CP Violation and Flavour

Lectures 1, 2

Dottorato in Fisica – XXI Ciclo

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Lectures: outline

1. Introduction: C, P, T symmetries and historical perspective

2. CP Violation (CPV) and mixing in K, B, D mesons; the Standard Model and the Cabibbo-Kobayashi-Maskawa (CKM) mechanism

3. Status of CPV in B mesons
    1. Mixing; direct CPV in B decays
    2. Unitarity Triangle (UT): measurements of sin2β, sin2α, γ
    3. Overall UT fits

4. Status of CPV in K mesons

5. Searches for CPV in D mesons and in Electric Dipole Moments

6. Conclusions and outlook

Contents

- Theory and formalism: a brief reminder only
  - After the initial discovery in K mesons…
  - … B mesons are specially suited for stringent experimental tests of a detailed pattern of theoretical expectations, including large asymmetries in some channels;
  - … D mesons are promising for New Physics searches, since the Standard Model predicts small CP effects

- Emphasis on present experiments and B mesons
  - Observables, experimental facilities and methods
  - Standard Model expectations (CKM mechanism) as organizing principle of a very rich phenomenology
  - Summary of experimental results, with emphasis on:
    - understanding their limits in precision and accuracy
    - possible windows for New Physics

Introduction

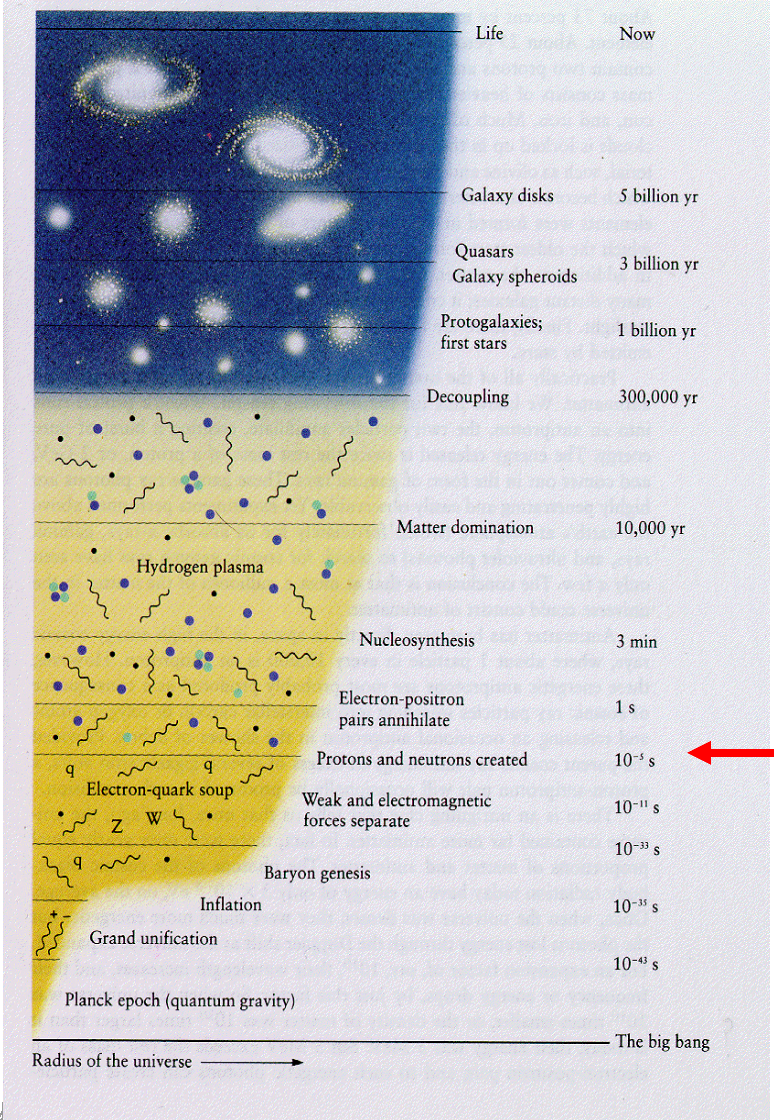
Why is CP Violation (CPV) interesting?

C, P, T symmetries

Historical perspective on CPV and Flavour

Why CPV ?

The anti-matter “puzzle”

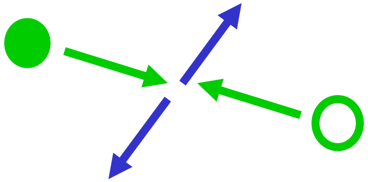


The present Universe is dominated by
- matter (particles) and
- radiation (photons);
Anti-matter (anti-particles) became very rare ! Why?

Evolution of Universe
In the Big Bang theory:
expansion, cooling

10^{-5} s $T \cong 10^{13}$K $E \cong 1$ GeV

Soup of particles, antiparticles, radiation
in thermal equilibrium



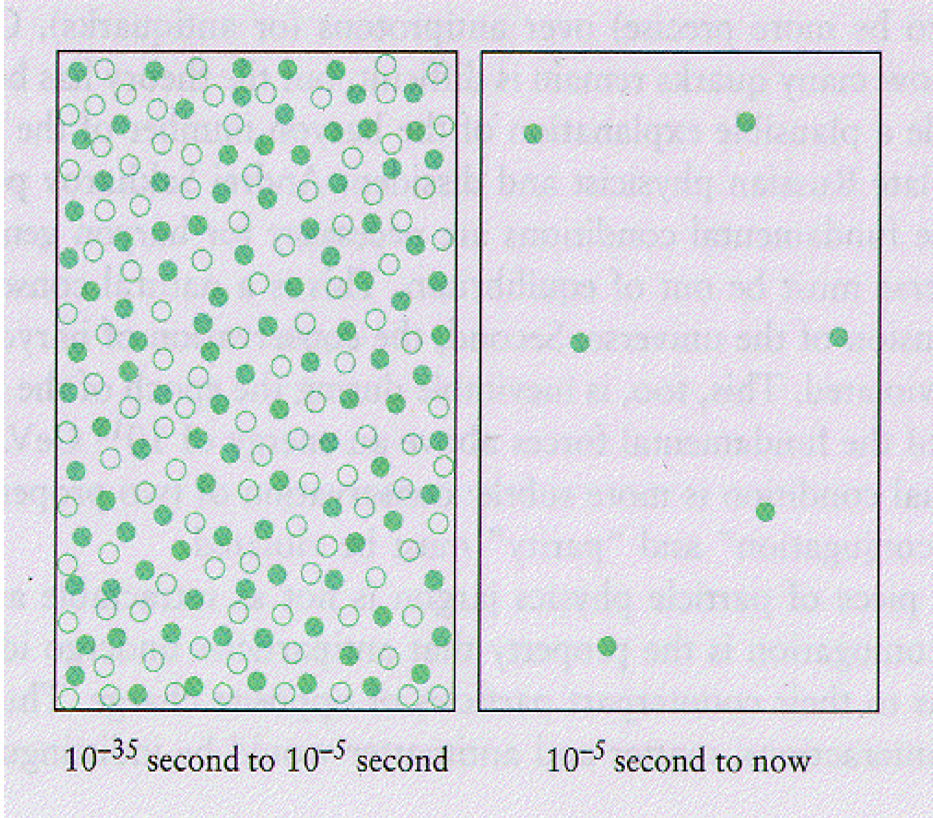
$E = mc^2$

Anti-matter: where did it go?

● $\approx 10^{88} + 10^{80}$

○ $\approx 10^{88}$

● $\approx 10^{80}$



10^{-35} second to 10^{-5} second

10^{-5} second to now

matter
antimatter
radiation

$\longrightarrow$

matter
radiation

$$n_B \approx 10^{-7}\,cm^{-3}, \quad n_\gamma \approx 400\,cm^{-3}$$

At present:

matter (baryons)/photons

$$\frac{n_B}{n_\gamma} \approx 10^{-9\pm1}$$

Quark-antiquark “primordial” asymmetry:

$$\boxed{\frac{n_B}{n_\gamma} \approx \frac{n_q - n_{\bar{q}}}{n_q + n_{\bar{q}}} \approx 10^{-9}}$$

Quark-antiquark asymmetry: when and why ?

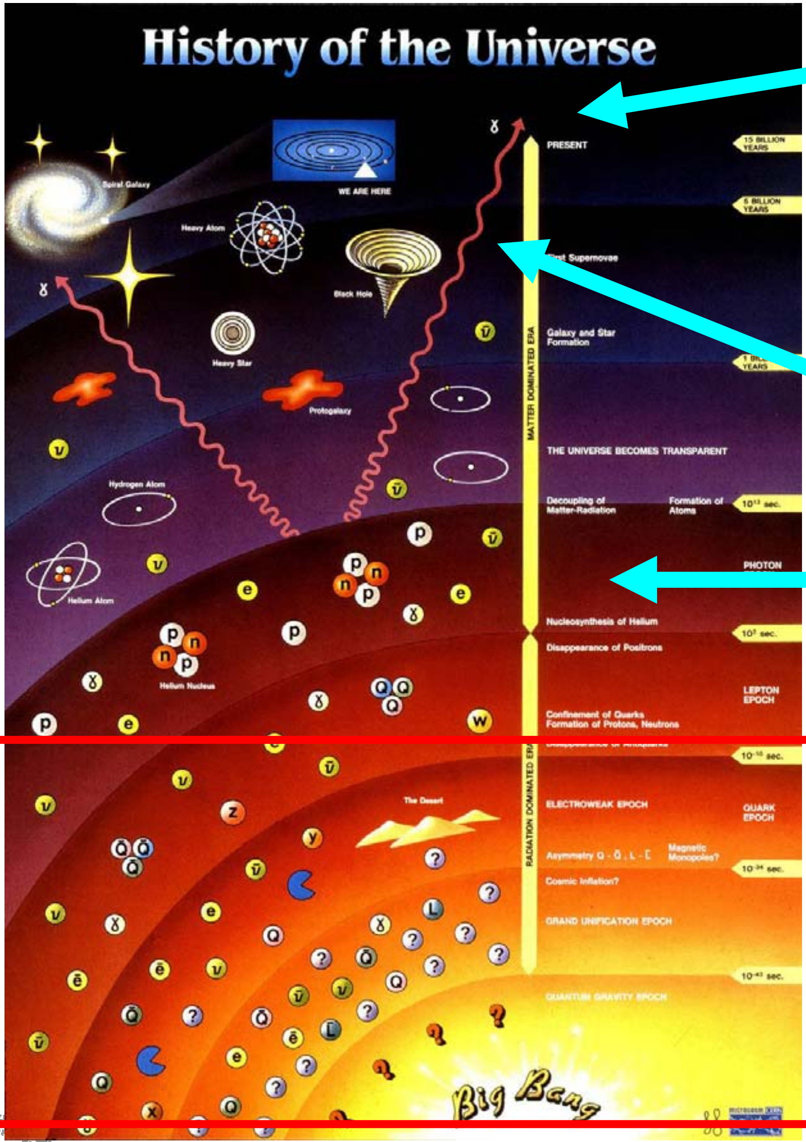
“baryogenesis”, t ≈ 10^{-35} s

(1) non-equilibrium

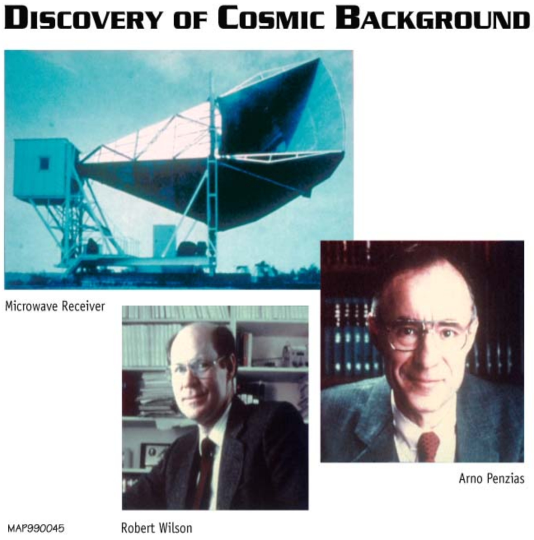
(2) B: not conserved

(3) CP symmetry: violated

Big Bang: strong evidence



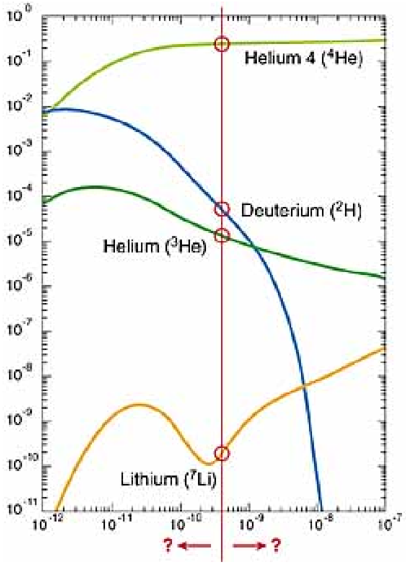
Black-body radiation:
The universe is filled
with (T = 3K) photons



Expanding universe:
speed is measured!
(red shift)

Light elements:

Nucleosynthesis OK



The transition to
Baryon asymmetry
requires an explanation
at the level of
fundamental interactions!

Baryogenesis: Sakharov conditions

- CPT: expectations

$$\text{particle } X \Leftrightarrow \text{anti-particle } \overline{X}, \quad m_X = m_{\overline{X}}\,, \quad \Gamma_X = \Gamma_{\overline{X}}\,, \quad Q_X = -Q_{\overline{X}}$$

$$\Rightarrow \text{expect} \quad n_X = n_{\overline{X}} \ \text{ at thermal equilibrium (but : initial conditions...?)}$$

- Sakharov conditions

  1) B violation

  2) C, CP violation

  3) Off-equilibrium !

$$\text{initially}: \quad B = 0\,, \quad \text{finally}: \quad B \neq 0$$

$$C\text{-invariance} \quad \Rightarrow \quad P(i \to f) = P(\bar{i} \to \bar{f})$$

$$T\text{-invariance} \quad \Rightarrow \quad P(i(\vec{r}_i, \vec{p}_i, \vec{s}_i) \to f(\vec{r}_j, \vec{p}_j, \vec{s}_j)) =$$

$$= P(f(\vec{r}_i, -\vec{p}_i, -\vec{s}_i) \to i(\vec{r}_j, -\vec{p}_j, -\vec{s}_j))$$

- Standard Model of elementary particles and GUT extensions?

  ... Have the basic ingredients, but fail to explain baryogenesis by many orders of magnitude ⇒ “new physics” is required

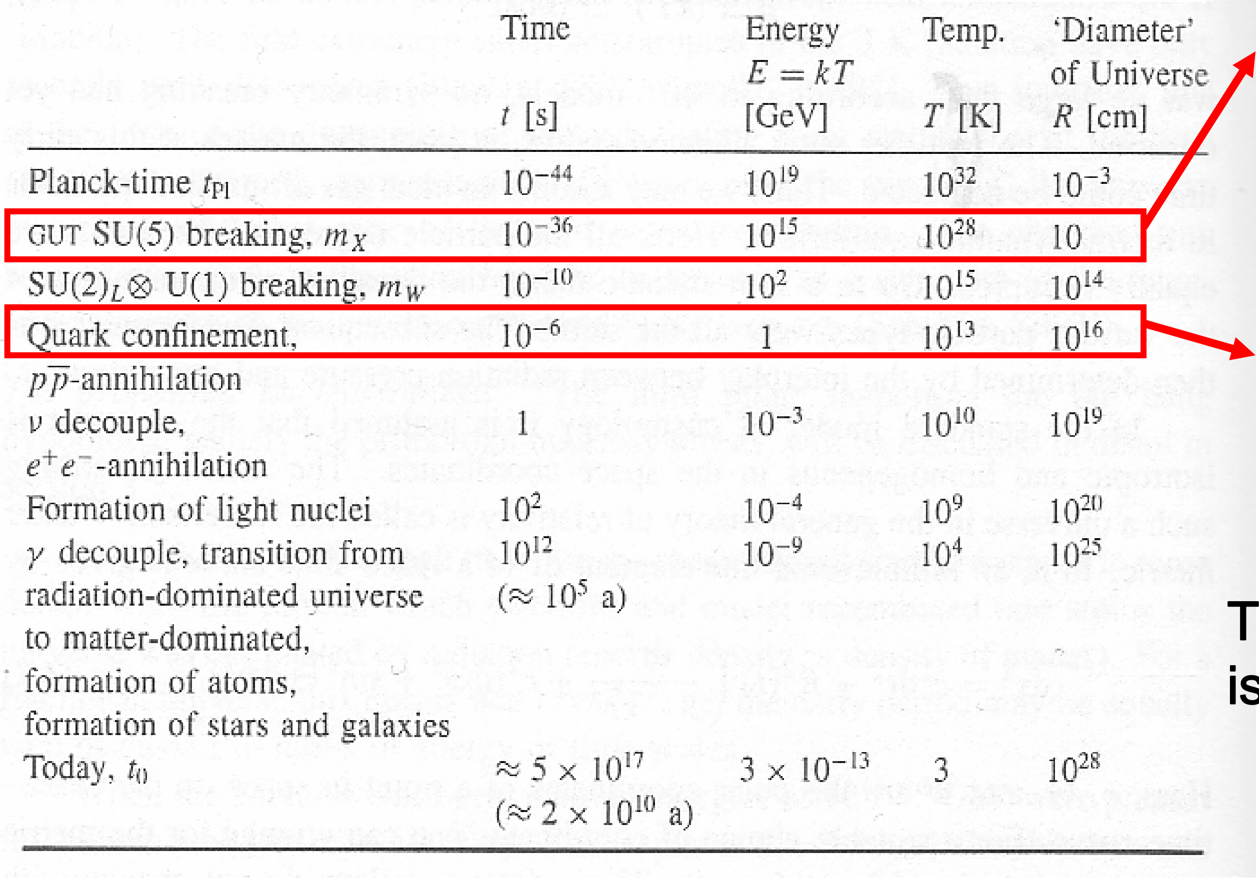
  See: A.Riotto, Theories of Baryogenesis, hep-ph/9807454

Baryogenesis (SM, GUT)

CPV ⇒ different rates for
(off thermal equilibrium)

$$X \to u + u \quad X \to \bar{d} + e^{+} \quad Y \to \bar{d} + \bar{\nu}_e \quad Y \to \bar{u} + e^{+}$$

$$\bar{X} \to \bar{u} + \bar{u} \quad \bar{X} \to d + e^{-} \quad \bar{Y} \to d + \nu_e \quad \bar{Y} \to u + e^{-}$$



	Time t [s]	Energy $E = kT$ [GeV]	Temp. T [K]	'Diameter' of Universe R [cm]
Planck-time t_{Pl}	10^{-44}	10^{19}	10^{32}	10^{-3}
GUT SU(5) breaking, m_X	10^{-36}	10^{15}	10^{28}	10
$SU(2)_L \otimes U(1)$ breaking, m_W	10^{-10}	10^{2}	10^{15}	10^{14}
Quark confinement, $p\bar{p}$-annihilation	10^{-6}	1	10^{13}	10^{16}
ν decouple, $e^{+}e^{-}$-annihilation	1	10^{-3}	10^{10}	10^{19}
Formation of light nuclei	10^{2}	10^{-4}	10^{9}	10^{20}
γ decouple, transition from radiation-dominated universe to matter-dominated, formation of atoms, formation of stars and galaxies	10^{12} ($\approx 10^{5}$ a)	10^{-9}	10^{4}	10^{25}
Today, t_0	$\approx 5 \times 10^{17}$ ($\approx 2 \times 10^{10}$ a)	3×10^{-13}	3	10^{28}

(1) GUT bosons X, Y decay and "decouple": a net quark-antiquark asymmetry remains

(2) Excess quarks form hadrons (baryons)

The predicted asymmetry is too small !

$$\boxed{\frac{n_B}{n_\gamma} \approx \frac{n_q - n_{\bar{q}}}{n_q + n_{\bar{q}}} \approx 10^{-18}}$$

CP Violation!

