frac{1}{q\mu_n N_D} \quad \ldots\ n\text{-type semiconductor}$$

$$\rho = \frac{1}{q\mu_p N_A} \quad \ldots\ p\text{-type semiconductor}$$

$$\mathscr{E} = \frac{1}{q}\frac{dE_c}{dx} = \frac{1}{q}\frac{dE_v}{dx} = \frac{1}{q}\frac{dE_i}{dx}$$

"band bending"

$$V = -\frac{1}{q}(E_c - E_{ref})$$

Quasi-Fermi Level Relationships

$$F_N \equiv E_i + kT \ln\left(\frac{n}{n_i}\right) \qquad \mathbf{J}_N = \mu_n n \nabla F_N$$

"Quasi-Fermi" ???

$$F_P \equiv E_i - kT \ln\left(\frac{p}{n_i}\right) \qquad \mathbf{J}_P = \mu_p p \nabla F_P$$

Quasi-Fermi levels: definition

- Fermi level E_F:
  - Unique, and *meaningful only in equilibrium conditions* !
  - Specifies both (free) electrons and holes concentrations
  - For non-degenerate semiconductors (calling E_i the "intrinsic" Fermi level): one-to-one correspondence between E_F and both concentrations (n,p)

$$n = n_i e^{(E_F - E_i)/kT} \qquad p = n_i e^{(E_i - E_F)/kT}$$

- In non-equilibrium conditions: E_F not defined! but:
  - Practically convenient to deduce quickly the concentrations by inspection from the energy band diagrams
  - Formally introduce *two different* "Quasi-Fermi Levels" F_N and F_P, *determined by definition* by the non-equilibrium concentrations n and p:

$$n \equiv n_i e^{(F_N - E_i)/kT} \quad \Leftrightarrow \quad F_N \equiv E_i + kT \ln(n/n_i)$$

$$p \equiv n_i e^{(E_i - F_P)/kT} \quad \Leftrightarrow \quad F_P \equiv E_i - kT \ln(p/n_i)$$

Quasi-Fermi levels: an example

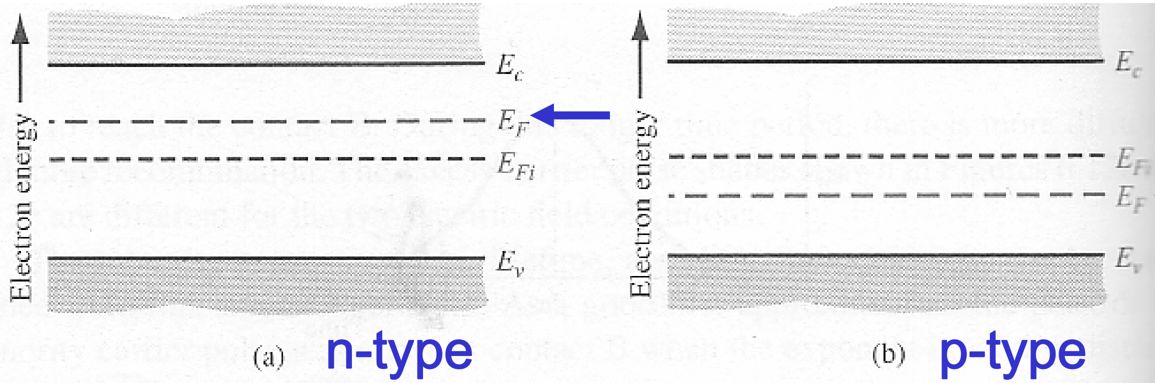


Figure 6.14 | Thermal-equilibrium energy-band diagrams for (*a*) n-type semiconductor and (*b*) p-type semiconductor.

