“Complementi di Fisica”
Lecture 2

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Reminder:
Classical Particles and Waves

Particles

Forces, acceleration, velocity (momentum), trajectory

Potential energy and kinetic energy

Waves

Wave equation and solutions

Wave superpositions

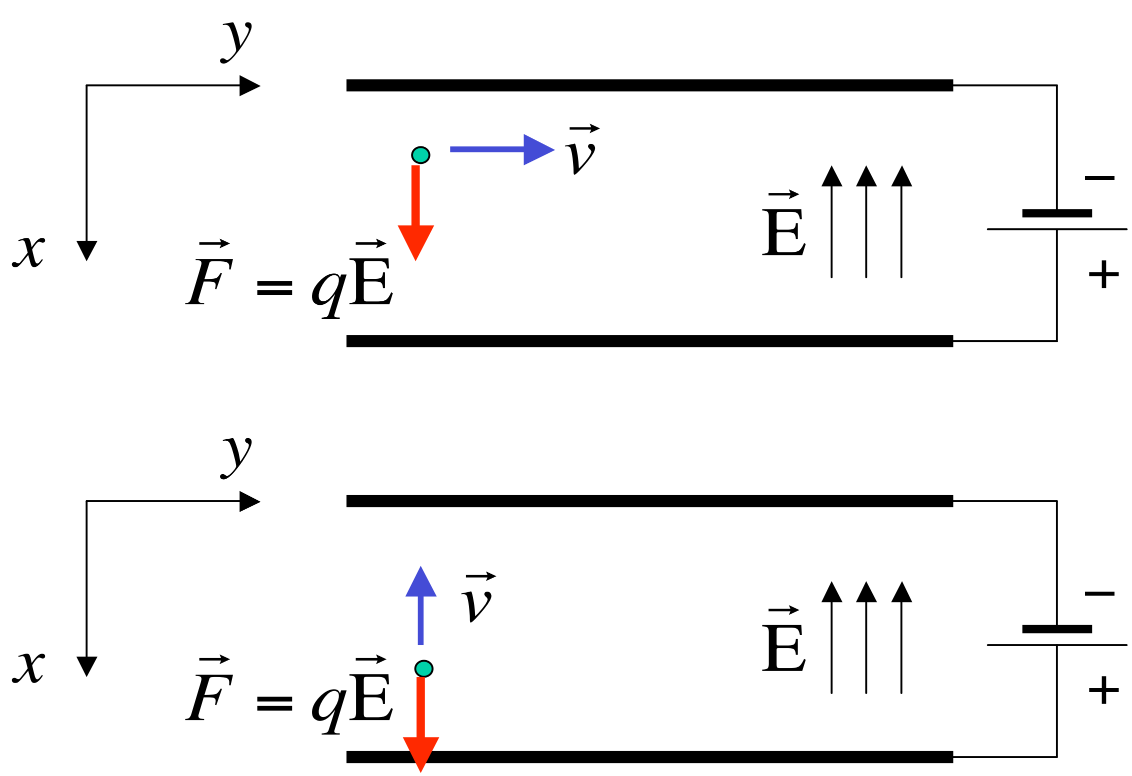
Energy flow

Particles: from forces…

Newton's Law:

$$\sum \vec{F} = \frac{d\vec{p}}{dt} = m\frac{d\vec{v}}{dt}$$

differential equation linking forces with rate of change of momentum



a specific example: electrostatic force on an electron (mass m, charge q<0)

$$\vec{F} = q\vec{E}$$

Electrostatic field:

$$E_x = -\frac{dV}{dx}$$

Electrostatic potential:

$$\Delta V = V(x) - V(0) = -\int_0^x E_x dx'$$

(defined as force and work per unit charge)