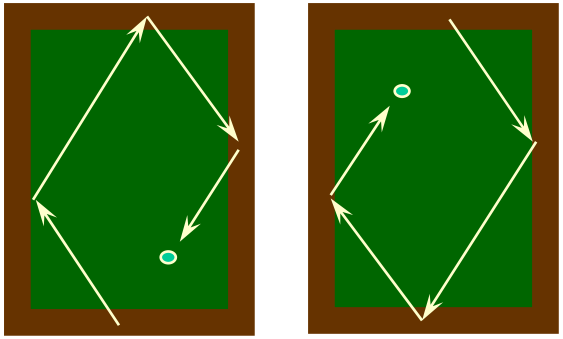
- CP violation has been observed in weak interactions (K and B mesons: mixing and decay)
- The Standard Model (SM) has a recipe for CPV ("CKM mechanism"), but no fundamental understanding of its origin
- Baryogenesis requires additional sources of CPV, beyond SM + GUT, that alone would predict $n_B/n_\gamma \approx 10^{-18}$ rather than the observed $n_B/n_\gamma \approx 10^{-9}$
- CPV is a very interesting probe of fundamental properties both of basic interactions among particles and of the universe evolution

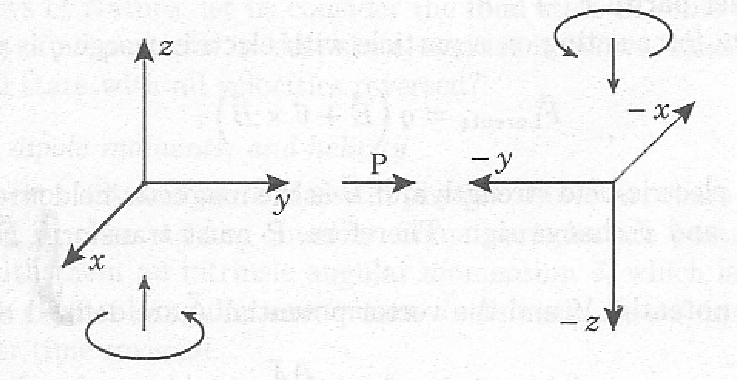
C, P, T: “discrete” symmetries

P and *T* in classical physics

Parity *P*

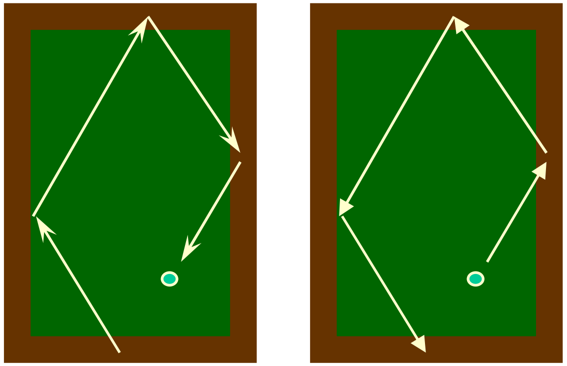
$$\boxed{\vec{r} \leftrightarrow -\vec{r}}$$





Time reversal *T*

$$\boxed{t \leftrightarrow -t}$$



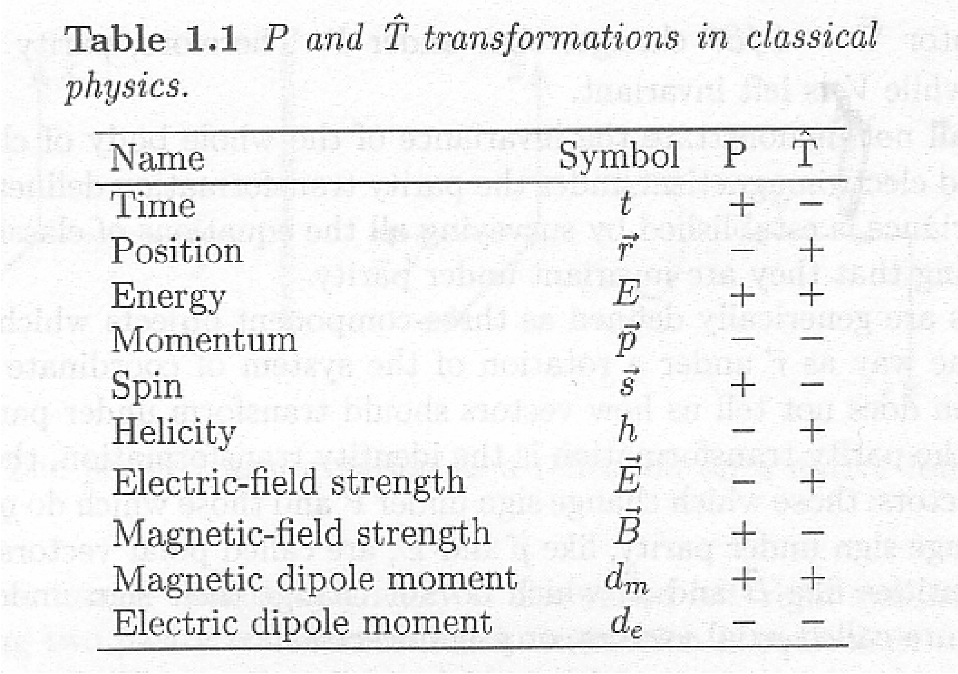
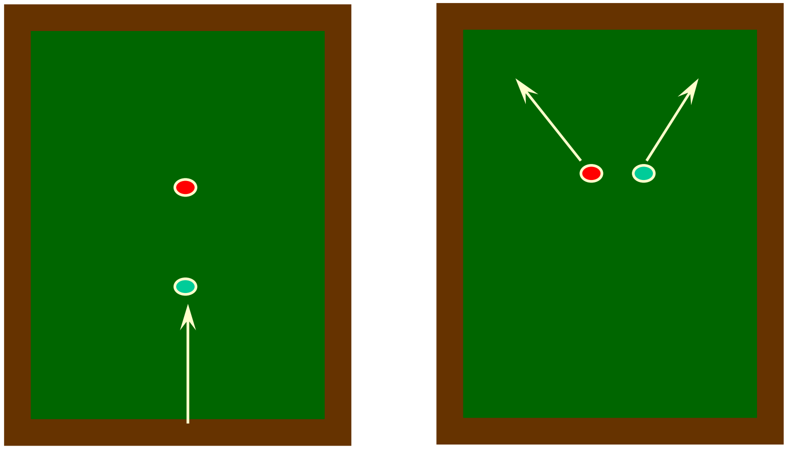


Table 1.1 *P and $\hat{T}$ transformations in classical physics.*

Name	Symbol	P	$\hat{T}$
Time	t	+	−
Position	$\vec{r}$	−	+
Energy	E	+	+
Momentum	$\vec{p}$	−	−
Spin	$\vec{s}$	+	−
Helicity	h	−	+
Electric-field strength	$\vec{E}$	−	+
Magnetic-field strength	$\vec{B}$	+	−
Magnetic dipole moment	d_m	+	+
Electric dipole moment	d_e	−	−

All the equations of classical physics are invariant for *P, T* transformations

What about the “time arrow”?

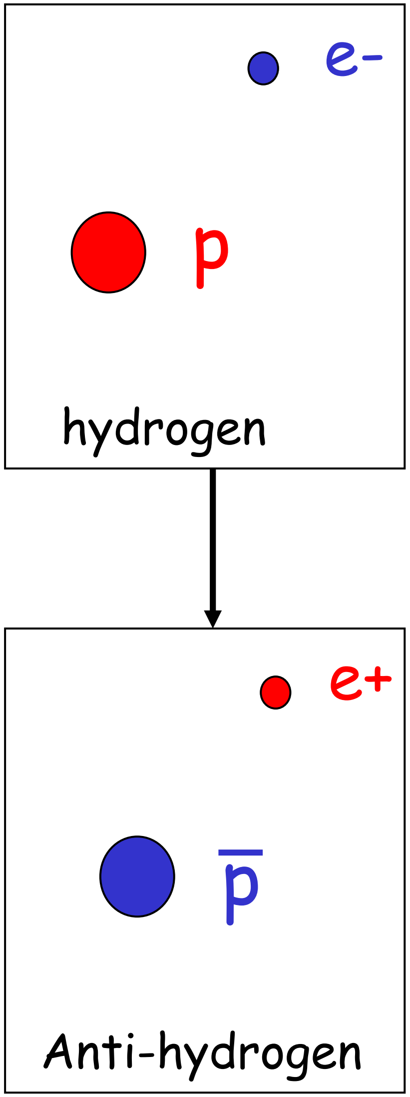


2nd Law of Thermodynamics: non-equilibrium macroscopic systems evolve towards states with higher entropy (configurations microscopically more probable)

This is explained by statistical mechanics and does not imply in any way a time-reversal asymmetry in the fundamental laws of microscopic physics

Charge Conjugation *C*

- Charge conjugation *C* :
  particle ↔ antiparticle
  - No analogue in classical physics
  - Relativistic quantum theory:
    - for every particle there is an anti-particle
    - particle and antiparticle have identical mass and lifetime
    - particle and antiparticle have opposite charges
      - Electric charge, baryon and lepton number, flavour quantum numbers such as strangeness, etc.
- *C* symmetry (if realized) implies that:
  - A system where all the particles are substituted with the corresponding antiparticles behaves as the original system
  - It is a matter of convention which of them we call "particles" and which we call "antiparticles"



C, *P* and *CP*

- Electromagnetic and strong interactions:
  - both *C*- and *P*-symmetric
- Weak interactions
  - violate *maximally* both the *C* and *P* symmetry
  - are *approximately* symmetric for the combined *CP* transformation

h = -1
(L-handed)
OK

h = +1
does NOT
occur

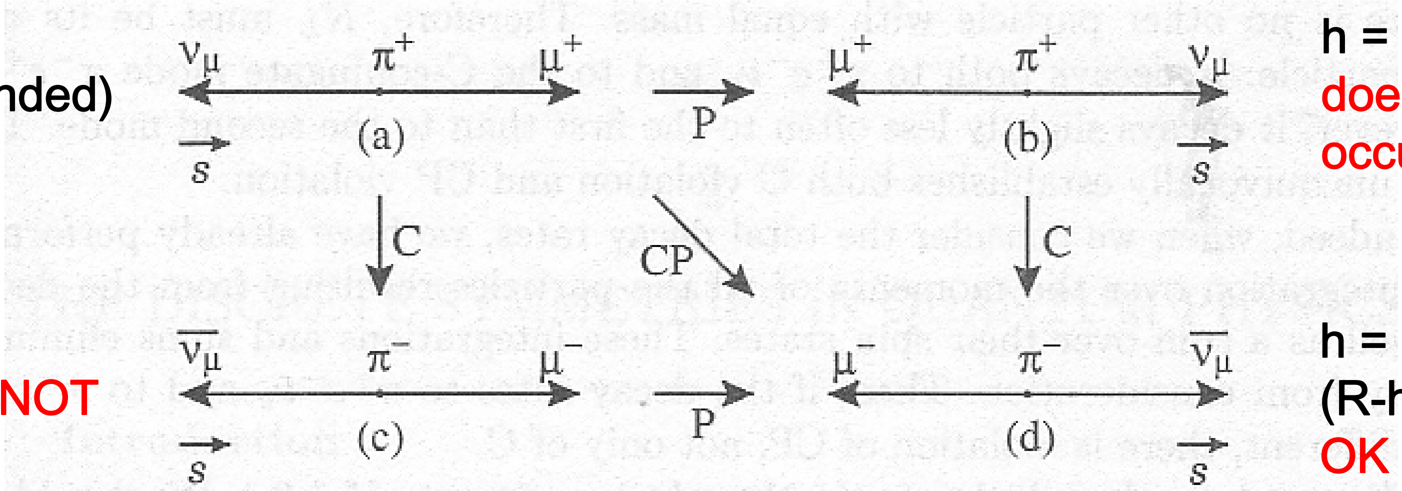


FIG. 1.4. C, P, and CP transformation of the decay $\pi^+ \rightarrow \mu^+\nu_\mu$.

h = -1
does NOT
occur

h = +1
(R-handed)
OK

Quantum Mechanics: *C*, *P*

C, *P* operators applied twice reproduce the initial state (up to a phase)

$$P^2 = 1 \qquad\qquad C^2 = 1$$

$$P^\dagger P = 1 \;\Rightarrow\; P^\dagger = P \qquad\qquad C^\dagger C = 1 \;\Rightarrow\; C^\dagger = C$$

unitary Hermitian (observable) unitary Hermitian (observable)

Eigenstates of *P* (*C*) are characterized by the eigenvalue "parity" ("C-parity"), a multiplicative quantum number

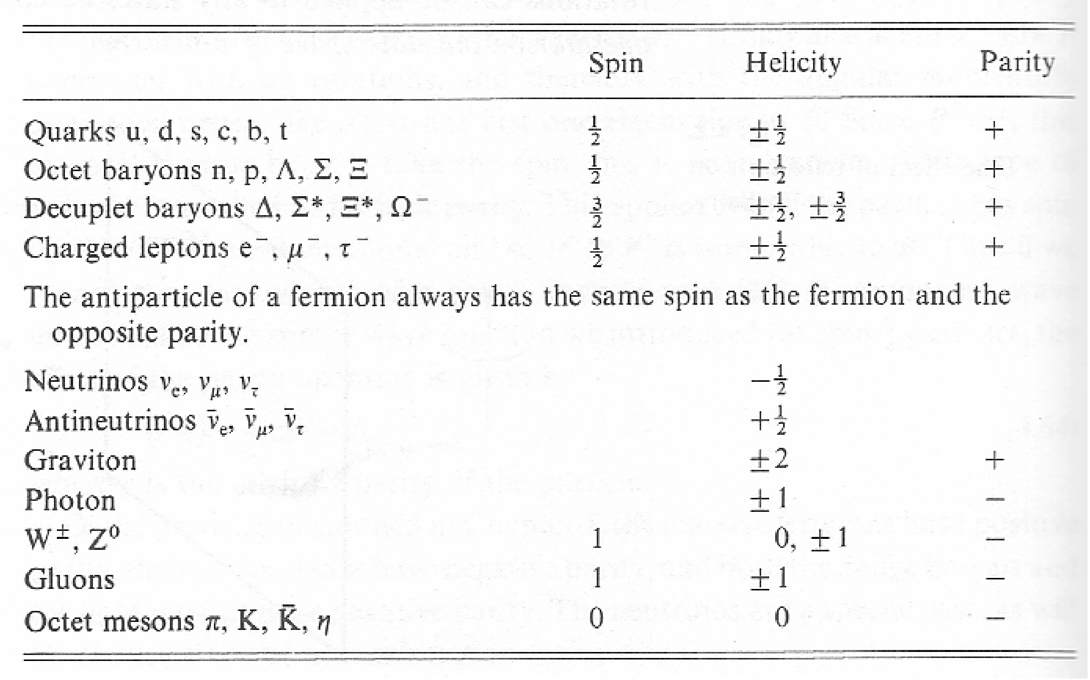
$$\text{if } \;|\psi\rangle\; \text{ is an eigenstate:} \qquad\qquad \text{if } \;|\psi\rangle\; \text{ is an eigenstate:}$$

$$P|\psi\rangle = \eta_P|\psi\rangle = (\pm 1)|\psi\rangle \qquad\qquad C|\psi\rangle = \eta_C|\psi\rangle = (\pm 1)|\psi\rangle$$

NB: not all single particle states are *C* eigenstates! Can you guess which?

“intrinsic” parities

- When single particle states are eigenstates:
  - *P* operator:
“intrinsic parity” $\eta_P = \pm 1$
  - *C* operator:
“intrinsic C-parity” $\eta_C = \pm 1$
- Assignments: conventional for some particles; for the others: parity conservation, angular momentum etc.
- C-parity is only defined for particles that are *“totally neutral”* (all charges = 0, not only the electrical charge)



	Spin	Helicity	Parity
Quarks u, d, s, c, b, t	$\frac{1}{2}$	$\pm\frac{1}{2}$	+
Octet baryons n, p, Λ, Σ, Ξ	$\frac{1}{2}$	$\pm\frac{1}{2}$	+
Decuplet baryons $\Delta, \Sigma^*, \Xi^*, \Omega^-$	$\frac{3}{2}$	$\pm\frac{1}{2}, \pm\frac{3}{2}$	+
Charged leptons e^-, μ^-, τ^-	$\frac{1}{2}$	$\pm\frac{1}{2}$	+
The antiparticle of a fermion always has the same spin as the fermion and the opposite parity.			
Neutrinos ν_e, ν_μ, ν_τ		$-\frac{1}{2}$	
Antineutrinos $\bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau$		$+\frac{1}{2}$	
Graviton		± 2	+
Photon		± 1	−
$W^\pm, Z^0$	1	$0, \pm 1$	−
Gluons	1	± 1	−
Octet mesons $\pi, K, \bar{K}, \eta$	0	0	−

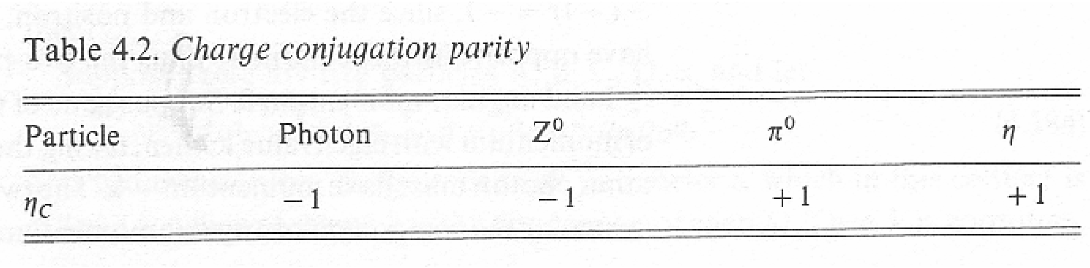


Table 4.2. *Charge conjugation parity*

Particle	Photon	Z^0	π^0	η
η_C	−1	−1	+1	+1

Quantum Mechanics: *T* is "anti-unitary"

- Time reversal in Quantum Mechanics:

  $t \to -t$ and $i \to -i$ (complex conjugation)

  - a simplified argument based on the invariance of the Schrodinger equation justifies the definition of the time reversal operator *T* as:

$$T = U\,K$$

U: Unitary
$t \to -t$

K: complex conjugation of all c-numbers standing on its right

CPT Theorem

A quantum field theory that

1. satisfies Lorentz and space-time translation invariance,
2. posesses a lowest “vacuum” state in its energy spectrum,
3. obeys “microcausality” (commutation or anticommutation relations between all distinct fields),

will be *CPT*-invariant: $[CPT, H] = 0$

Direct consequence:

every particle has the same mass and lifetime as its antiparticle

magnetic moments are equal in size, opposite direction

CPT invariance and *CP* violation ⇒ *T* violation

CP (*T*) Violation: “cherchez la phase”!

