"Complementi di Fisica"
Exercises

Livio Lanceri
Università di Trieste

Trieste, 04-01-05

Problem-solving teams

Nome	Cognome	CdL	team
Lorenzo	COMEL	mat.	1
Luigi	GALASSO	mat.	1
Giulio	GRASSI	eln	2
Giovanni	BIANCUZZI	eln	2
Francesco	PIVETTA	eln	2
Luca	ZANELLA	eln	3
Bojan	SIMONETA	eln	3
Matej	BUDIN	tlc	3
Alberto	MARCHESAN	tlc	4
Giulia	CEROVAZ	tlc	4
Mohamad	ABBAS	tlc	4

Lecture 1 (conduction, crystals)- exercises

- Exercise 1.1: review dimensions and units for the electric field and electric potential; check the dimensions and units given for resistivity, conductivity, mobility.
- Exercise 1.2: for a typical conductor at room temperature, (i.e. aluminum (Al): $\sigma = 4 \times 10^5\ (\Omega\ \mathrm{cm})^{-1}$), compare the thermal velocity with the drift velocity for a typical applied electric field, and find the orders of magnitude of μ, τ and λ
- Exercise 1.3: What is the distance between nearest neigbours in Si crystals?
- Exercise 1.4: If a plane has intercepts 2a, 3a, 4a along the three axes, find its Miller indices.
- Exercise 1.5: Find the number of atoms per cm^2 in Si in the (100), (110), and (111) planes.

Lecture 2 (equilibrium concentrations and Fermi levels for intrinsic semiconductors) - exercises

- Exercise 2.1: Integrate the product of the density function and Fermi function g(E)F(E) to obtain the carrier concentrations n and p.
- Exercise 2.2: Estimate orders of magnitude for the conductivity of Si (pure and with realistic defects)
- Exercise 2.3: At room temperature (300K) the effective density of states in the valence band is 1.04×10^{19} cm^{-3} for silicon and 7×10^{19} cm^{-3} for gallium arsenide; find the corresponding effective masses of holes. Compare these masses with the free-electron mass.
- Exercise 2.4: Calculate the location of the intrinsic Fermi level E_i in silicon at liquid nitrogen temperature (77K), at room temperature (300K), and at 100°C (let $m_p=0.5m_0$ and $m_n=0.3\ m_0$). Is it reasonable to assume that E_i is at the center of the forbidden gap?

- (the use of MATLAB or similar programs to perform computations, plot functions etc. is encouraged; for instance: plot the Fermi function for different values of the temperature T).

Lecture 3 (equilibrium concentrations and Fermi level for extrinsic semiconductors) - exercises

- **Exercise 3.1:** A silicon sample at T=300K contains an acceptor impurity concentration of $N_A=10^{16}$ cm^{-3}. Determine the concentration of donor impurity atoms that must be added so that the silicon is n-type and the Fermi energy is 0.20 eV below the conduction band edge.
- **Exercise 3.2:** Find the electron and hole concentrations and Fermi level in silicon at 300K (a) for 1×10^{15} boron atoms/cm^3 and (b) for 3×10^{16} boron atoms /cm^3 together with 2.9×10^{16} arsenic atoms/cm^3.
- **Exercise 3.3:** Calculate the Fermi level of silicon doped with 10^{15}, 10^{17} and 10^{19} phosphorus atoms/cm^3, assuming complete ionization. From the calculated Fermi level, check if the assumption of complete ionization is justified for each doping. Assume that the ionized donors density is given by $N_D^+ = N_D(1-F(E_D))$.

Lecture 4 (drift of carriers) - exercises

- **Exercise 4.1:** Find the electron and hole concentrations, mobilities and resistivities of silicon samples at 300K, for each of the following impurity concentrations: (a) 5×10^{15} boron atoms/cm^3; (b) 2×10^{16} boron atoms/cm^3 together with 1.5×10^{16} arsenic atoms/cm^3; and (c) 5×10^{15} boron atoms/cm^3, together with 10^{17} arsenic atoms/cm^3, and 10^{17} gallium atoms/cm^3.
- **Exercise 4.2:** For a semiconductor with a constant mobility ratio $b \equiv \mu_n\mu_p > 1$ independent of impurity concentration, find the maximum resistivity ρ_m in terms of the intrinsic resistivity ρ_i and of the mobility ratio.
- **Exercise 4.3:** A semiconductor is doped with N_D ($N_D \gg n_i$) and has a resistance R_1. The same semiconductor is then doped with an unknown amount of acceptors $_{NA}$ ($N_A \gg N_D$), yielding a resistance of $0.5R_1$. Find N_A in terms of N_D if the ratio of diffusivities for electrons and holes is $D_n/D_p=50$.