… to trajectories

Newton's Law:

$$\sum \vec{F} = \frac{d\vec{p}}{dt} = m\frac{d\vec{v}}{dt}$$

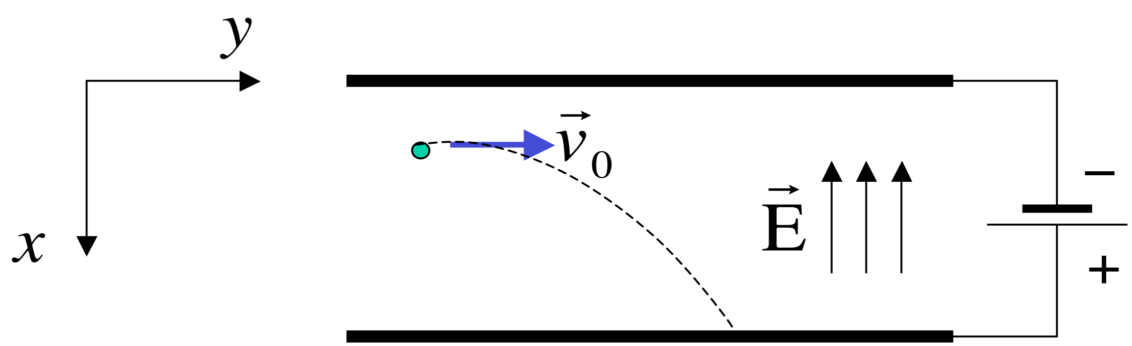
plus initial conditions

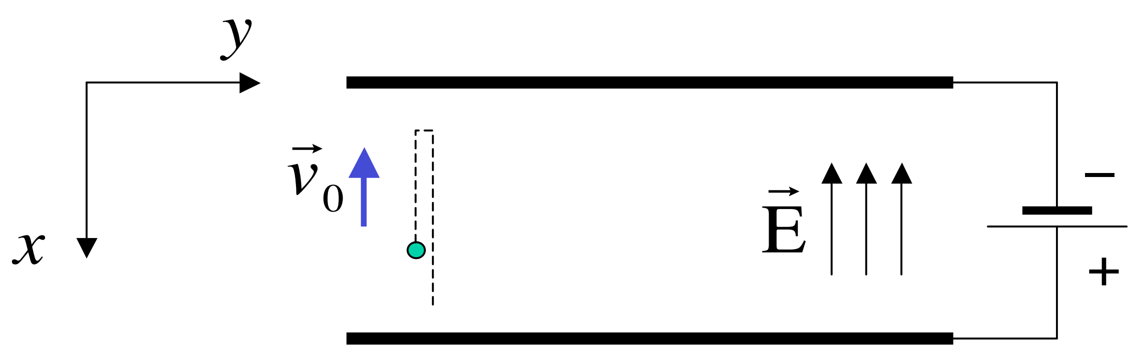
$$\vec{r}_0 = \vec{r}(t_0), \quad \vec{v}_0 = \vec{v}(t_0)$$