If a process is described by *two* amplitudes *with a relative phase*

$$M = A_1 + e^{i\delta} A_2$$

Time reversal *T* in QM:

t→ - t   and   i→ - i (complex conj.)

$$M' = (CP)M = A_1 + e^{-i\delta} A_2$$

CP Violation is a consequence of the interference term:

$$|M|^2 = |A_1 + e^{i\delta} A_2|^2 \ \underset{CP}{\overset{T}{\longrightarrow}}\ |M'|^2 = |A_1 + e^{-i\delta} A_2|^2 \neq |M|^2$$

CPV and Flavour

Historical (experimental) perspective

“strange” K mesons in cosmic rays

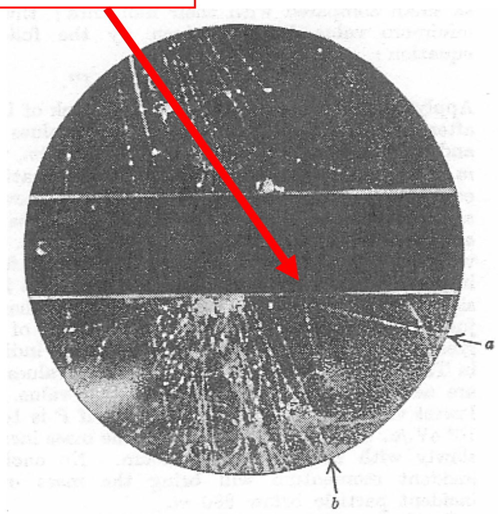
- 1944, L.Leprince-Ringuet & M.Lheritier
- 1947, G.D.Rochester & C.C.Butler [Nature, 160, 855 (1947)]

Cloud chamber exposed to cosmic rays

“V particles”: first evidence of “strange” matter, not present on earth, unstable

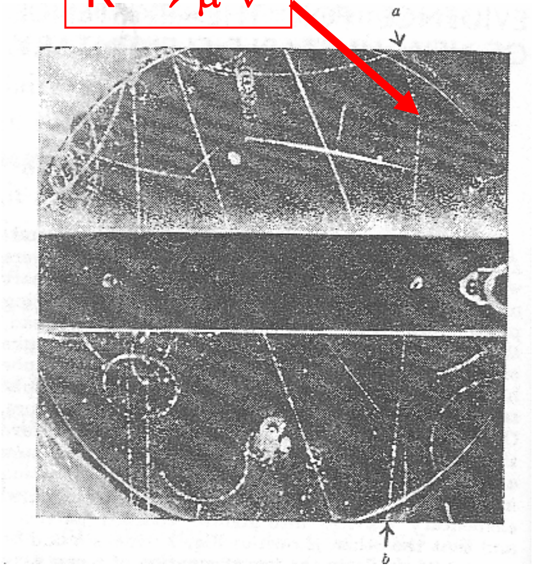
Neutral particle, mass 393 to 818 MeV/c^2

$K^0 \rightarrow \pi^+\pi^-$



Charged particle, mass 500 MeV/c^2 to m_p

$K^+ \rightarrow \mu^+\nu$



Strangeness: K^0 (S = +1), Λ (S = –1)

Experiments at accelerators:

associate "strong" production (strangeness conserving)

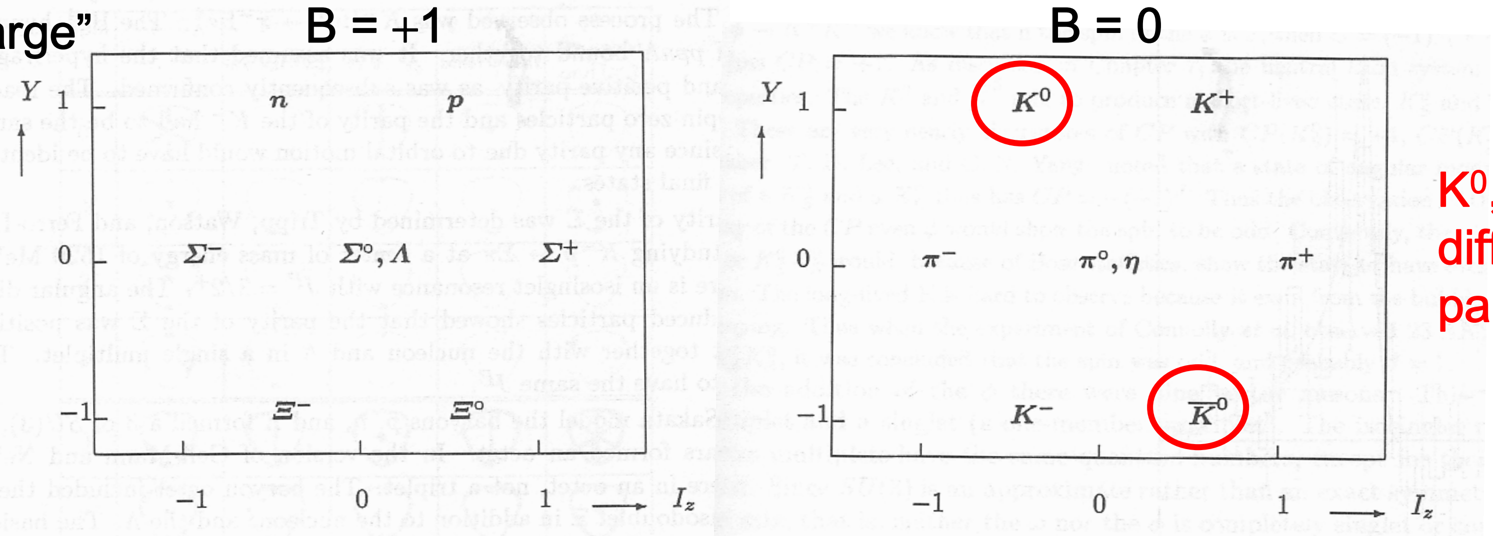
$\sigma\ (\pi^- p \rightarrow K^0\Lambda) \approx 1$ mb $\approx \sigma_{tot} / 40$

"weak" decay (strangeness violating)

$\tau\ (\Lambda \rightarrow \pi^- p) \approx 10^{-10}$ s >> 10^{-23} s

as slow as $\pi^- p \rightarrow \pi^0\Lambda$, strangeness-violating

"hypercharge"
Y = B + S



K^0, $\overline{K}^0$: different particles!

Strangeness: K^0 (S = +1), $\overline{K}^0$ (S = –1)

1955, Gell-Mann & Pais:

predicted *oscillations* and *long-lived neutral K*

Also $\overline{K}^0$ is produced, for instance in $K^- p \to \Lambda K^0 \overline{K}^0$

Equal *m* (CPT), undefined τ: not “physical” states

$K^0 \neq \overline{K}^0$ strong interaction (S) eigenstates, not weak or CP eigenstates

$K^0 \to \pi^+\pi^-$, $\overline{K}^0 \to \pi^+\pi^-$ common weak decays ⇒ coupled!

$K^0 \to \pi^- e^+ \nu$, $\overline{K}^0 \to \pi^+ e^- \overline{\nu}$ different weak “semileptonic” decays
⇒ a superposition can be analyzed!

K_1^0, K_2^0 “physical” (full hamiltonian) eigenstates, different *m*, well-defined τ

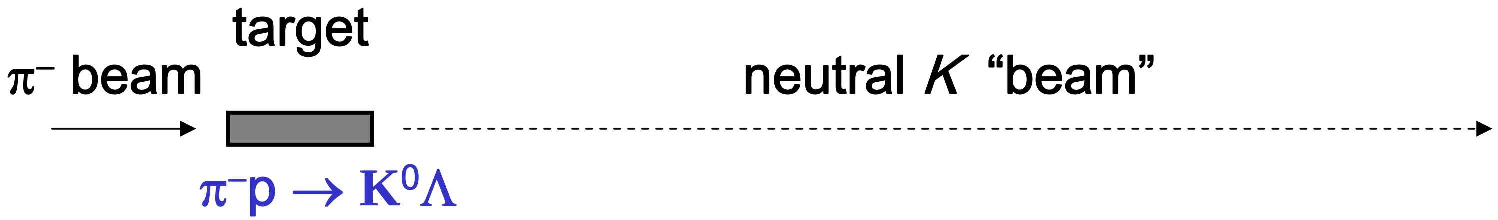
$$K_1^0 = \frac{1}{\sqrt{2}}\left(K^0 + \overline{K}^0\right) \xrightarrow{CP=+1} \pi^+\pi^-$$

relatively short-lived

$$K_2^0 = \frac{1}{\sqrt{2}}\left(K^0 - \overline{K}^0\right) \xrightarrow{CP=-1} \pi^+\pi^-\pi^0$$

relatively long-lived
(less available phase-space)

Quantum Mechanics "laboratory" !



neutral K beam composition:

after a few K_1^0 lifetimes:

"strong" eigenst. $K^0 \dashrightarrow 1/\sqrt{2}\left(K^0 - \overline{K}^0\right)$

"mass" eigenst. $1/\sqrt{2}\left(K_2^0 - K_1^0\right) \dashrightarrow K_2^0$

short-lived K_1^0 component: disappears faster !

observed decays:

semileptonic $\pi^- e^+ \nu \dashrightarrow \pi^- e^+ \nu\left(\approx 50\%\right), \pi^+ e^- \overline{\nu}\left(\approx 50\%\right)$

hadronic $K_1^0 \to \underset{\text{mostly}}{\pi^+\pi^-}, \underset{\text{a few}}{\pi^+\pi^-\pi^0} \dashrightarrow K_2^0 \to \underset{\approx\text{ only}}{\pi^+\pi^-\pi^0}$

Strangeness oscillations: K^0_S & K^0_L

1956, Lande et al. [Phys.Rev.103, 1901 (1956)]:

observation of $K^0_2 \approx K^0_L$ with cloud chamber exposed to the Brookhaven Cosmotron 3-GeV beam

Lifetime estimated in the $10^{-9} - 10^{-6}$ s range ($K^0_1 \approx K^0_S$: 10^{-10} s)

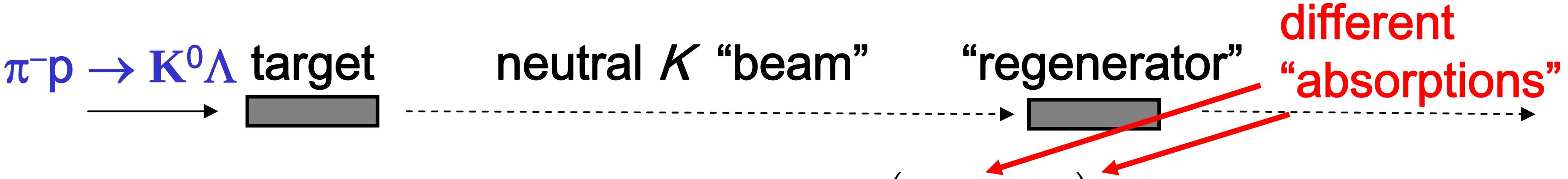
Lifetime, (recently) measured values:

$$\tau_S \equiv 1/\Gamma_S = (8.927 \pm 0.009) \times 10^{-11}\ \text{s}$$

$$\tau_L \equiv 1/\Gamma_L = (5.17 \pm 0.04) \times 10^{-8}\ \text{s}$$

factor 600 !

1955, Pais & Piccioni: predict coherent "regeneration" of a K^0_1 component in a beam of K^0_2 going through a target (matter)



K composition: $K^0 \dashrightarrow K^0_2 \propto \left(K^0 - \overline{K}^0\right) \qquad \approx K^0_2 + rK^0_1$

decays: $2\pi \dashrightarrow 3\pi \qquad 3\pi, 2\pi$

Regeneration of K^0_S …

1960, F.Muller et al [Phys.Rev.Lett. 4, 418 (1960)]: Regeneration and Mass Difference of Neutral K Mesons

- Beginning of systematic study of regeneration
- Different materials and thicknesses
- Regeneration depends also on Δm $\Rightarrow \Delta m$ measurement

… paved also the way for the discovery of CP Violation:

- in some experiments there were hints of “anomalous” regeneration (2π decays of K_2^0), possibly compatible with backgrounds

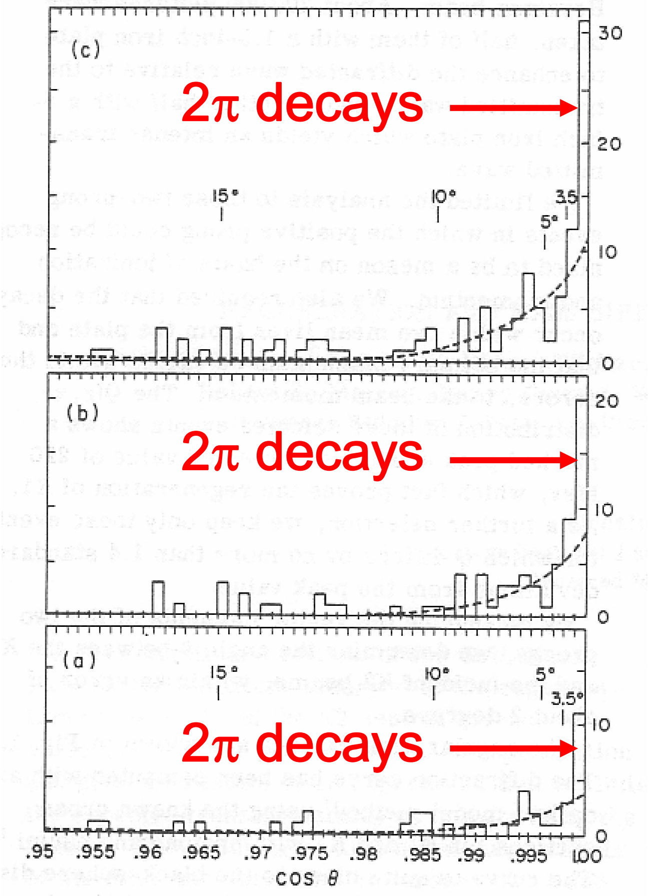
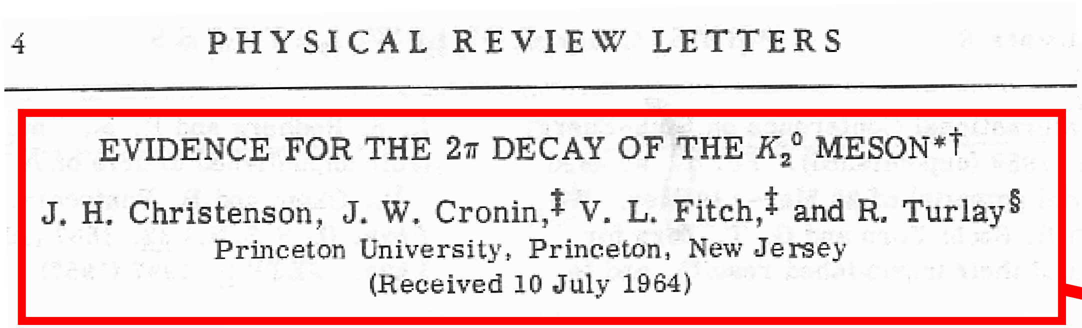


FIG. 1. Histograms of number of $K1$ decay events per 0.001 interval of cosθ (θ is the angle between the direction of the primary $K2$ beam and the regenerated $K1$). (a) Data for the 1.5-inch plate; (b) data for the 6-inch plate; (c) combined data for the two plates. The curves are diffraction angular distributions normalized in the 0.980 to 0.998 interval for cos θ.

… and the discovery of CP violation



4 PHYSICAL REVIEW LETTERS

EVIDENCE FOR THE 2π DECAY OF THE K_2^0 MESON*†

J. H. Christenson, J. W. Cronin,‡ V. L. Fitch,‡ and R. Turlay§
Princeton University, Princeton, New Jersey
(Received 10 July 1964)

- 1964, Christenson et al.
  - Careful control of material to subtract regeneration background
  - Double-arm spectrometer, spark chambers

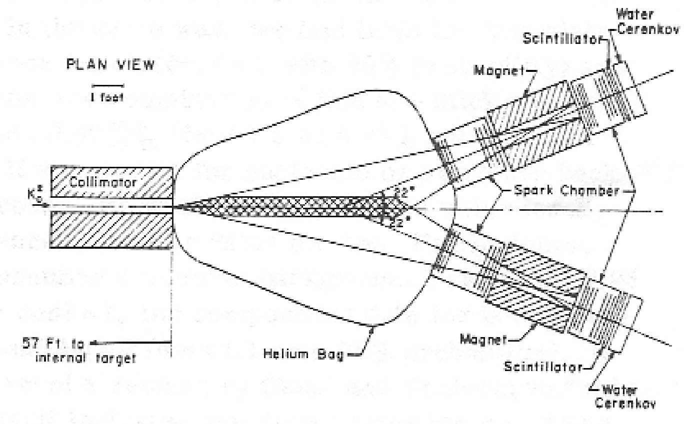


FIG. 1. Plan view of the detector arrangement.