Lecture 5 (diffusion of carriers) - exercises

- **Exercise 5.1:** An intrinsic Si sample is doped with donors from one side such that $N_D=N_0\exp(-ax)$. (a) Find an expression for the built-in electric field $E(x)$ at equilibrium over the range for which $N_D \gg n_i$. (b) Evaluate $E(x)$ when $a = 1\mu m^{-1}$.
- **Exercise 5.2:** An n-type Si slice of thickness L is inhomogeneusly doped with phosphorous donor whose concentration profile is given by $N_D(x) = N_0 + (N_L - N_0)(x/L)$. What is the formula for the electric potential difference between the front and the back surfaces when the sample is at thermal and electric equilibria regardless of how the mobility and diffusivity vary with position? What is the formula for the equilibrium electric field at a plane x from the front surface for a constant diffusivity and mobility?

Lecture 6 (generation, recombination, continuity) - exercises

- **Exercise 6.1:** Calculate the electron and hole concentration under steady-state illumination in an n-type silicon with $G_L=10^{16}cm^{-3}s^{-1}$, $N_D=10^{15}cm^{-3}$, and $\tau_n=\tau_p=10$ μs.
- **Exercise 6.2:** An n-type silicon sample has $2x10^{16}$ arsenic atoms/cm^3, $2x10^{15}$ bulk recombination centers/cm^3, and 10^{10} surface recombination centers/cm^2. (a) Find the bulk minority carrier lifetime, the diffusion length, and the surface recombination velocity under low-injection conditions. The values of σ_p and σ_s are $5x10^{-15}$ and $2x10^{-16}$ cm^2, respectively. (b) If the sample is illuminated with uniformly absorbed light that creates 10^{17} electron-hole pairs/(cm^{2}s), what is the hole concentration at the surface?
- **Exercise 6.3:** The total current in a semiconductor is constant and is composed of electron drift current and hole diffusion current. The electron concentration is constant and equal to 10^{16} cm^{-3}. The hole concentration is given by $p(x)=10^{15}\exp(-x/L)$ cm^{-3} ($x>0$), where L = 12μm. The hole diffusion coefficient is D_p=12cm2/s and the electron mobility is μ_n=1000cm^2/(Vs). The total current density is J = 4.8 A/cm^2. Calculate (a) the hole diffusion current density as a function of x, (b) the electron current density versus x, and (c) the electric field versus x.

Lecture 9 - exercises

- **Exercise 9.1:** In (SZE 2.5.1), nonpenetrating illumination of a semiconductor bar was found to cause a steady state, excess-hole concentration of $\Delta p_n(x) = \Delta p_{n0} \exp(-x/L_p)$. Given low-level injection conditions, and noting that $p = p_0 + \Delta p_n$, we can say that $n \approx n_0$ and $p \approx p_0 + \Delta p_{n0} \exp(-x/L_p)$.
  (a) Find the quasi-Fermi levels $F_N(x)$ and $F_P(x)$ as functions of x.
  (b) Show that $F_P(x)$ is a linear function of x when $\Delta p_n(x) >> p_0$.
  (c) Sketch the energy band diagram under equilibrium (no illumination) and in illuminated steady-state conditions, assuming negligible electric field.
  (d) Is there a hole current in the illuminated bar, under steady state conditions? Explain.
  (e) Is there an electron current in the illuminated bar, under steady state conditions? Explain.

Lecture 10, 11 - exercises

- **Exercise 10.1:** Suppose that a 60 W lightbulb radiates primarily at a wavelength of about 1000 nm. Find the number of photons emitted per second.
- **Exercise 10.2:** When electromagnetic radiation of wavelength 270 nm falls on an aluminum surface, photoelectrons are emitted. The most energetic are stopped by a potential difference of 0.406 volts. Find the work function of aluminum in electron-volts.
- **Exercise 11.1:** Find the DeBroglie wavelengths of an electron with kinetic energy of 1 eV, 1 keV, 10 MeV; of a neutron with kinetic energy kT, where T=300K; a neutron with kinetic energy of 10 MeV
- **Exercise 11.2:** Find the DeBroglie wavelength of an electron with kinetic energy of 100 eV. Supposing a beam of such electrons is sent on a crystal with spacing between atomic planes a = 1.0 nm, at what scattering angle would you expect the first diffraction maximum? (assume the Bragg condition for constructive interference)