Unique solution:
"trajectory" =
= position vs time

$$\vec{r} = \vec{r}(t)$$

$$x = x(t), \quad y = y(t), \quad z = z(t)$$





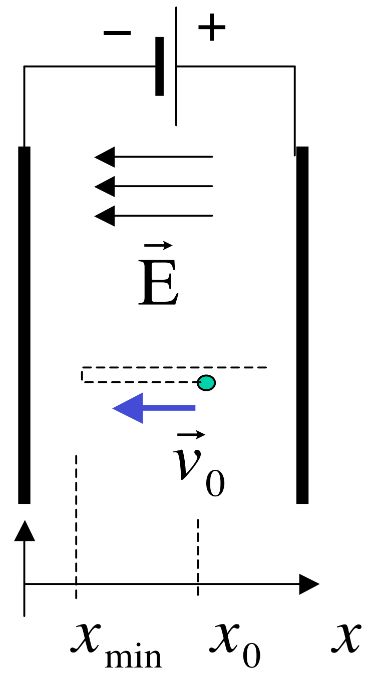
Particles: energy

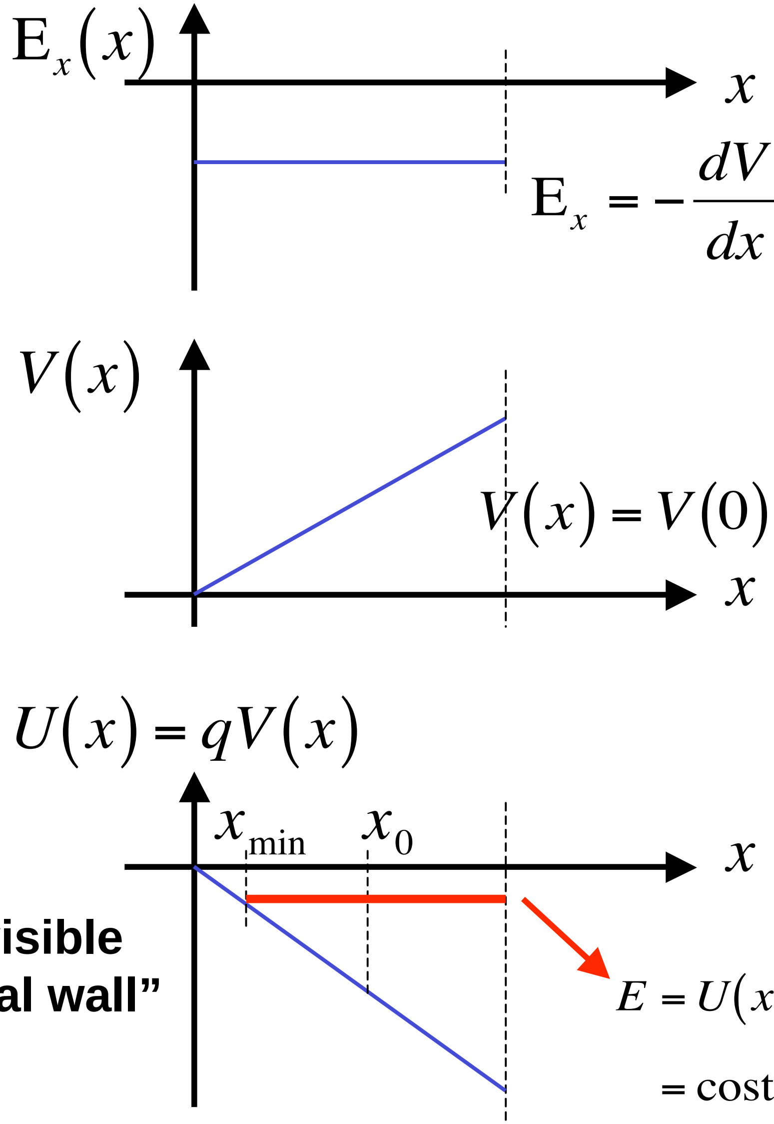
If forces are conservative:

$$E = K + U = \frac{p^2}{2m} + U = \text{cost.}$$

Kinetic Energy **Potential Energy**

Example:
electron
(q < 0)





$$E_x = -\frac{dV}{dx}$$

$$V(x) = V(0) - \int_0^x E_x dx'$$

an invisible "potential wall"

$$E = U(x_0) + \frac{1}{2} m v_0^2 = \text{cost.}$$

Waves: equation…

This differential equation with partial derivatives describes several different phenomena:

$$\frac{\partial^2 f}{\partial z^2} = \frac{1}{u^2}\frac{\partial^2 f}{\partial t^2}$$

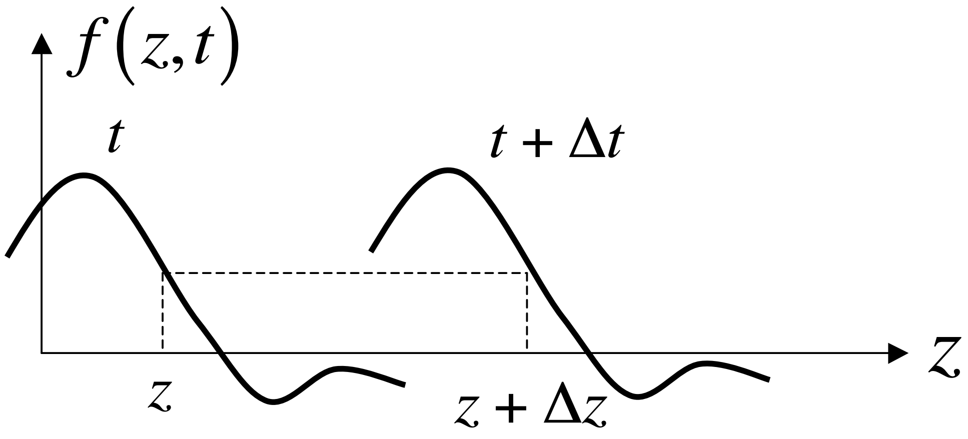
z	**space coordinate**
t	**time**
u	**constant parameter (propagation velocity)**
f	**e.m. waves: electric (magnetic) field** **sound waves: local gas pressure** **transverse waves on a rope: displacement** **etc…**