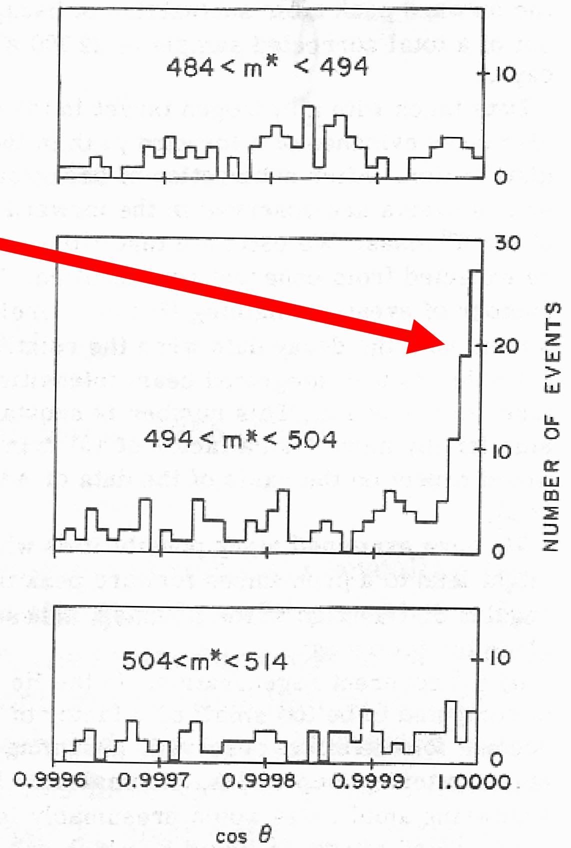


FIG. 3. Angular distribution in three mass ranges for events with $\cos\theta > 0.9995$.

Kobayashi and Maskawa

2 families of 2 quarks:
4 couplings
2x2 unitary matrix:
No phase!

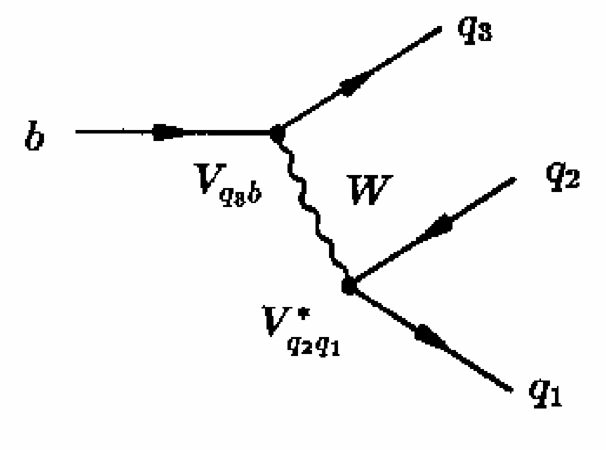
↓

To obtain a phase
(source of CPV):

3 families of 2 quarks!
9 couplings
3x3 unitary matrix

Discovered!

- couplings V_{qq} among up and down quarks (3 families): 9 complex numbers, may have an irreducible phase



$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ \bigcirc V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

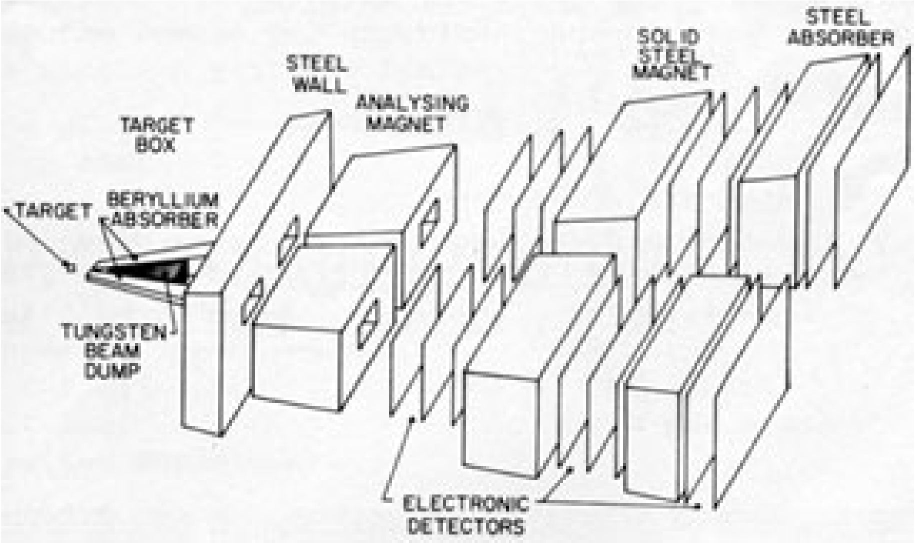
- phase $\Rightarrow$ CP violation

Theoretical predictions

- Kobayashi & Maskawa: CP violation can be explained by a phase in the quark mixing matrix, but then a third family of quarks is required!

- Later, after B meson discovery: Carter, Bigi & Sanda suggested that, if the SM explanation of CP Violation effects is correct, then large CP asymmetries should be seen in “rare” B decays

Experiment: from the discovery of b quarks…

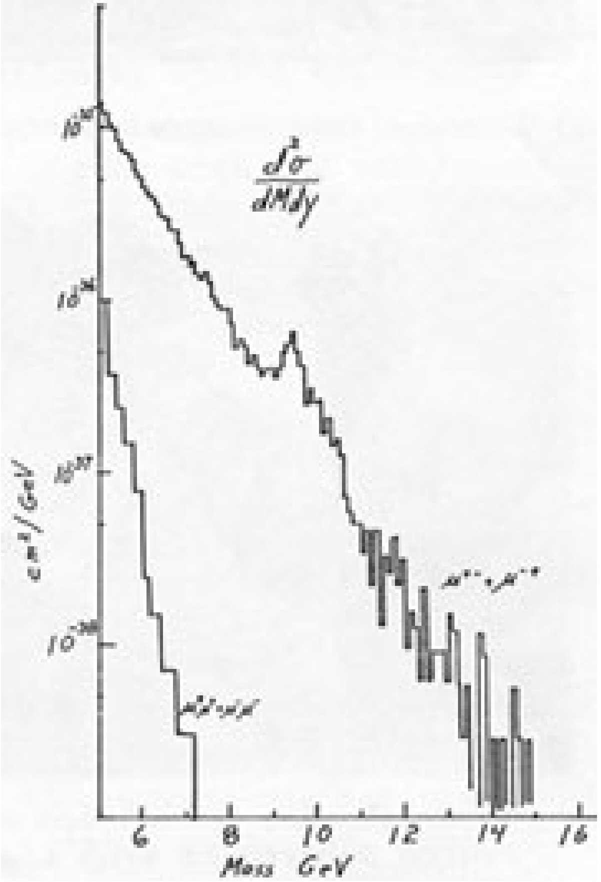


Summer 1977 at FNAL:

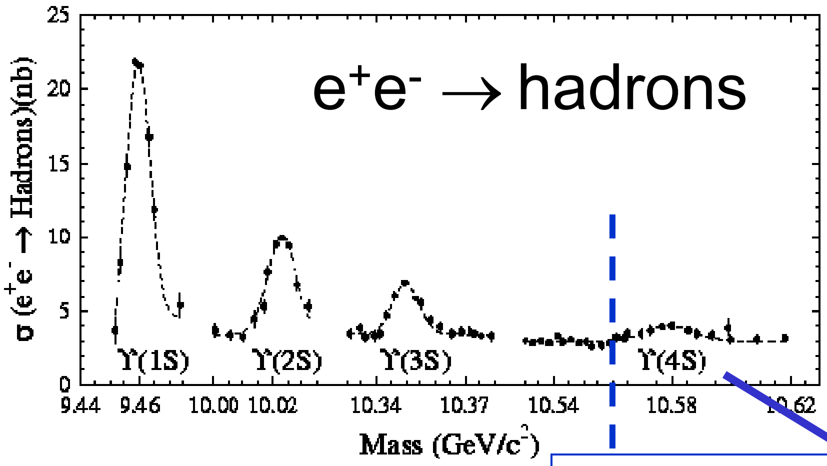
Discovery of $\Upsilon(9.46) \rightarrow \mu^+\mu^-$

interpreted as $1^3S_1\ b\bar{b}$

"Observation of a Dimuon Resonance at 9.5 GeV in 400 GeV Proton-Nucleus Collisions,"
PRL 39, p. 252, (1977)



… and the discovery of B=($\bar{b}$q) mesons…



B$\bar{B}$ threhsold

CESR at Cornell:
"naked beauty"

$$e^+e^- \rightarrow \Upsilon(4S) \rightarrow B\bar{B}$$

CLEO Collaboration, "Observation of Exclusive Decay Modes of b-Flavored Mesons", *PRL* 50, p. 881 (1983)

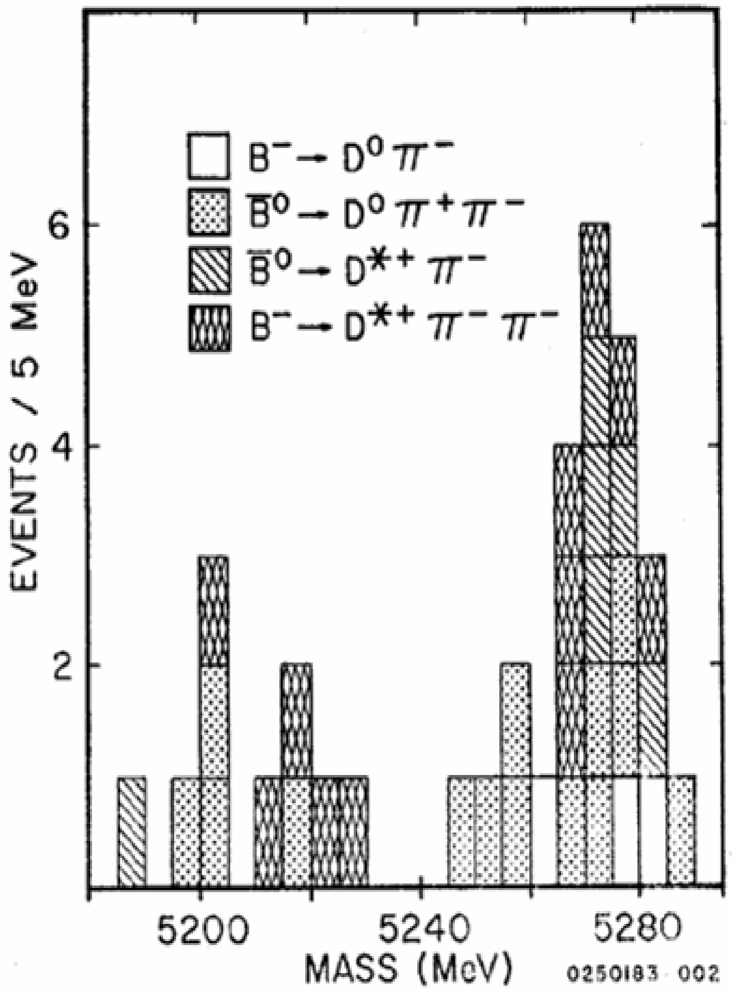
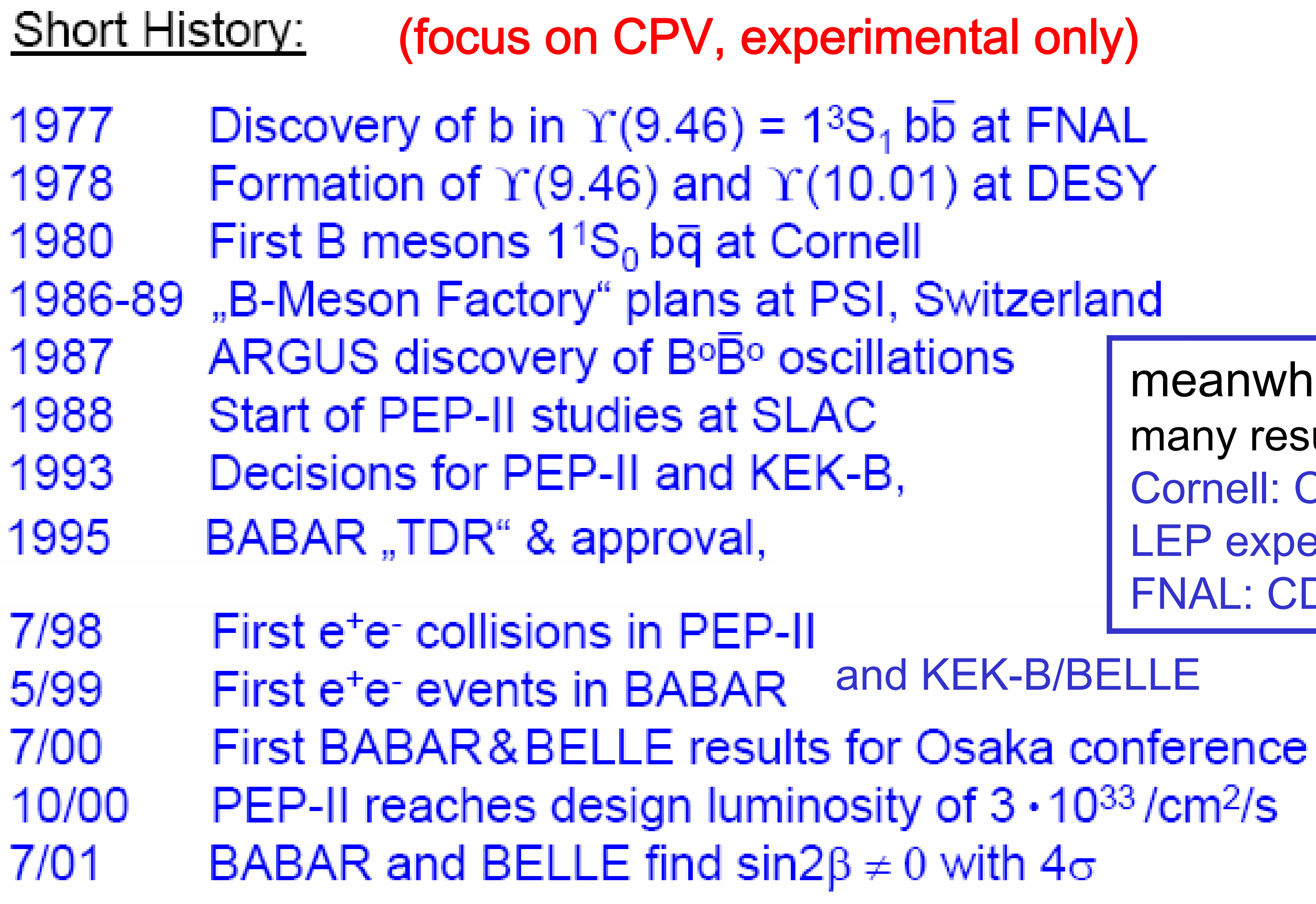


FIG. 2

…to the birth of the “B-factories”…



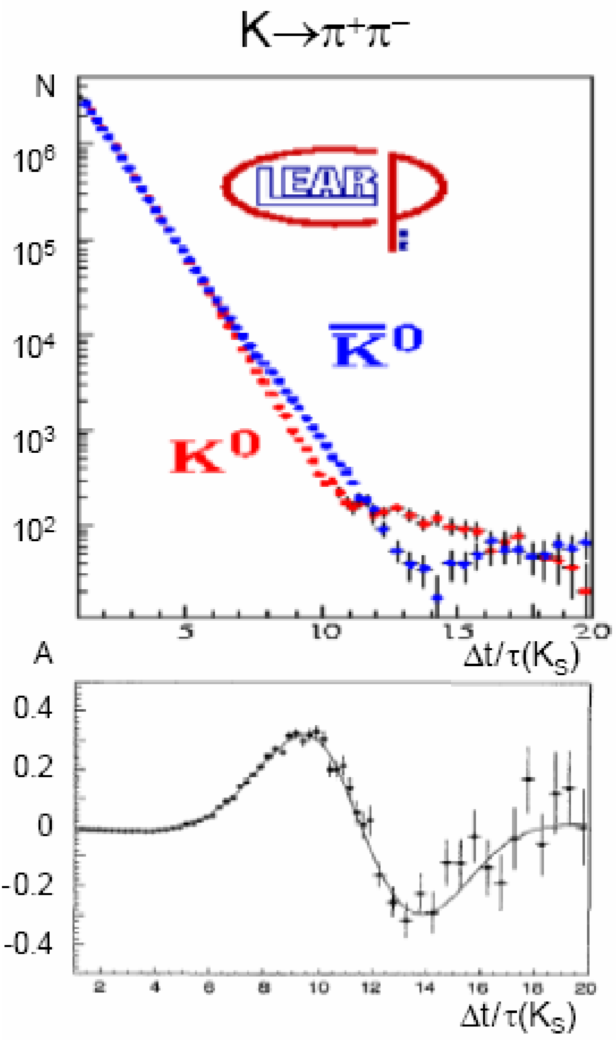
Short History: (focus on CPV, experimental only)

1977	Discovery of b in $\Upsilon(9.46) = 1^3S_1\ b\bar{b}$ at FNAL
1978	Formation of $\Upsilon(9.46)$ and $\Upsilon(10.01)$ at DESY
1980	First B mesons $1^1S_0\ b\bar{q}$ at Cornell
1986-89	„B-Meson Factory“ plans at PSI, Switzerland
1987	ARGUS discovery of $B^0\bar{B}^0$ oscillations
1988	Start of PEP-II studies at SLAC
1993	Decisions for PEP-II and KEK-B,
1995	BABAR „TDR“ & approval,
7/98	First e^+e^- collisions in PEP-II
5/99	First e^+e^- events in BABAR and KEK-B/BELLE
7/00	First BABAR&BELLE results for Osaka conference
10/00	PEP-II reaches design luminosity of $3 \cdot 10^{33}\ /cm^2/s$
7/01	BABAR and BELLE find $\sin 2\beta \neq 0$ with 4σ

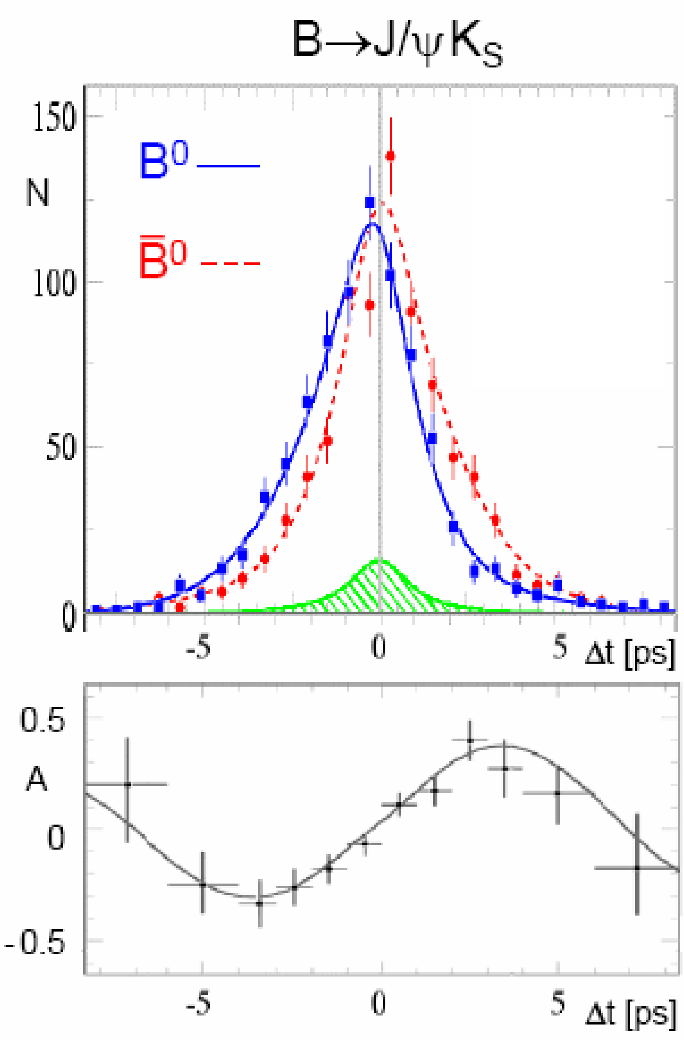
meanwhile:
many results from
Cornell: CLEO,
LEP experiments,
FNAL: CDF, D0

…and the observation of CPV in the B_d system!