Lectures 12, 13 - exercises

- **Exercise 12.1:** The attractive gravitational force between an electron and a proton is $Gm_p m_e/r^2$, where m_e=0.9x10^{-30} kg, m_p=1.67x10-27 kg, and G=6.67x10^{-11} m^3/kg.s^2. What is the lowest Bohr gravitational radius?
- **Exercise 12.2:** Suppose that an hydrogen atom in its ground state absorbs a photon whose wavelength is 15 nm. Will the atom be ionized? If so, what will be the kinetic energy of the electron when it gets far away from its atom of origin?
- **Exercise 12.3:** The muon, with mass $m_\mu=209m_e$, acts as a heavy electron, and can bind to a proton forming a "muonic atom". Calculate the ionization energy of this atom in its ground state, ignoring reduced-mass effects.
- **Exercise 13.1:** Consider the time-independent wave-function $C\exp(-x^2/2a^2)$. Determine the normalization constant C. Calculate the expectation values $\langle x\rangle$, $\langle x^2\rangle$, $\langle p\rangle$,$\langle p^2\rangle$ for this wave-function, evaluate the corresponding "uncertainties" (uncertainty$^2 \equiv$ variance)

Lecture 14, 15 - exercises

- Exercise 14.1: Consider a particle of mass m, bound in a one-dimensional "infinite potential well" of width a, and assume that its wave function is the ground energy eigenfunction, with $n=1$. Compute the corresponding uncertainties in position Δx and momentum Δp_x. (Hint: this problem is discussed in Bernstein, example 6-4, p.166-167)

- Exercise 15.1: Consider a gaussian wave packet specified at $t=0$ by $\phi(k)=C\exp(-a^2x^2)$, where C is a suitable normalization constant, k is the wave number and a is a parameter with dimensions $[a]=[L]$. Write the wave function $\Psi(x,0)$ at $t=0$ and find the corresponding uncertainties in position Δx and momentum Δp_x. (Hint: this problem is discussed in Bernstein, example 7-3, 7-5).

- Exercise 15.2: Study the time evolution of a gaussian wave packet, and in particular (a) the velocity and (b) show that the width of the packet increases with time. (Hint: see the next "back-up" slides)

Lecture 16, 17 - exercises

- **Exercise 16.1:** Consider the derivation of "bound" solutions for the finite well; following the track given in this lecture, fill in the calculations leading to the equation for the energy eigenvalues for the "even" solutions. Find the numerical energy eigenvalue for the lowest energy "even" state, assuming a = 500Å and V_0 = 10eV.

- **Exercise 16.2:** Following the method described in this lecture (see also back-up slides for details), derive the transmission amplitude T for a "square" potential barrier for $E < V_0$ and the approximate expression for the tunneling probability $|T|^2$. Compute the numerical value of the transmission (tunneling) probability for a particle with energy E = 9eV, incident on a "square" potential barrier (V_0 = 10 eV, a = 50Å and 100Å)

- **Exercise 17.1:** (a) Check that the two forms given for the Bloch wave functions in the Bloch theorem are indeed equivalent. (b) Explain in words what is meant by "Brillouin zones"

Lecture 18 - exercises

- Exercise 18.1: Define in words what is meant by a “Brillouin zone”.
- Exercise 18.2: Briefly explain why the current associated with the motion of electrons in a totally filled energy band (a band in which all allowed states are occupied) is identically zero.
- Exercise 18.3: Compare the values of effective masses for electrons in Si that you found on textbooks or in the literature. Are they all equal? By how much do they differ? What may be the origin of the differences?
- Exercise 18.4: What is the value of the band gap E_G that you expect for Si at T=500K ? (At 300K it is E_G = 1.125 eV). What may be the origin of the change with temperature?

Lecture 19 - exercises

- **Exercise 19.1:** Consider a simplified model of a conductor with non-interacting conduction electrons in a 3-d infinite well. Find the Fermi energy and the average inter-electron spacing. Apply the results to the case of aluminum (A=27), assuming: density $\rho = 2.7 \times 10^3$ kg/m^3, and three free electrons per atom (hint: see Bernstein, par.10-5 and example 10-5).

- **Exercise 19.2:** Write down the results of this lecture on the density of states for the conduction and valence bands and on the Fermi probability density function. Compare them with those used in previous lectures to compute the concentration of carriers in semiconductors at a given temperature. OK? Explain the reason for introducing the effective mass in the density of states as obtained from the "infinite well" box model.

Lectures 20, 21 - exercises

- Exercise 20.1: From slide 25, figure 4.6: determine the order of magnitude of phonon energies in Silicon in the different branches, at $k=k_{max}$. What is the order of magnitude for k_{max} in Silicon (1st Brillouin zone)?

- Exercise 21.1: Write down the expression of conductivity and mobility in the classical Drude model. What changes in these expressions in the quantum theory of conductivity for metals? And for semiconductors?

- Exercise 20.2: Write down the Boltzmann transport equation and the drift-diffusion continuity equation. Discuss qualitatively the meaning of each term in these equations.