… and solutions

$$f(\xi), \quad \xi = z \pm ut \quad \text{is a solution} \quad \forall f \,, \quad \text{easy to check :}$$

$$\frac{\partial^2 f}{\partial z^2} = \frac{d^2 f}{d\xi^2}, \quad \frac{\partial^2 f}{\partial t^2} = u^2 \frac{d^2 f}{d\xi^2} \quad \Rightarrow \quad \frac{\partial^2 f}{\partial z^2} = \frac{1}{u^2}\frac{\partial^2 f}{\partial t^2}$$

Interpretation? Family of functions of z: varying the parameter t, they *"move" along z with speed u*



$$u = \frac{\Delta z}{\Delta t} \quad \Rightarrow \quad \Delta z = u\Delta t \quad \Rightarrow$$

$$f(z,t) = f(z - ut) =$$
$$= f(z + \Delta z - \Delta z - ut) =$$
$$= f(z + \Delta z - u\Delta t - ut) =$$
$$= f(z + \Delta z, t + \Delta t)$$

Sinusoidal waves

Sinusoidal waves: particularly useful

- **if the source of the wave is a harmonic oscillator, and:**
- **can be used in Fourier analysis to approximate any wave**

real form:

$$f(z,t) = f_0 \cos[k(z-ut)] = f_0 \cos(kz - kut) = f_0 \cos(kz - \omega t)$$

dimensions:

$$[z] = [ut] = [L]$$
$$[k] = [L^{-1}]$$

$$[ku] = [\omega] = [t^{-1}] = [T^{-1}]$$

$$u = \frac{\omega}{k} = v_f \quad \text{“phase velocity”}$$

complex exponential form:

$$f(z,t) = f_0 e^{ik(z-ut)} =$$
$$= f_0[\cos(kz - \omega t) + i \sin(kz - \omega t)]$$

Sinusoidal waves

Wave-length λ and wave-number *k*

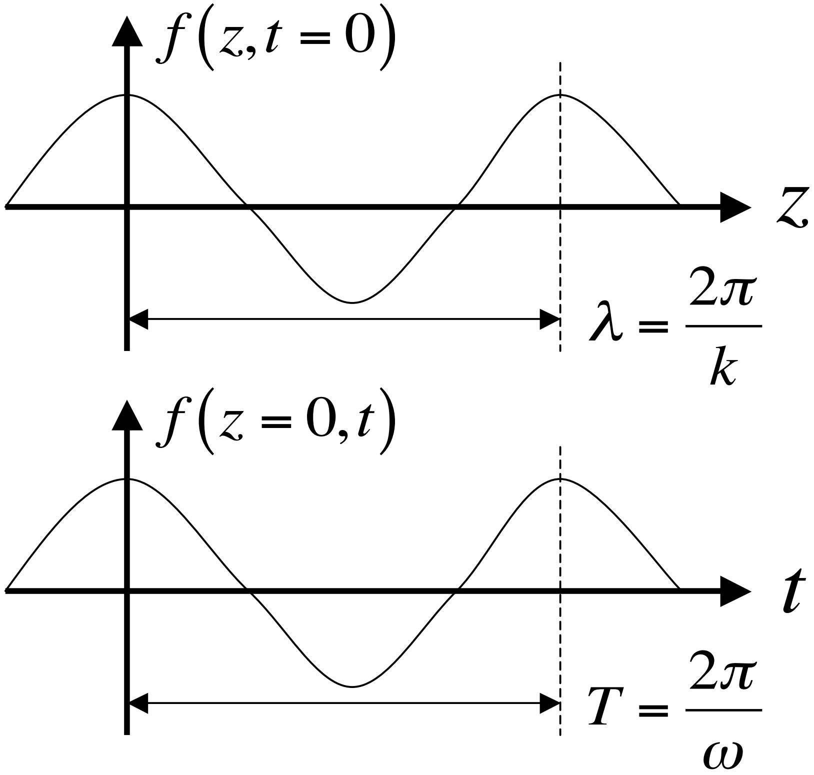
(real form for simplicity)

At fixed $t = 0$

$$f(z,t=0) = f_0 \cos(kz)$$

$$\lambda = \frac{2\pi}{k}$$

λ **"space period"**
k **"space frequency"**



Period *T* and frequency ν

At fixed $z = 0$

$$f(z=0,t) = f_0 \cos(\omega t)$$

$$T = \frac{2\pi}{\omega} = \frac{1}{\nu}$$

T **"time period"**
ν **"time frequency"**

Phase velocity: $$v_f = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda\nu$$