K mesons



B_d mesons



plot from *BABAR* similar result from *BELLE*

…this is our starting point

Charged and neutral pseudoscalar mesons: a reminder

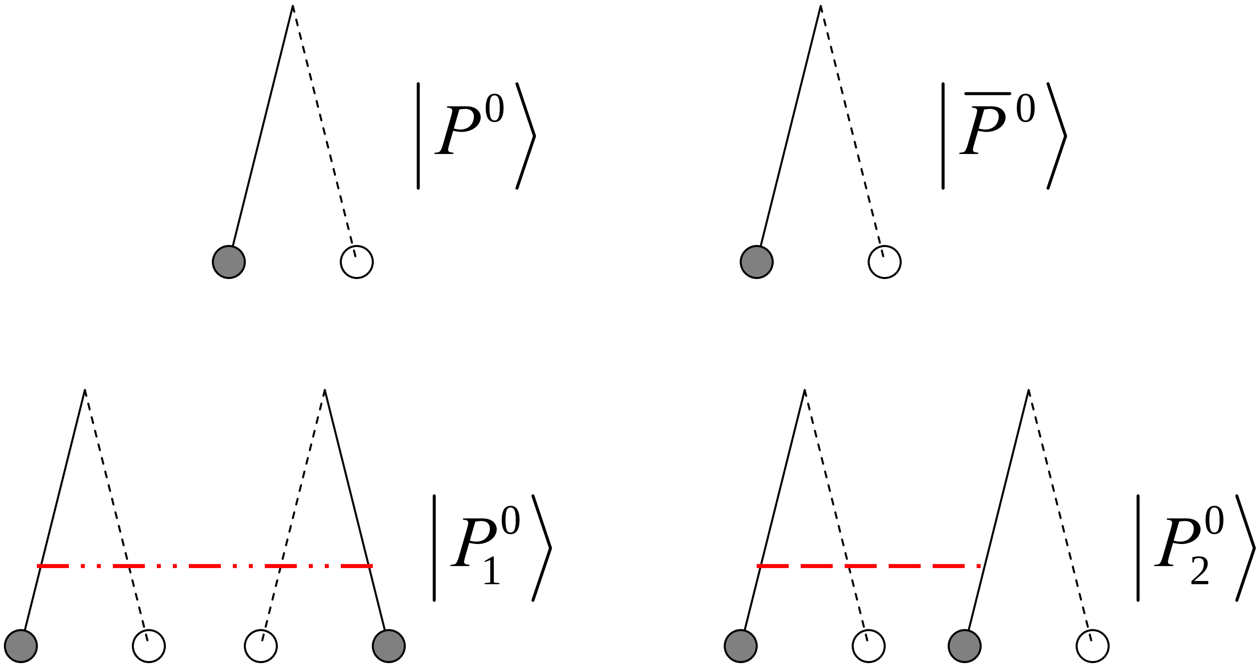
Decays and mixing

Time evolution and CP-violating observables

Theoretical interpretation: CKM

Mixing and decays

Coupled oscillators, with damping



Coupling ⇒ frequency (energy, mass) splitting (2 “normal modes”)
Damping ⇒ one of the two modes lasts longer…

Decays

Charged and neutral pseudoscalar mesons (*P = K, D, B*)
Some examples (we will start by discussing B mesons, in particular):

$$K^0 = (\bar{s}d) \qquad B_d^0 = (\bar{b}d) = B^0 \qquad B^+ = (\bar{b}u)$$

$$D^0 = (c\bar{u}) \qquad B_s^0 = (\bar{b}s) \qquad B^- = (b\bar{u})$$

Decay amplitudes for $P \rightarrow f$ and CP-conjugated states:

$$A_f = \langle f|H|P\rangle, \quad \bar{A}_f = \langle f|H|\bar{P}\rangle$$

$$A_{\bar{f}} = \langle \bar{f}|H|P\rangle, \quad \bar{A}_{\bar{f}} = \langle \bar{f}|H|\bar{P}\rangle$$

Decays and mixing: example from *K*

decay = “damping”

mixing = “coupling”

“tree”

“penguin”

“box”

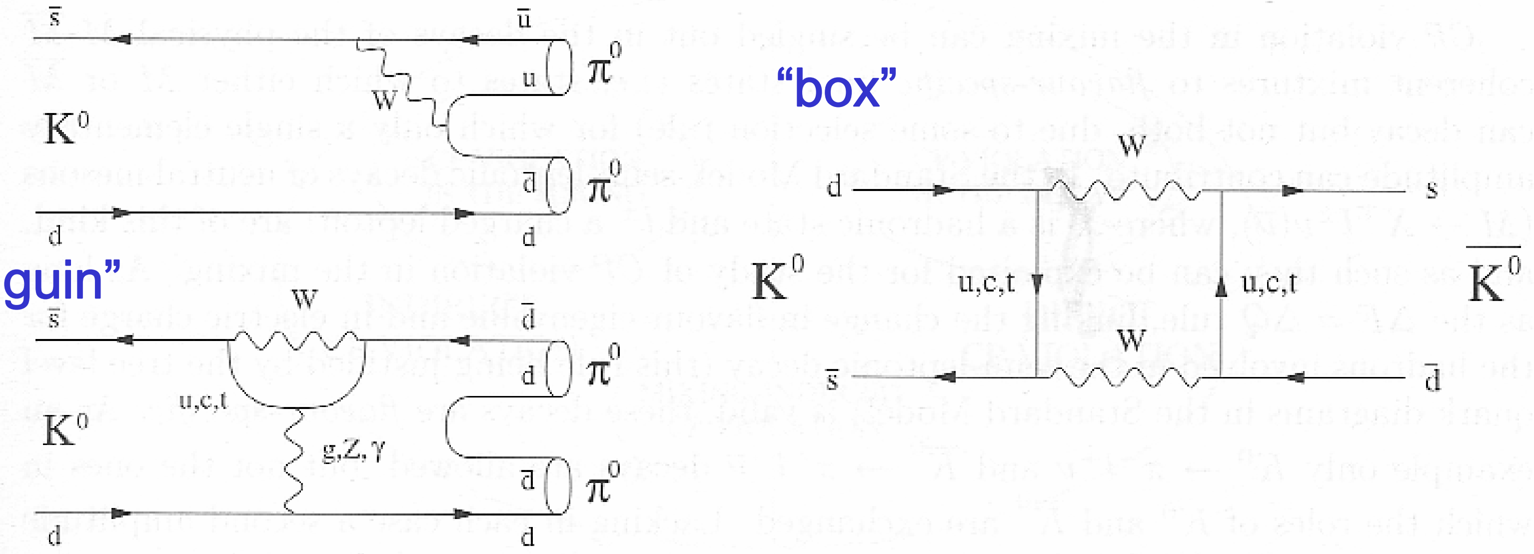


Fig. 1. – Left: the “tree” and “penguin” diagrams originating $K^0 \rightarrow \pi^+\pi^-$ decays in the Standard Model. Right: The “box” diagram originating K^0-$\overline{K}^0$ transitions in the Standard Model.

Mixing for *P = K, D, B*

Problem: find the time evolution of a neutral pseudoscalar meson P:

t = 0: superposition of strong “flavour” eigenstates (P^0, $\overline{P}^0$)

t > 0: also states n_1, n_2, n_3, … to which P may decay

$$\left|\psi(t)\right\rangle = a(t)\left|P^0\right\rangle + b(t)\left|\overline{P}^0\right\rangle + c_1(t)\left|n_1\right\rangle + c_2(t)\left|n_2\right\rangle + c_3(t)\left|n_3\right\rangle + \ldots$$

Effective Hamiltonian approximation:

“dispersive”

“absorptive”

$$i\frac{d}{dt}\begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = H\begin{pmatrix} a(t) \\ b(t) \end{pmatrix}; \quad P^0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \overline{P}^0 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}; \quad H_{ij} = \overset{\downarrow}{M_{ij}} - i\Gamma_{ij}/2$$

non-hermitian

hermitian

Strategy:

Basis: “flavour” eigenstates, unperturbed “strong” hamiltonian

Do not try to compute $c_1(t)$, $c_2(t)$, …: only a(t) and b(t)

⇒ two-component wave function; hamiltonian in 2nd order perturbation theory

⇒ use proper time t (particle rest-frame)

⇒ find “mass eigenstates”, that evolve as “physical states” (…) ⇒ diagonalize H !

Hamiltonian & Perturbation Theory …

What is the matrix H_{ij}? From 2nd order perturbation theory:

$$M_{ij} = m_0\delta_{ij} + \langle i|H_W|j\rangle + \sum_n P\frac{\langle i|H_W|n\rangle\langle n|H_W|j\rangle}{m_0 - E_n}$$

H_W: weak perturbation
n: intermediate
virtual states

$$\Gamma_{ij} = 2\pi\sum_n \delta(m_0 - E_n)\langle i|H_W|n\rangle\langle n|H_W|j\rangle$$

n: physical states
to which both can decay

⇒ complex numbers, to be evaluated by the theory of weak int.
⇒ Assuming CPT invariance, they reduce to 2 real and 2 complex

$$CPT \Rightarrow M_{11} = M_{22} = m_0\ ; \quad \Gamma_{11} = \Gamma_{22} = \gamma$$

diagonal: real,
P^0 mass and lifetime

$$\text{hermiticity} \Rightarrow M_{21} = M_{12}^*\ ; \qquad \Gamma_{21} = \Gamma_{12}^*$$

off-diagonal: complex,
represent mixing
(off- and on-shell states)

General formalism, including CPT violation: see i.e. *Kirkby & Nir, PDG*

Diagonalization: eigenvalues

In the “flavour eigenstates” basis:

$$H = \begin{pmatrix} m_0 - \frac{i}{2}\gamma & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & m_0 - \frac{i}{2}\gamma \end{pmatrix} = \begin{pmatrix} \mu_0 & p^2 \\ q^2 & \mu_0 \end{pmatrix} \Rightarrow$$

Diagonal form, basis: “mass eigenstates”

$$H' = \begin{pmatrix} \mu_1 & 0 \\ 0 & \mu_2 \end{pmatrix}$$

Secular equation, giving the eigenvalues

$$\det\begin{pmatrix} \mu_0 - \mu & p^2 \\ q^2 & \mu_0 - \mu \end{pmatrix} = 0 \Rightarrow$$

$$\Rightarrow \mu_{1,2} = \mu_0 \pm pq = m_0 - \frac{i}{2}\gamma \pm \sqrt{\left(M_{12} - \frac{i}{2}\Gamma_{12}\right)\left(M_{12}^* - \frac{i}{2}\Gamma_{12}^*\right)}$$

Mass and width differences (*H = “heavy”, L = “light”*):

$$\Delta\mu = \Delta m - \frac{i}{2}\Delta\Gamma = 2pq \qquad \Delta m^2 - 1/4\,\Delta\Gamma^2 = 4|M_{12}|^2 - |\Gamma_{12}|^2$$

$$\Delta m = m_H - m_L \quad \Delta\Gamma = \Gamma_H - \Gamma_L \qquad \Delta m \Delta\Gamma = 4\Re\mathrm{e}\left(M_{12}\Gamma_{12}^*\right)$$

Diagonalization: “mass” eigenstates

“mass eigenstates” (P_L, P_H) ≈ CP eigenstates (P_1, P_2): expressed in terms of “flavour eigenstates” (P^0, $\overline{P}^0$)

$$\left|P_L^0\right\rangle = p\left|P^0\right\rangle + q\left|\overline{P}^0\right\rangle = \frac{1}{\sqrt{1+\left|\tilde{\varepsilon}\right|^2}}\left(\tilde{\varepsilon}\left|P_1\right\rangle + \left|P_2\right\rangle\right)$$

$$\tilde{\varepsilon} = \frac{p-q}{p+q}$$

(complex !)

$$\left|P_H^0\right\rangle = p\left|P^0\right\rangle - q\left|\overline{P}^0\right\rangle = \frac{1}{\sqrt{1+\left|\tilde{\varepsilon}\right|^2}}\left(\left|P_1\right\rangle + \tilde{\varepsilon}\left|P_2\right\rangle\right)$$

$$|q|^2 + |p|^2 = 1$$

CP eigenstates (P_1, P_2):

$$\left|P_1^0\right\rangle = \frac{1}{\sqrt{2}}\left|P^0\right\rangle + \frac{1}{\sqrt{2}}\left|\overline{P}^0\right\rangle$$

$$\left|P_2^0\right\rangle = \frac{1}{\sqrt{2}}\left|P^0\right\rangle - \frac{1}{\sqrt{2}}\left|\overline{P}^0\right\rangle$$

$$\frac{q}{p} = \sqrt{\frac{2M_{12}^* - i\Gamma_{12}^*}{2M_{12} - i\Gamma_{12}}} = \frac{\Delta\mu}{2M_{12} - i\Gamma_{12}}$$

$$\delta = |p|^2 - |q|^2 = \left\langle P_L \middle| P_H \right\rangle$$

$$|p|^2 = \frac{1+\delta}{2} \qquad |q|^2 = \frac{1-\delta}{2}$$

CP symmetry ⇒ “mass” = “CP” eigenstates, $\tilde{\varepsilon} = \delta = 0$

Time evolution

(assuming CPT as a good symmetry, for simplicity)…

Time evolution of the “physical” mass eigenstates:

$$\left|P_L^0(t)\right\rangle = e^{-t\Gamma_L/2}\ e^{-itm_0}\ e^{+it\Delta m/2}\left|P_L^0(0)\right\rangle$$

$$\left|P_H^0(t)\right\rangle = e^{-t\Gamma_H/2}\ e^{-itm_0}\ e^{-it\Delta m/2}\left|P_H^0(0)\right\rangle$$

If at t = 0 the state is not a mass eigenstate but some superposition of them (for instance: a flavour state *P^0*), then the time evolution is simply the corresponding appropriate combination

Dimensionless parameters

Taking $\hbar$ = c =1, these quantities can be expressed using the same units (for example, MeV or s^{-1}):

$$\Gamma \equiv \frac{\Gamma_L + \Gamma_H}{2}, \quad \Gamma_L \equiv \frac{1}{\tau_L}, \quad \Gamma_H \equiv \frac{1}{\tau_H}, \quad \Delta\Gamma \equiv \Gamma_H - \Gamma_L , \quad \Delta m \equiv m_H - m_L$$

The following dimensionless parameters often appear in time evolution equations:

$$x \equiv \frac{\Delta m}{\Gamma}$$ related to oscillations “frequency”

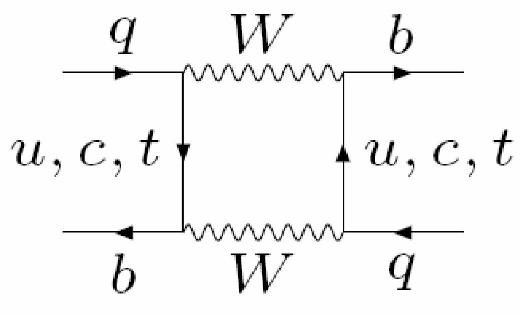
$$y \equiv \frac{\Delta\Gamma}{2\Gamma}$$ related to oscillations “damping”

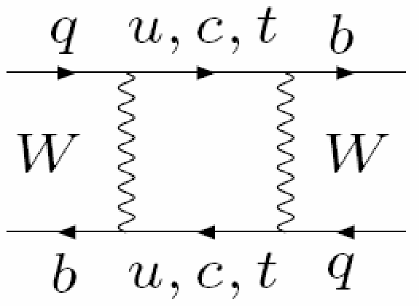
B mesons: time evolution and CP-violating observables

q, p, Δm and $\Delta\Gamma$ for B_d and B_s

$B_d^0 = (\bar{b}\, d)$

$B_s^0 = (\bar{b}\, s)$





$\bar{B}_d^0 = (b\bar{d})$

$\bar{B}_s^0 = (b\bar{s})$

In the SM for B mesons:

M_{12} dominated by the top quark

Γ_{12} few common on-shell states

$$\Gamma_{12}/M_{12} << 1$$

$$\Rightarrow \Delta m \approx 2|M_{12}| \qquad \Delta\Gamma \approx \frac{2\Re\mathrm{e}(M_{12}\Gamma_{12}^*)}{|M_{12}|} << \Delta m \qquad \frac{q}{p} = -\frac{\Delta m - i/2\Delta\Gamma}{2M_{12} - i\Gamma_{12}} \approx -\frac{|M_{12}|}{M_{12}}$$

CP-violating parameter: $\delta = |p|^2 - |q|^2 = \langle P_H | P_L \rangle = \dfrac{2\Im\mathrm{m}(M_{12}^*\Gamma_{12})}{(\Delta m)^2 + |\Gamma_{12}|^2} \approx 10^{-3}$

Time evolution of neutral *B* mesons - 1

(assuming CPT as a good symmetry, for simplicity)…

Time evolution of mass eigenstates:

$$\left|B_L^0(t)\right\rangle = e^{-t\Gamma_B/2}\ e^{-itM_B}\ e^{+it\Delta m_B/2}\left|B_L^0(0)\right\rangle$$

$$\left|B_H^0(t)\right\rangle = e^{-t\Gamma_B/2}\ e^{-itM_B}\ e^{-it\Delta m_B/2}\left|B_H^0(0)\right\rangle$$