Thermal equilibrium Fermi level E_F

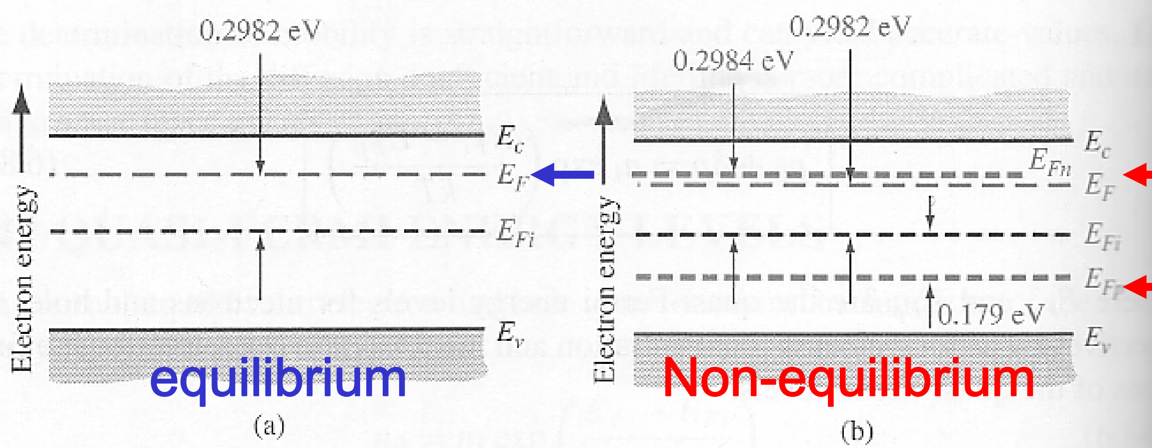


Figure 6.15 | (a) Thermal-equilibrium energy-band diagram for $N_d = 10^{15}$ cm^{-3} and $n_i = 10^{10}$ cm^{-3}. (b) Quasi-Fermi levels for electrons and holes if 10^{13} cm^{-3} excess carriers are present.

Non-equilibrium Quasi-Fermi levels F_N, F_P

Quasi-Fermi levels and currents

- Interesting practical consequence: current density
  - For simplicity, one-dimensional case:

$$J_{p,x} \equiv q\mu_p p E_x - qD_p \frac{\partial p}{\partial x}$$

  - Substituting the concentration p(x) in terms of the quasi-Fermi level F_P (dependent on x!) and using the Einstein relationship, one obtains:

$$J_{p,x} \equiv \mu_p p \frac{\partial F_P}{\partial x} \qquad \left(\text{similarly:} \quad J_{n,x} \equiv \mu_n n \frac{\partial F_N}{\partial x} \right)$$

  - A quasi-Fermi level that varies with position in a band diagram immediately indicates that current is flowing in the semiconductor! (see exercise 9.1 for an application)

Device simulations

- In real life, device designers use programs performing numerical integrations in discrete space and time steps, to obtain (*):

Process simulator

↓

doping profiles N_D, N_A

+ boundary conditions, external fields and "excitations"

↓

Device simulator

↓

n, p

$\mathrm{el.\,field}$

$\vec{J}_n, \vec{J}_p$

(*) carrier concentrations, fields, currents in:
(1) equilibrium,
(2) steady state,
(3) transients

$$\vec{\nabla} \bullet \vec{E} = -\nabla^2 V = \frac{\rho}{\varepsilon}$$

$$\rho = -n + p - N_A + N_D$$

$$\vec{J}_n = -qn\mu_n \vec{\nabla} V + qD_n \vec{\nabla} n$$

$$\vec{J}_p = -qn\mu_n \vec{\nabla} V - qD_n \vec{\nabla} n$$

$$\frac{1}{q}\vec{\nabla} \bullet \vec{J}_n = (R - G) + \frac{\partial n}{\partial t}$$