Light (e.m.) waves

From Maxwell equations for electric and magnetic fields:
- e.m. fields can propagate in a vacuum!
- in the wave equation fror e.m., derived from Maxwell eq.s:

$$u = v_f = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = c \approx 3 \times 10^8\, m/s$$

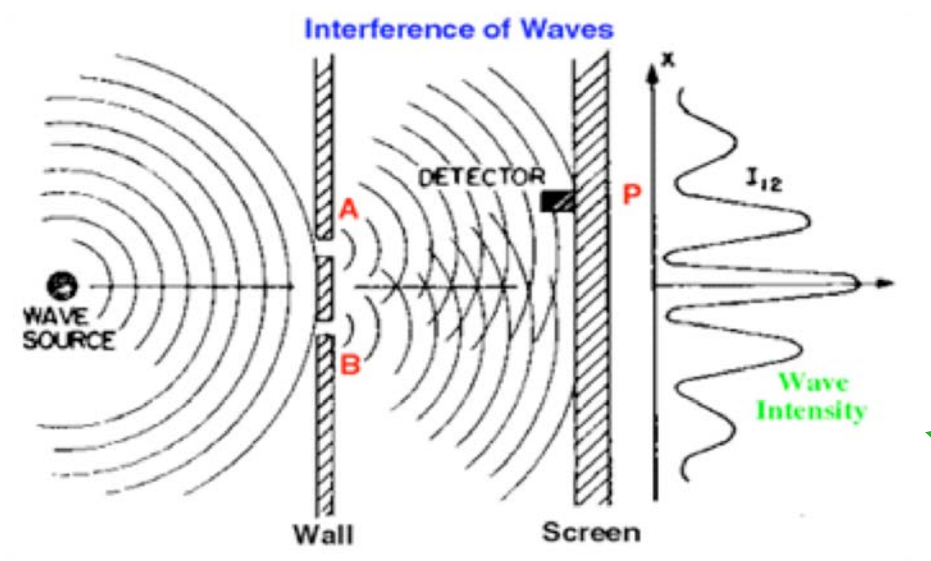
Table 1.3. THE ELECTROMAGNETIC SPECTRUM

Type of Radiation		*Frequency*	*Wavelength*	*Quantum Energy*
"Wave" region	radio waves	10^9 Hz and less	300 mm and longer	0.000004 eV and less
	microwaves	10^9 Hz to 10^{12} Hz	300 mm to 0.3 mm	0.000004 eV to 0.004 eV
"Optical" region	infrared	10^{12} Hz to 4.3×10^{14} Hz	300 μ to 0.7 μ	0.004 eV to 1.7 eV
	visible	4.3×10^{14} Hz to 5.7×10^{14} Hz	0.7 μ to 0.4 μ	1.7 eV to 2.3 eV
	ultraviolet	5.7×10^{14} Hz to 10^{16} Hz	0.4 μ to 0.03 μ	2.3 eV to 40 eV
"Ray" region	x-rays	10^{16} Hz to 10^{19} Hz	300 Å to 0.3 Å	40 eV to 40,000 eV
	gamma rays	10^{19} Hz and above	0.3 Å and shorter	40,000 eV and above

Note: The numerical values are only approximate and the division into the various regions are for illustration only. They are quite arbitrary.

Waves: diffraction

Wave equation: linear
⇒ linear combinations of solutions are good solutions
⇒ "superposition principle"
⇒ example 1: diffraction



Electric field from point source i

$$\vec{E}_i(\vec{r},t) = \vec{E}_0(\vec{r})e^{j(\omega t - \vec{k}\bullet\vec{r})}$$