Time evolution of initially (t=0) pure flavour eigenstates:

$$\left|B_{phys}^0(t)\right\rangle = h_+(t)\left|B^0\right\rangle + \frac{q}{p}h_-(t)\left|\overline{B}^0\right\rangle$$

$$\left|\overline{B}_{phys}^0(t)\right\rangle = \frac{p}{q}h_-(t)\left|B^0\right\rangle + h_+(t)\left|\overline{B}^0\right\rangle$$

$$h_+(t) = e^{-t\Gamma_B/2}\ e^{-itM_B}\ \cos(t\,\Delta m_B/2)$$

$$h_-(t) = i\left[e^{-t\Gamma_B/2}\ e^{-itM_B}\ \sin(t\,\Delta m_B/2)\right]$$

Time evolution of neutral B mesons - 2

Flavour oscillations: for initially pure $B^0(t=0)$,
probability for finding $B^0\left(\overline{B}^0\right)$ at time t, assuming $|q/p|=1$

$$|h_\pm(t)|^2 = \frac{1}{2} e^{-t\Gamma_B} \left[1 \pm \cos(t\,\Delta m_B)\right] \quad \Rightarrow \quad a_{mix}(t) = \cos(t\,\Delta m) = \cos(x\Gamma t)$$

Time-integrated ratio and time-integrated oscillation probability:

$$r = \frac{N\left(\overline{B}^0\right)}{N\left(B^0\right)} = \frac{\int_0^\infty dt\,|h_-(t)|^2}{\int_0^\infty dt\,|h_+(t)|^2} = \frac{x^2}{2+x^2}, \quad \chi = \frac{r}{1+r} = P\left(B^0 \to \overline{B}^0\right), \quad x \equiv \frac{\Delta m}{\Gamma}$$

Observable by looking at self-flavour tagging semileptonic or hadronic decays! For example:

$$B^0 \to D^{*-} l^+ \nu \qquad \overline{B}^0 \to D^{*+} l^- \overline{\nu}$$

$$B^0 \to D^- \pi^+ \qquad \overline{B}^0 \to D^+ \pi^-$$

$$B_s^0 \to D_s^- l^+ \nu \qquad \overline{B}_s^0 \to D_s^+ l^- \overline{\nu}$$

Time evolution of neutral *B* mesons - 3

(assuming CPT as a good symmetry, for simplicity)…

Time-dependent decay rate for $B^0_{phys}\to f$:

$$\frac{d\Gamma\left(B^0_{phys}(t)\to f\right)}{dt}=\left|\left\langle f\left|H\right|B^0_{phys}(t)\right\rangle\right|^2=$$

"decay"

"oscillation, then decay"

$$=\frac{e^{-\Gamma t}}{2}\left[\begin{array}{l}\left(1+\cos(\Delta m\,t)\right)\left|A_f\right|^2+\\ +\left(1-\cos(\Delta m\,t)\right)\left|\frac{q}{p}\right|^2\left|\overline{A}_f\right|^2-\\ -2\Im\mathrm{m}\left(\frac{q}{p}A_f^*\overline{A}_f\right)\sin(\Delta m\,t)\end{array}\right]=\frac{e^{-\Gamma t}}{2}\left|A_f\right|^2\left[\begin{array}{l}\left(1+\cos(\Delta m\,t)\right)+\\ +\left(1-\cos(\Delta m\,t)\right)\left|\lambda_f\right|^2-\\ -2\Im\mathrm{m}\left(\lambda_f\right)\sin(\Delta m\,t)\end{array}\right]$$

"interference"

$$\lambda_f=\frac{q}{p}\frac{\overline{A}_f}{A_f}$$

Time evolution of neutral *B* mesons - 4

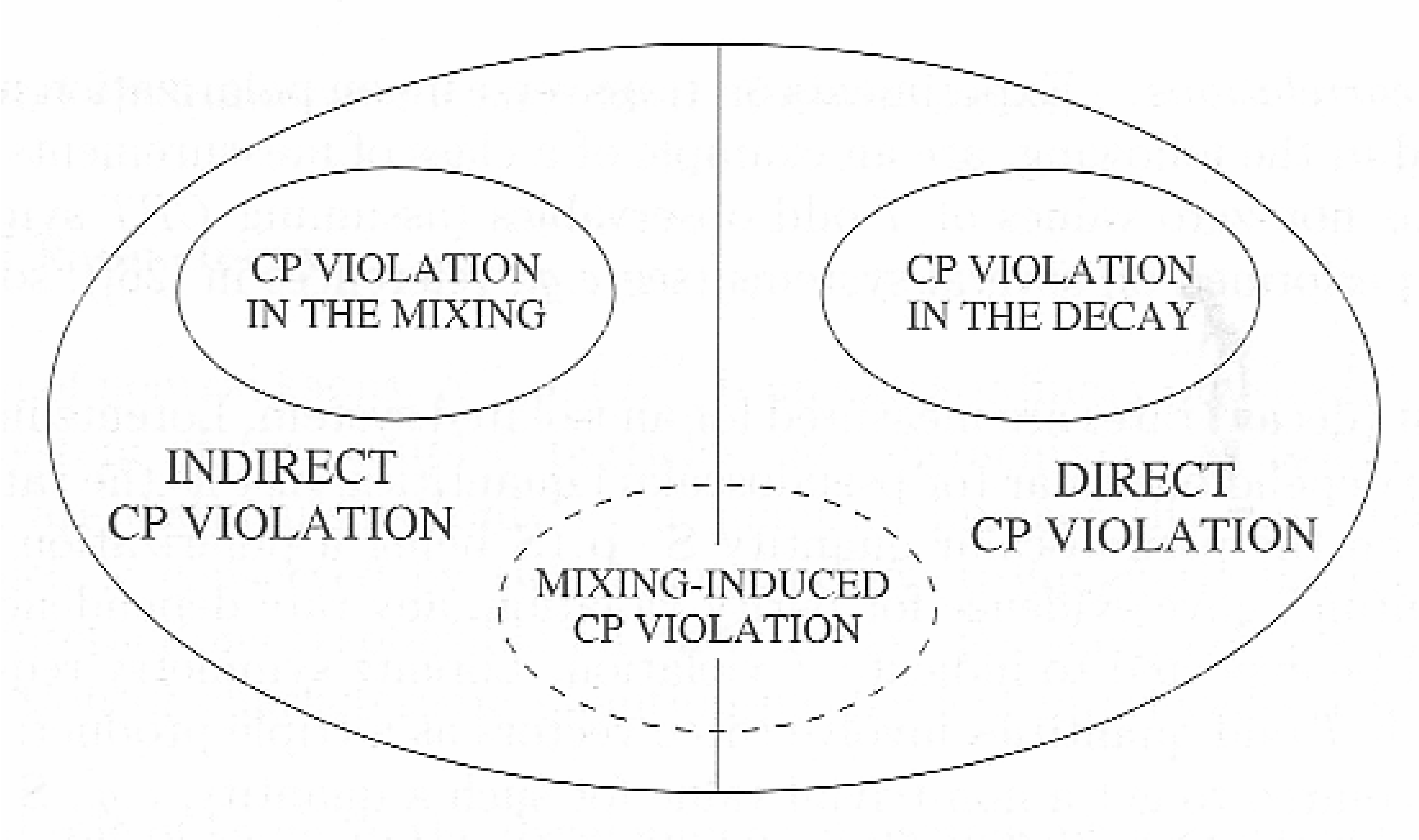
Combining similar expressions for: $\overline{B}^0_{phys} \to f\ ,\quad B^0_{phys} \to \bar{f}\ ,\quad \overline{B}^0_{phys} \to \bar{f}$
observable *CP-violating asymmetries* can be derived

Important special case:
neutral pseudoscalar mesons produced coherently in pairs,
from the decay of a vector resonance: $V \to P^0\overline{P}^0$,
and subsequent decays to final states $f_1\ , f_2$ at times $t_1\ , t_2$
for instance: $\Upsilon(4S) \to B^0\overline{B}^0 \qquad \phi \to K^0\overline{K}^0$

The corresponding time-dependence of decay rates and asymmetries have similar forms, with : $\Delta t \equiv t_2 - t_1$

For a complete discussion including CPT: see e.g.: *Kirkby & Nir, PDG*

Classification of CP-violating effects



Classification of CP-violating effects

CPV in decay:

$$\left|\overline{A}_{\bar f}/A_f\right| \neq 1$$

$$A_{CP,f^\pm} \equiv \frac{\Gamma\left(P^- \to f^-\right) - \Gamma\left(P^+ \to f^+\right)}{\Gamma\left(P^- \to f^-\right) + \Gamma\left(P^+ \to f^+\right)} = \frac{\left|\overline{A}_{f^-}/A_{f^+}\right|^2 - 1}{\left|\overline{A}_{f^-}/A_{f^+}\right|^2 + 1}$$

CPV in mixing:

$$\left|q/p\right| \neq 1$$

$$A_{SL}(t) \equiv \frac{d\Gamma/dt\left(\overline{P}^0_{phys} \to l^+ X\right) - d\Gamma/dt\left(P^0_{phys} \to l^- X\right)}{d\Gamma/dt\left(\overline{P}^0_{phys} \to l^+ X\right) + d\Gamma/dt\left(P^0_{phys} \to l^- X\right)} =$$

$$= \frac{1 - \left|q/p\right|^4}{1 + \left|q/p\right|^4}$$

CPV in the interference decay-mixing (“mixing-induced”):

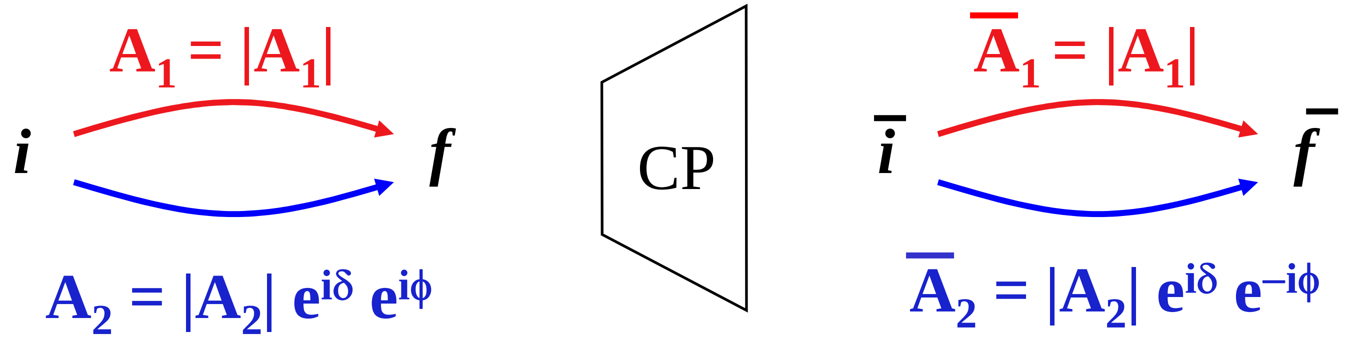
$$\Im\mathrm{m}\left(\lambda_f\right) \neq 0$$

$$\lambda_f \equiv \frac{q}{p}\frac{\overline{A}_f}{A_f}$$

For example: decays to CP eigenstates f_{CP}

$$A_{f_{CP}}(t) \equiv \frac{d\Gamma/dt\left(\overline{P}^0_{phys} \to f_{CP}\right) - d\Gamma/dt\left(P^0_{phys} \to f_{CP}\right)}{d\Gamma/dt\left(\overline{P}^0_{phys} \to f_{CP}\right) + d\Gamma/dt\left(P^0_{phys} \to f_{CP}\right)}$$

Observables: “direct” CP asymmetry - 1



$A_1 = |A_1|$

$A_2 = |A_2|\, e^{i\delta}\, e^{i\phi}$

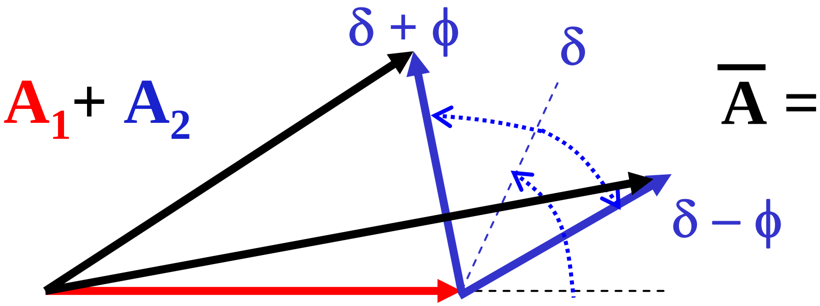
$\bar{A}_1 = |A_1|$

$\bar{A}_2 = |A_2|\, e^{i\delta}\, e^{-i\phi}$

$\delta \to \delta$ (CP-conserving)
$\phi \to -\phi$ (CP-violating)

Time-integrated “direct” CP asymmetry requires two amplitudes and $\delta \neq 0$:

$A = A_1 + A_2$



$\bar{A} = \bar{A}_1 + \bar{A}_2 \neq A$

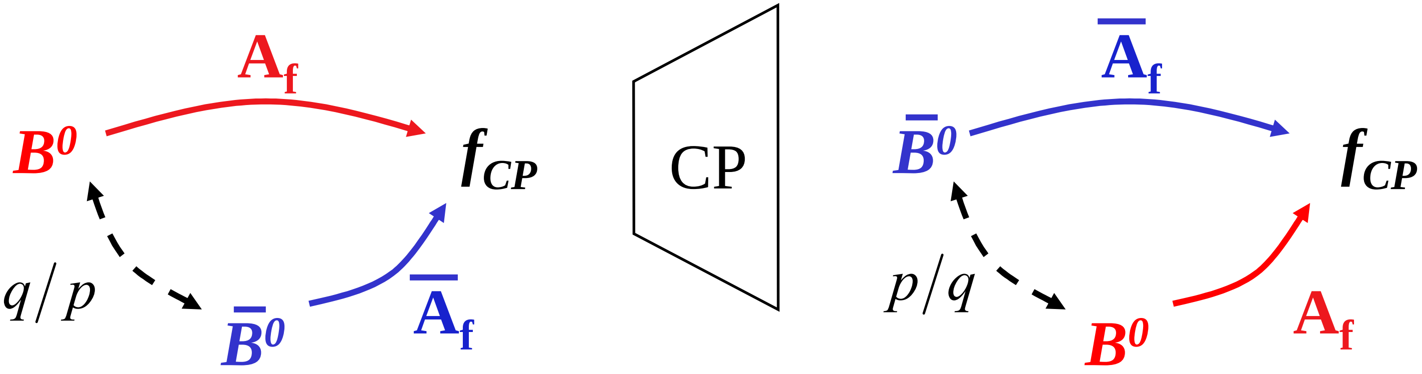
Observables: “direct” CP asymmetry - 2

Time-integrated “direct” CP asymmetry (“CP violation in decay”):

$$A_{CP} \equiv \frac{\Gamma\left(i \to f\right) - \Gamma\left(\bar{i} \to \bar{f}\right)}{\Gamma\left(i \to f\right) + \Gamma\left(\bar{i} \to \bar{f}\right)} = \frac{2\left|A_1\right|\left|A_2\right| \sin\delta \sin\phi}{\left|A_1\right|^2 + \left|A_2\right|^2 + 2\left|A_1\right|\left|A_2\right| \cos\delta \cos\phi}$$

- the only possibile CPV effect for *charged* mesons decays !
- requires at least two amplitudes *and* $\delta \neq 0$

Time-dependent CP asymmetry - 1



Interference between mixing and decay to a CP eigenstate f_{CP}

$$\Rightarrow \quad \Gamma\left(B^0_{phys}(t) \to f_{CP}\right) \neq \Gamma\left(\overline{B}^0_{phys}(t) \to f_{CP}\right)$$

Flavor-tagged time-dependent decay rates are different!
they are governed by the “CP parameter”:

$$\lambda_{f_{CP}} = \eta_{f_{CP}} \frac{q}{p} \frac{\overline{A}_{f_{CP}}}{A_{f_{CP}}}$$

CP eigenvalue

$\approx e^{-i2\beta}$ from mixing

Amplitude ratio

Time-dependent CP asymmetry - 2

Decay distributions $f_+(f_-)$ when tag = $B^0(\bar{B}^0)$, pair-produced at $\Upsilon(4S)$

$$f_{CP,\pm}(\Delta t) = \frac{\Gamma}{4} e^{-\Gamma\Delta t}[1 \pm S_{f_{CP}} \sin\Delta m_d \Delta t \mp C_{f_{CP}} \cos\Delta m_d \Delta t]$$

Asymmetry

$$A_{f_{CP}}(\Delta t) = C_{f_{CP}} \cos(\Delta m_d \Delta t) - S_{f_{CP}} \sin(\Delta m_d \Delta t)$$

CP parameter

$$\lambda_{f_{CP}} = \eta_{f_{CP}} \frac{q}{p} \cdot \frac{\bar{A}_{\bar{f}_{CP}}}{A_{f_{CP}}}$$

$$\boxed{\begin{aligned} C_{f_{CP}} &= \frac{1 - |\lambda_{f_{CP}}|^2}{1 + |\lambda_{f_{CP}}|^2} \\ S_{f_{CP}} &= \frac{-2\,\mathrm{Im}\,\lambda_{f_{CP}}}{1 + |\lambda_{f_{CP}}|^2} \end{aligned}}$$

For single decay amplitude

$$= 0$$

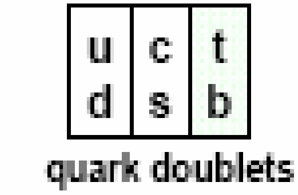
$$= -\mathrm{Im}\,\lambda_{f_{CP}}$$

“Theoretical interpretation”: CKM

The CKM paradigm in the SM

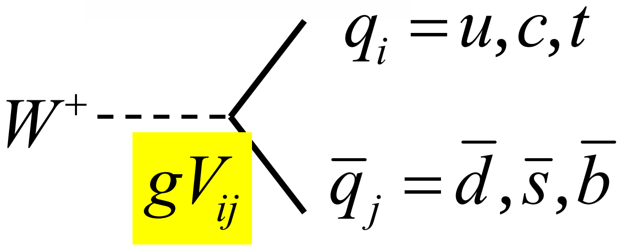
(1973) M.Kobayashi and T.Maskawa

- CP violation ⇒ third generation of quarks

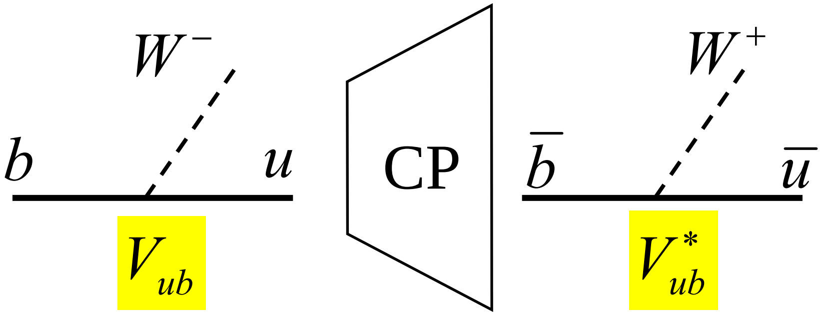


Cabibbo-Kobayashi-Maskawa matrix V

- couples quark charged currents to $W^{\pm}$
- mixes the left-handed (q_j=d,s,b) quark mass eigenstates to give weak eigenstates;
- unitary, with 4 independent parameters (e.g., 3 angles and 1 phase)
- complex elements: phase changes sign under CP
- interfering amplitudes can give observable CP-violating rate asymmetries