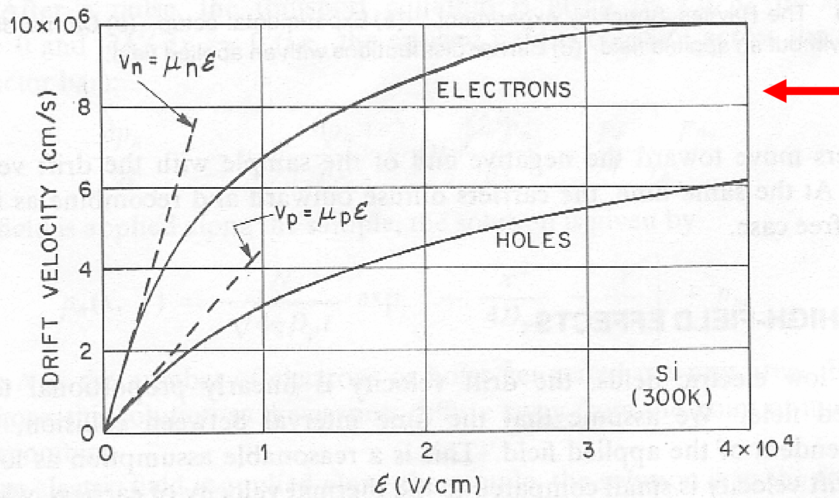
$$\frac{1}{q}\vec{\nabla} \bullet \vec{J}_p = (R - G) + \frac{\partial p}{\partial t}$$

High field effects

Drift velocity saturation

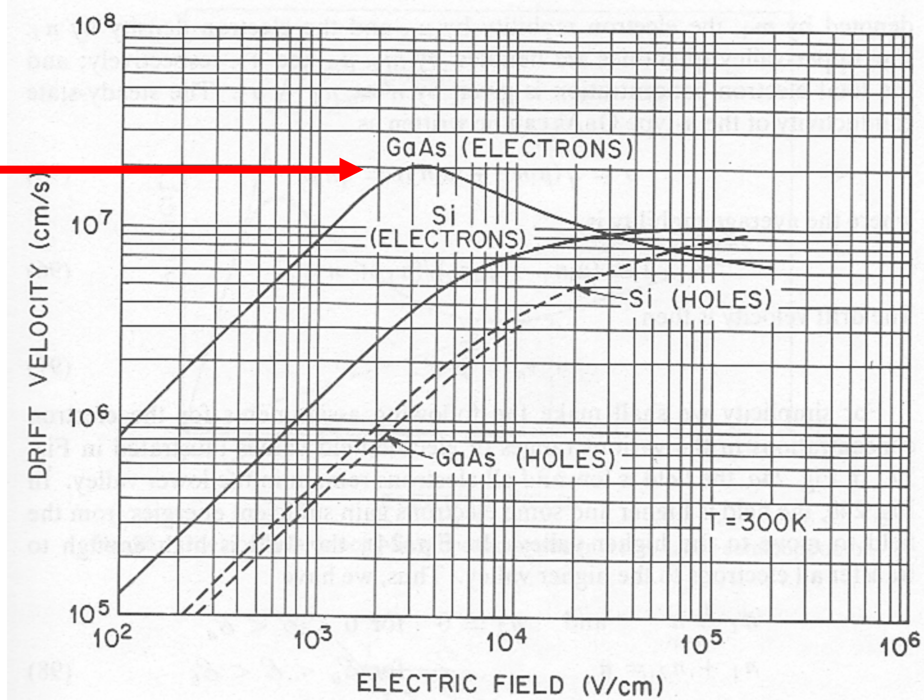
Avalanche processes

Drift velocity saturation



Silicon:
electrons and holes drift velocity
- increases linearly at small fields
- saturates (~ thermal velocity) at high fields

GaAs:
electrons and holes drift velocity
- Peculiar behavior
- see next slide for an explanation



Two-valley semiconductors

GaAs:

Two-valley model of E-k band diagram:

Different effective masses

Different mobilities

$$\mu = \frac{q\tau_c}{m^*}$$

$$\sigma = q(\mu_1 n_1 + \mu_2 n_2) = qn\overline{\mu}$$

$$\overline{\mu} \equiv (\mu_1 n_1 + \mu_2 n_2)/(n_1 + n_2)$$

$$v_n = \overline{\mu}E$$

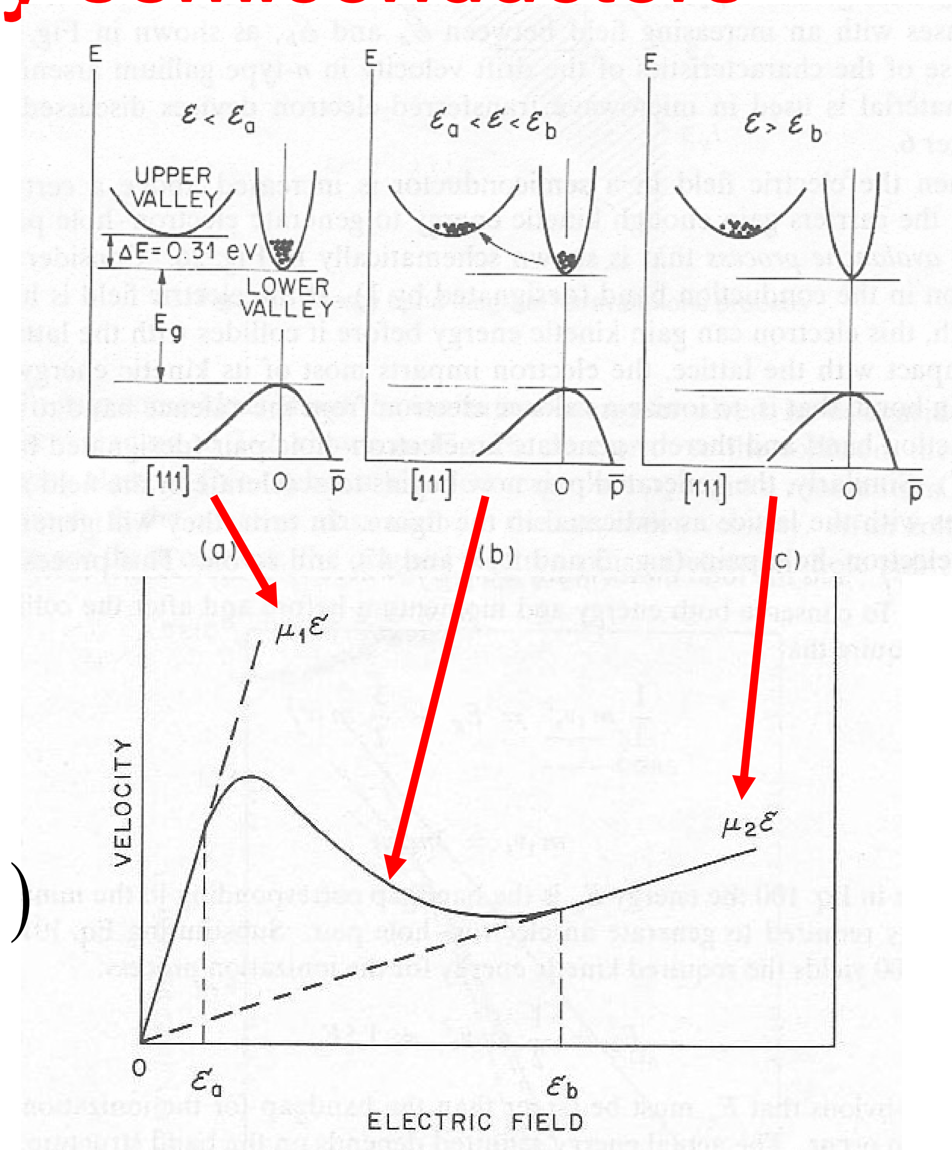


Fig. 25 One possible velocity-field characteristic of a two-valley semiconduct

Avalanche processes

To start an avalanche:
enough kinetic energy
to create an e-h pair

Order of magnitude estimate
from energy and momentum
conservation (process 1 → 2):