Superposition from slits A and B

$$\vec{E} = \vec{E}_A + \vec{E}_B$$

Intensity (~ energy)

$$\langle I(\vec{r})\rangle_t = \vec{E}(\vec{r},t)\vec{E}^*(\vec{r},t)$$

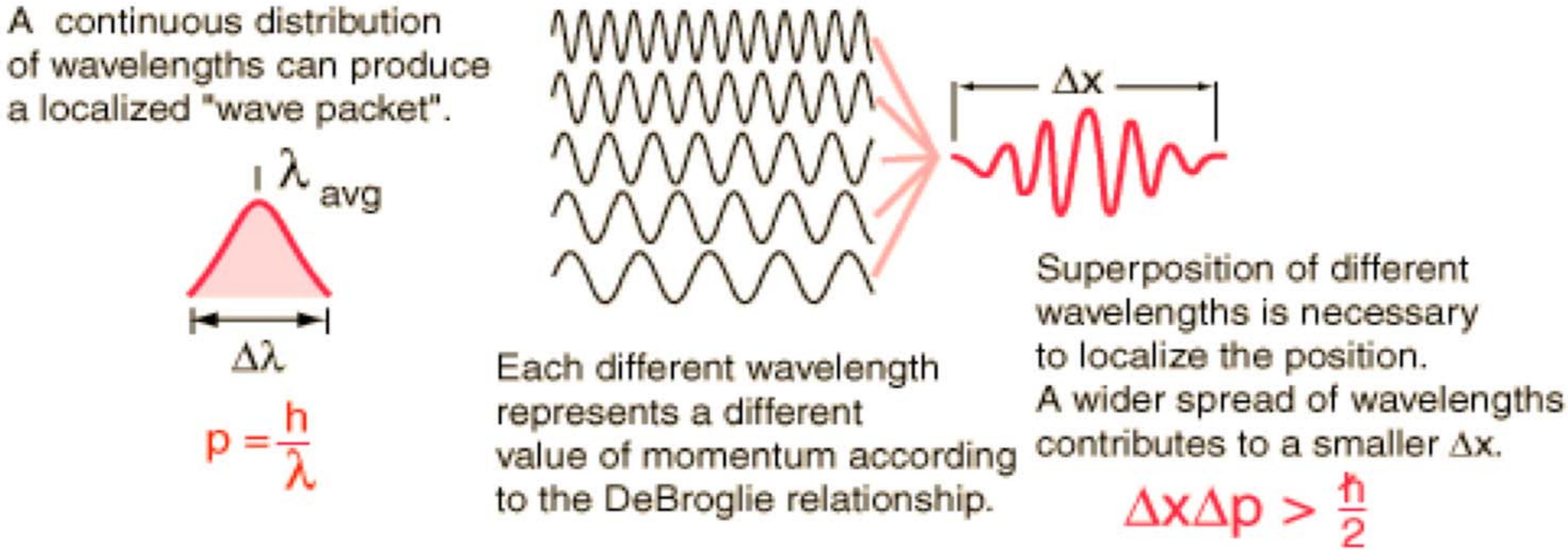
Waves: packets

Wave equation: linear

⇒ linear combinations of solutions are good solutions

⇒ "superposition principle"

⇒ example 2: wave packets



From: HyperPhysics (©C.R. Nave, 2003)

Waves: energy flow

Even if the medium does not move
(for instance gas for sound waves; no medium needed for e.m.)
waves carry both energy and momentum!

For e.m. waves (…), the energy flux is described by:

$$\left\langle \vec{S} \right\rangle = \frac{1}{2} \vec{\mathrm{E}}_0 \times \vec{H}_0 = I \frac{\vec{k}}{k} = I\hat{n} \qquad I = \frac{1}{2} E_0 H_0$$

"Poynting vector" [W/m^2] **"Irradiance"**

A beam of light exerts pressure *P* on a black surface: $P = \frac{I}{c}$

(This is related to the momentum of photons… more on this later!)

Classical Particles and Waves

Summary of the classical point of view:

Point-like particles (for instance electrons):

- ***localized* in space, with well defined *momentum* (velocity)**
- **predictable *trajectories*, if forces acting on them are known**
- **associated *energy* (potential and kinetic)**

Waves (for instance e.m. waves)

- ***distributed* over all space, if *wave number* (frequency) is well defined**
- **superpositions of waves**
  - ***"diffraction"* and other interference phenomena in *space***
  - ***"wave packets":* only *partially localized* in space, time and frequancy**
- **associated *energy flow***

Classical Particles and Waves

In the next lectures: experimentally,

Can waves (e.m., …) behave like "particles"?

Can particles (electrons, …) behave like "waves"?

Back-up slides

Lecture 2 - Homework

- **Exercise 2.1:** first exercise
- **Exercise 2.2:** second exercise
- **Exercise 2.3:** third exercise
- **Exercise 2.4:** fourth exercise
- **Exercise 2.5:** fifth exercise