$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



CKM matrix and Unitarity Triangle

“improved” Wolfenstein parameterization:

$$V_{\mathrm{CKM}} =$$

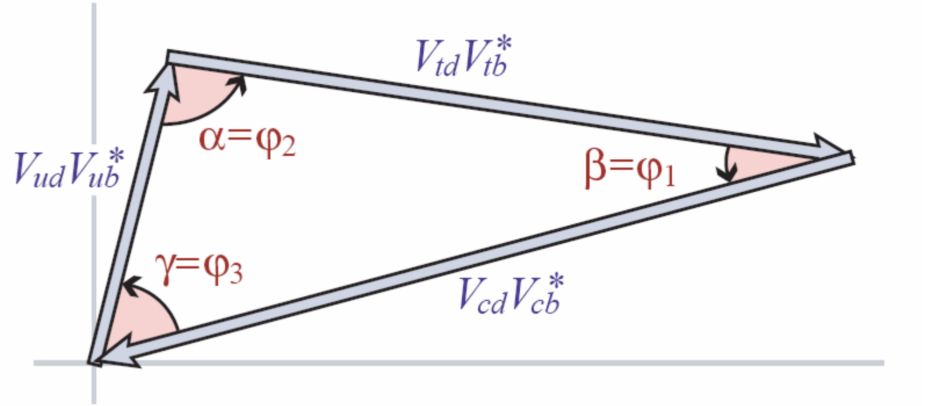
$$\begin{array}{cc}
\left(\begin{array}{ccc}
1-\frac{1}{2}\lambda^2-\frac{1}{8}\lambda^4 & \lambda & A\lambda^3(\rho-i\eta) \\
-\lambda+\frac{1}{2}A^2\lambda^5[1-2(\rho+i\eta)] & 1-\frac{1}{2}\lambda^2-\frac{1}{8}\lambda^4(1+4A^2) & A\lambda^2 \\
A\lambda^3[1-(1-\frac{1}{2}\lambda^2)(\rho+i\eta)] & -A\lambda^2+\frac{1}{2}A\lambda^4[1-2(\rho+i\eta)] & 1-\frac{1}{2}A^2\lambda^4
\end{array}\right) & \begin{array}{c} u \\ c \\ t \end{array} \\
\begin{array}{ccc} d & \qquad\qquad\qquad s & \qquad\qquad\qquad b \end{array} &
\end{array}$$

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0\ .$$

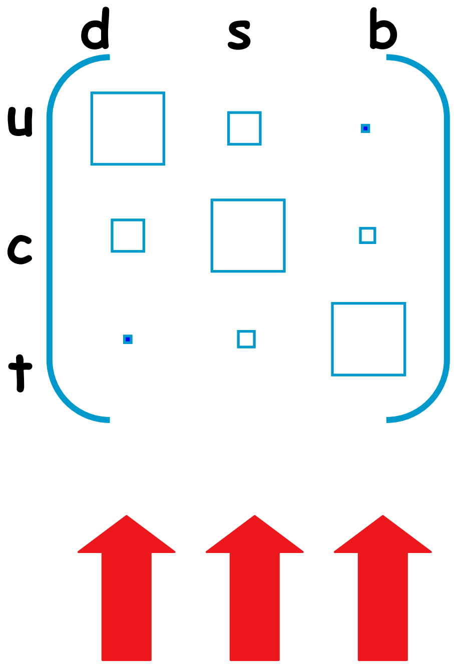
$$\alpha \equiv \varphi_2 \equiv \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right),$$

$$\beta \equiv \varphi_1 \equiv \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right),$$

$$\gamma \equiv \varphi_3 \equiv \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right),$$



The Unitarity Triangles



apply unitarity constraint to pairs of columns

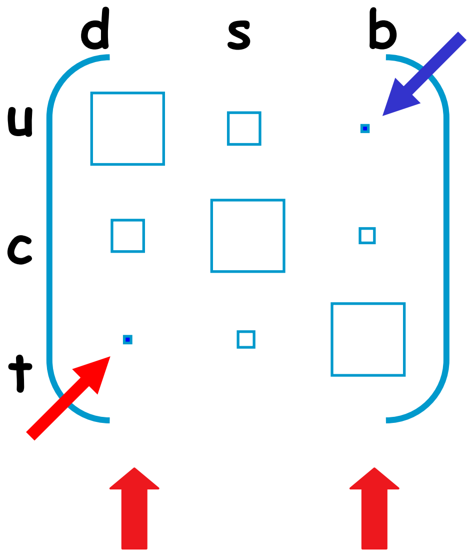
$d \cdot s^* = 0$ (K system)

$s \cdot b^* = 0$ (B_s system)

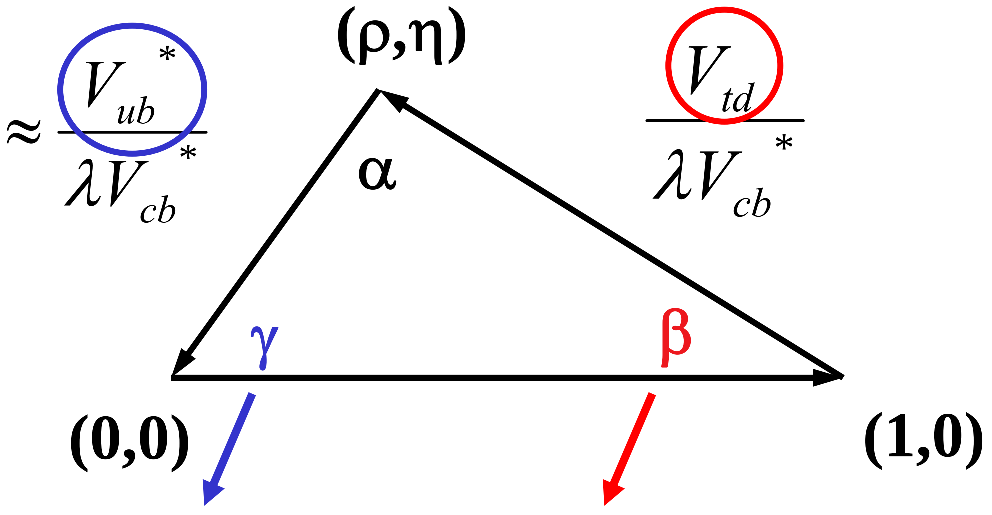
$d \cdot b^* = 0$ (B_d system)

These three triangles (and the three triangles corresponding to the rows) all have the same area. A nonzero area is a measure of CP violation and is an invariant of the CKM matrix.

The “normalized” Unitarity Triangle



apply unitarity constraint to
these two columns



$$\gamma \approx \arg V_{ub}^{*} \qquad \beta \approx -\arg V_{td} \qquad \alpha = \pi - \beta - \gamma$$

Orders of magnitude for
Wolfenstein parameters:

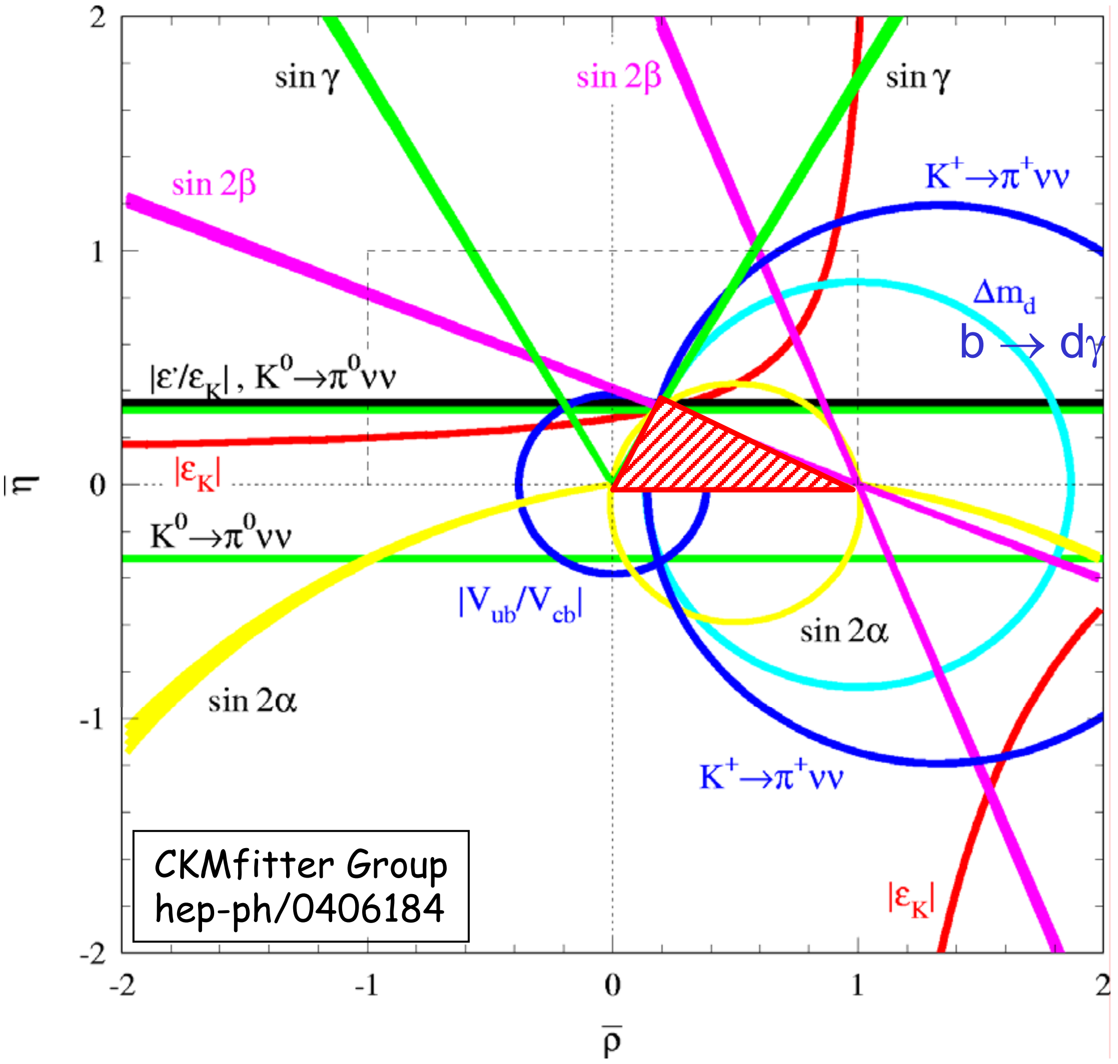
$$\boxed{\lambda \approx 0.22\,, \quad A \approx 0.8\,, \quad \sqrt{\rho^{2}+\eta^{2}} \approx 0.4}$$

$$\boxed{\begin{array}{l} \frac{V_{ud}V_{ub}^{*}}{V_{cd}V_{cb}^{*}} + 1 + \frac{V_{td}V_{tb}^{*}}{V_{cd}V_{cb}^{*}} = 0 \\ V_{cd} = \lambda\,, \quad V_{ud} \approx V_{tb} \approx 1 \end{array}}$$

CKM and Unitarity Angles: CPV roadmap

B meson mixing and decays probe 5 of the 9 elements of the CKM matrix

CP violating asymmetries directly access the CKM phase through the Unitarity Angles $\alpha(\phi_2)$, $\beta(\phi_1)$, $\gamma(\phi_3)$



CPV in the B sector: CKM angles

$V_{td} = |V_{td}|e^{-i\phi_1}$ (B^0-$\bar{B}^0$ mixing) $V_{td} = |V_{td}|e^{-i\beta}$

- Mixing-assisted CPV
  - Observation in $B^0 \to J/\psi K^0$ BaBar & Belle (2001)
- CPV in B^0-$\bar{B}^0$ mixing itself
  - Not seen yet

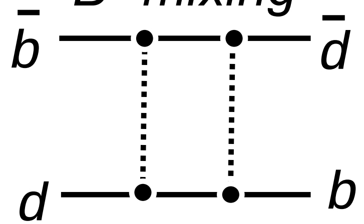
$V_{ub} = |V_{ub}|e^{-i\phi_3}$ ($b \to u$ decays) $V_{ub} = |V_{ub}|e^{-i\gamma}$

- Direct CPV (Interference with other diagrams)
  - Evidence in $B^0 \to \pi^+\pi^-$ Belle (2003), not seen by BaBar
  - Evidence in $B^0 \to K^+\pi^-$ BaBar & Belle (2004)

Both V_{td} and V_{ub} are involved

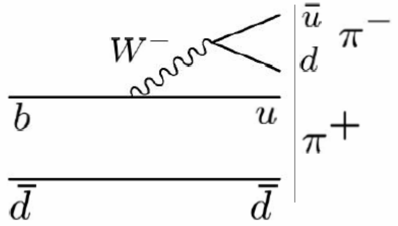
- Mixing-assisted CPV for final states containing V_{ub}
  - Evidence in $B^0 \to \pi^+\pi^-$ Belle (2003), not seen by BaBar

B^0 mixing



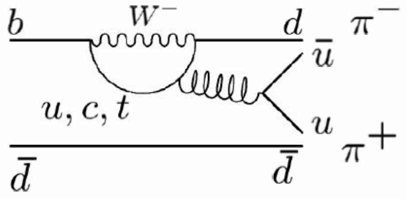
$q/p \approx e^{-i2\beta}$

B^0 decay: tree



$A \propto V_{ub}^* V_{ud}$ $\propto \lambda^3$

B^0 decay: penguin



$A \propto V_{td}^* V_{tb}$ $\propto \lambda^3$

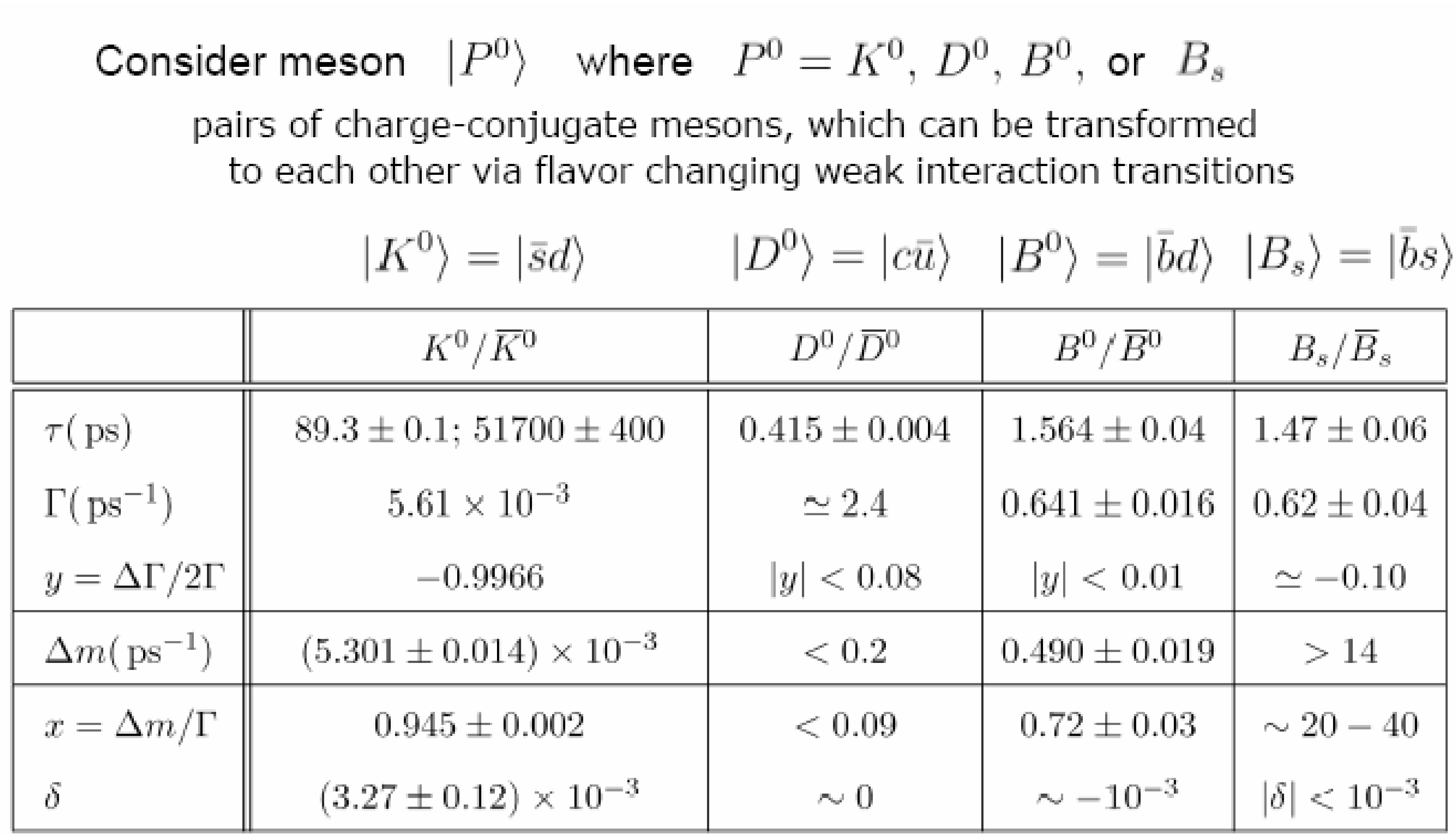
$P = K^0, D^0, B^0_d, B^0_s$

Peculiarities of pseudoscalar mesons

in terms of $\Delta m\,, \quad \Delta\Gamma\,, \quad x = \frac{\Delta m}{\Gamma}\,, \quad y = \frac{\Delta\Gamma}{2\Gamma}$

and of the expectations for *CP* effects

K^0, D^0, B^0_d, B^0_s: Δm and $\Delta\Gamma$



Consider meson $|P^0\rangle$ where $P^0 = K^0,\ D^0,\ B^0,$ or B_s

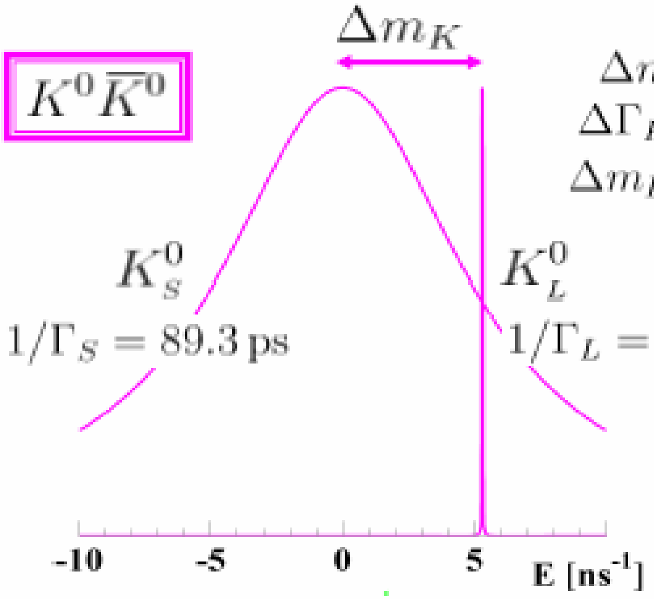
pairs of charge-conjugate mesons, which can be transformed to each other via flavor changing weak interaction transitions

$|K^0\rangle = |\bar{s}d\rangle$ $\quad |D^0\rangle = |c\bar{u}\rangle$ $\quad |B^0\rangle = |\bar{b}d\rangle$ $\quad |B_s\rangle = |\bar{b}s\rangle$

	$K^0/\overline{K}^0$	$D^0/\overline{D}^0$	$B^0/\overline{B}^0$	$B_s/\overline{B}_s$
$\tau(\mathrm{ps})$	$89.3 \pm 0.1;\ 51700 \pm 400$	0.415 ± 0.004	1.564 ± 0.04	1.47 ± 0.06
$\Gamma(\mathrm{ps}^{-1})$	5.61×10^{-3}	$\simeq 2.4$	0.641 ± 0.016	0.62 ± 0.04
$y = \Delta\Gamma/2\Gamma$	-0.9966	$\lvert y\rvert < 0.08$	$\lvert y\rvert < 0.01$	$\simeq -0.10$
$\Delta m(\mathrm{ps}^{-1})$	$(5.301 \pm 0.014) \times 10^{-3}$	< 0.2	0.490 ± 0.019	> 14
$x = \Delta m/\Gamma$	0.945 ± 0.002	< 0.09	0.72 ± 0.03	$\sim 20 - 40$
δ	$(3.27 \pm 0.12) \times 10^{-3}$	~ 0	$\sim -10^{-3}$	$\lvert\delta\rvert < 10^{-3}$

Time units (ps) for all quantities…!