$$\frac{1}{2}m_1 v_s^2 \approx E_g + 3\frac{1}{2}m_1 v_f^2$$

$$m_1 v_s \approx 3 m_1 v_f$$

$$E_0 = \frac{1}{2}m_1 v_s^2 \approx 1.5\,E_g$$

$$G_A = \frac{1}{q}\left(\alpha_n |J_n| + \alpha_p |J_p|\right)$$

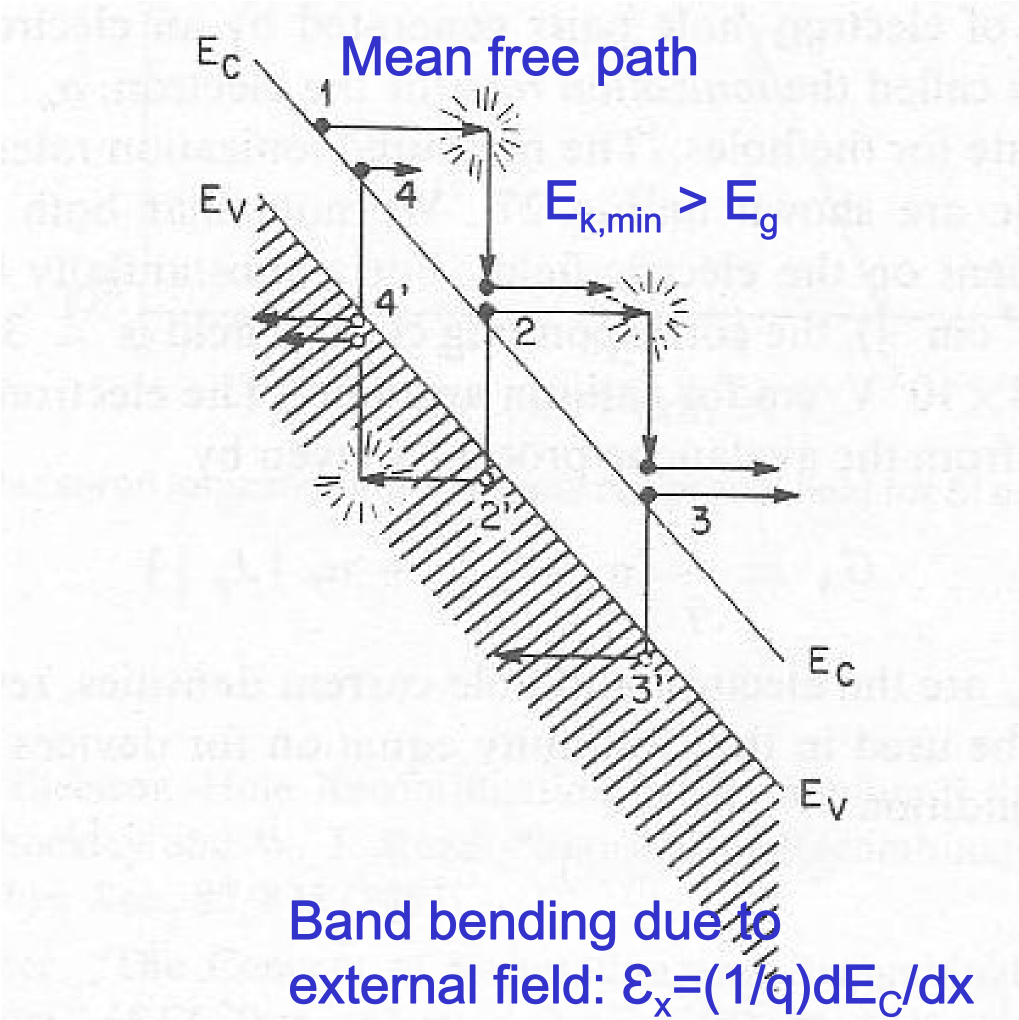


Fig. 26 Energy band diagram for avalanche process.

Ionization rates and generation rate

α_n, α_p ionization rates (cm^{-1})

e – h generation rate (cm^{-3}s^{-1})

$$G_A = \frac{1}{q}\left(\alpha_n |J_n| + \alpha_p |J_p|\right)$$

In Silicon:
$E_0 = 3.2\ E_g$ (el.)
$E_0 = 4.4\ E_g$ (h.)

$\mathcal{E} \sim 3\times10^5$ V/cm
For $\alpha \sim 10^4$ cm^{-1}

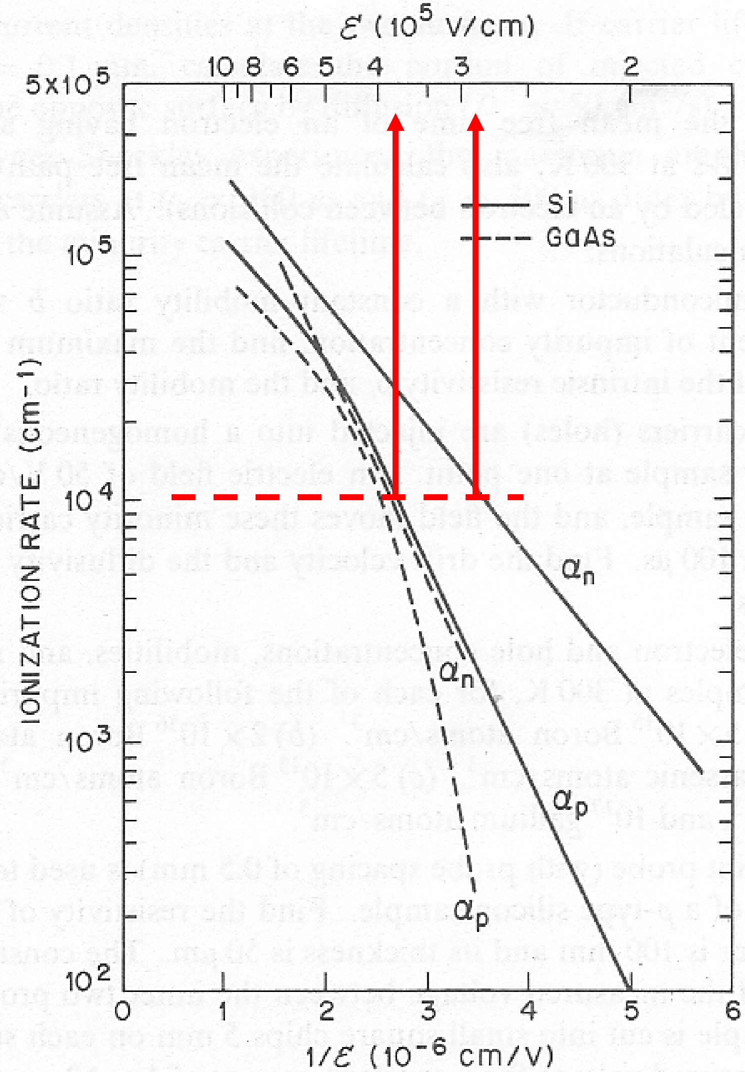


Fig. 27 Measured ionization rates versus reciprocal field for Si and GaAs.[13]

Lecture 9 - exercises

- **Exercise 9.1:** In (SZE 2.5.1), nonpenetrating illumination of a semiconductor bar was found to cause a steady state, excess-hole concentration of $\Delta p_n(x) = \Delta p_{n0} \exp(-x/L_p)$. Given low-level injection conditions, and noting that $p = p_0 + \Delta p_n$, we can say that $n \approx n_0$ and $p \approx p_0 + \Delta p_{n0} \exp(-x/L_p)$.
  (a) Find the quasi-Fermi levels $F_N(x)$ and $F_P(x)$ as functions of x.
  (b) Show that $F_P(x)$ is a linear function of x when $\Delta p_n(x) >> p_0$.
  (c) Sketch the energy band diagram under equilibrium (no illumination) and in illuminated steady-state conditions, assuming negligible electric field.
  (d) Is there a hole current in the illuminated bar, under steady state conditions? Explain.
  (e) Is there an electron current in the illuminated bar, under steady state conditions? Explain.