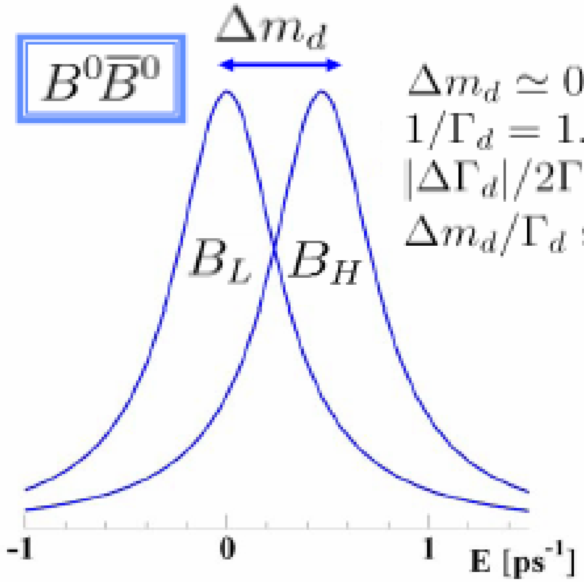
Exercise: check against the latest PDG and HFAG values



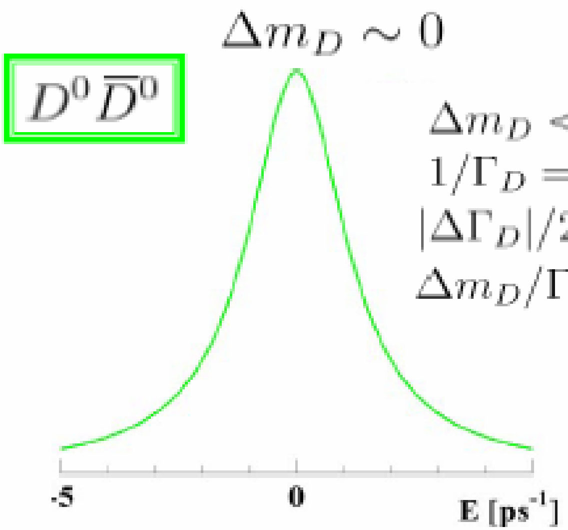
$1/\Gamma_S = 89.3\,\mathrm{ps}$

$1/\Gamma_L = 51.7\,\mathrm{ns}$

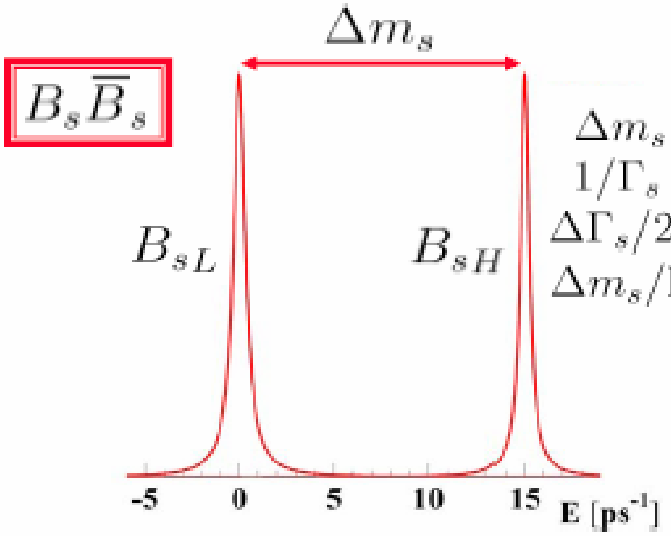
$\Delta m_K \simeq 5.3\,\mathrm{ns}^{-1}$
$\Delta\Gamma_K/2\Gamma_K \approx -1$
$\Delta m_K/\Gamma_K \simeq 0.945$



$\Delta m_d \simeq 0.5\,\mathrm{ps}^{-1}$
$1/\Gamma_d = 1.56\,\mathrm{ps}$
$|\Delta\Gamma_d|/2\Gamma_d \ll 1$
$\Delta m_d/\Gamma_d \simeq 0.75$



$\Delta m_D < 0.2\,\mathrm{ps}^{-1}$
$1/\Gamma_D = 0.415\,\mathrm{ps}$
$|\Delta\Gamma_D|/2\Gamma_D \ll 1$
$\Delta m_D/\Gamma_D < 0.09$

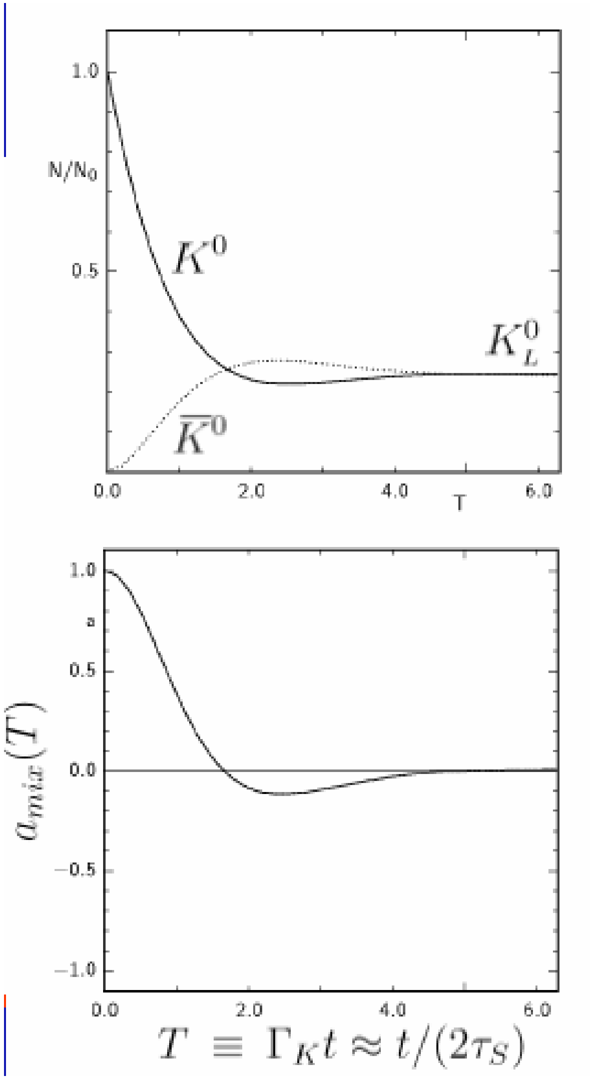


$\Delta m_s > 14\,\mathrm{ps}^{-1}$
$1/\Gamma_s = 1.47\,\mathrm{ps}$
$\Delta\Gamma_s/2\Gamma_s \approx -0.10$
$\Delta m_s/\Gamma_s \sim 20-40$

[from a seminar by G.Hamel de Monchenault] mixing

K^0 system

CPV



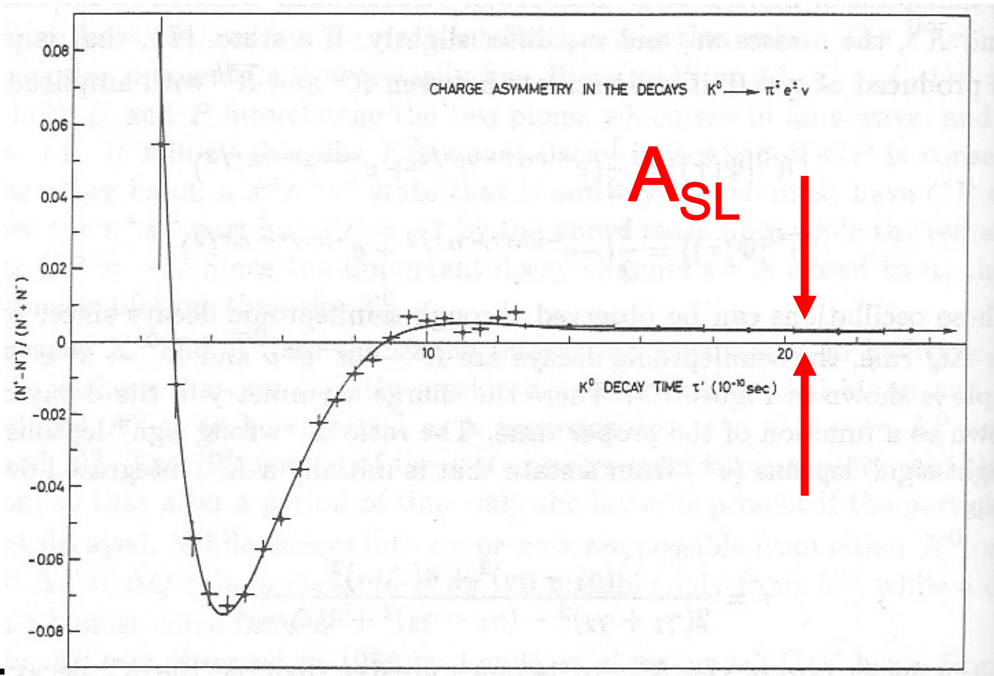
$$x_K \cong 0.95$$

$$y_K \cong -0.996$$

Both of order unity!
Only K^o_L is left after
≈ one oscillation

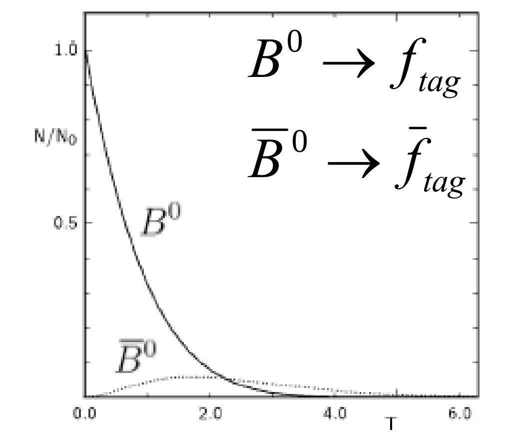
CPV is small…

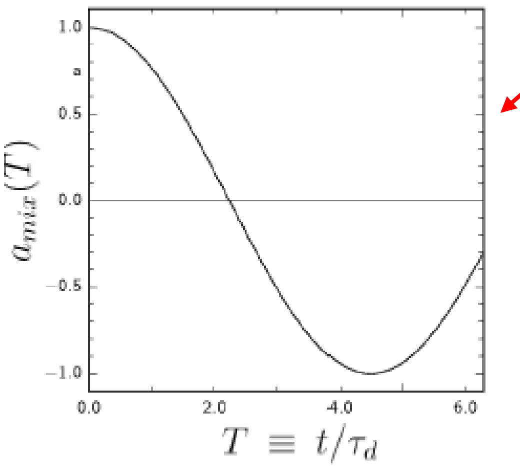
$$\delta_K = \frac{2\,\mathrm{Re}(\varepsilon_K)}{1+|\varepsilon_K|^2} \cong 3\times10^{-3}$$



B_d system

mixing





To a very good approx., equal decay widths and no CPV in mixing

$y_d \approx 0$

$\delta_d \approx 0$

$$a_{mix}(t) = \cos(\Delta m\, t)$$
$$= \cos(x_d\, t/\tau_d)$$

In the simplest case, time-dependent CP asymmetry:

$$a^{CP}_{f_{CP}}(t) = \mathrm{Im}(\lambda_{f_{CP}})\sin(\Delta m\, t)$$

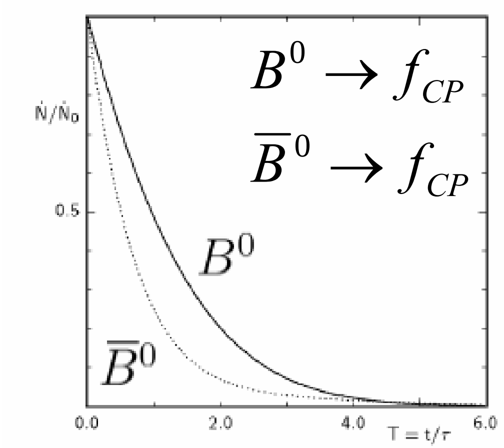
Time-integrated (incoherent!):

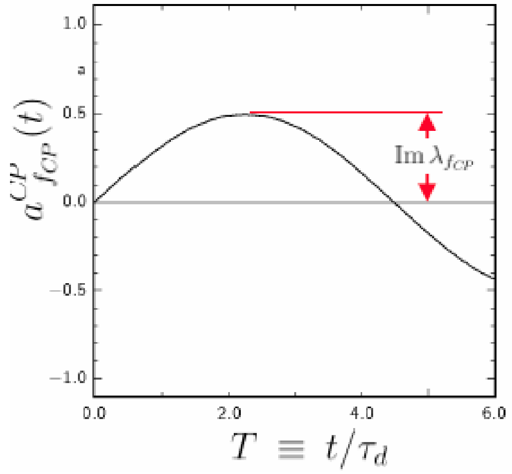
$$A^{CP}_{f_{CP}} = \frac{x_d}{1+x_d^2}\mathrm{Im}(\lambda_{f_{CP}})$$

$$x_d = 0.72 \pm 0.03$$

$$\chi_d = \frac{x_d^2}{2(1+x_d^2)} \cong 18\%$$

CPV

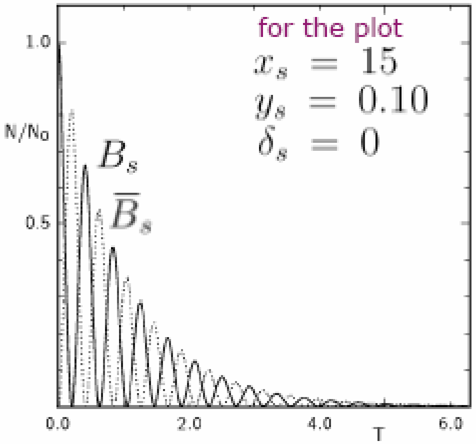


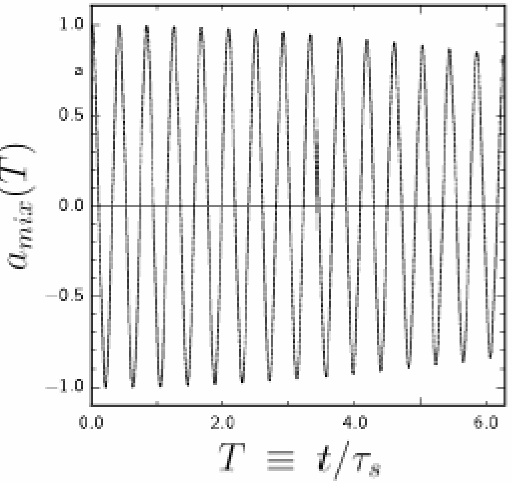


$$\frac{x_d}{1+x_d^2} \cong 0.47$$

B_s system

mixing





x_s is very large

y_s small, perhaps not negligible

$$x_s > 21 \quad (95\%\ CL)$$

$$2y_s < 0.46 \ (95\%\ CL)$$

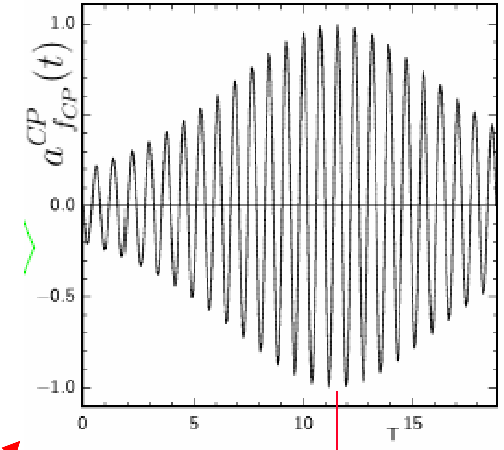
Mixing probability close to 50%

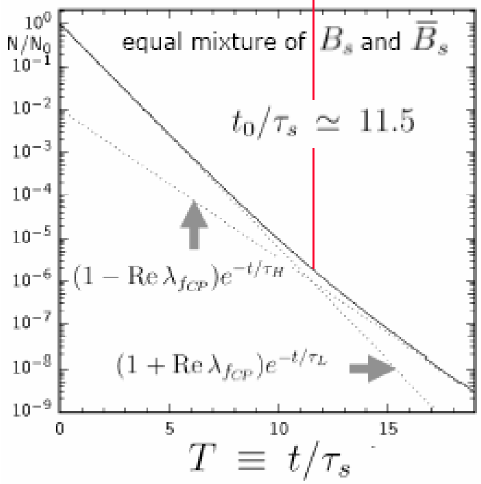
$$\chi_s = \frac{x_s^2 + y_s^2}{2(1+x_s^2)} > 0.4988$$

Time-dependent CP-asymmetry: sinusoidal function, modulated by a function f(t); 100% at the max.!

$$a_{f_{CP}}^{CP}(t) = \mathrm{Im}(\lambda_{f_{CP}}) \sin(\Delta m\, t) f(t)$$

CPV



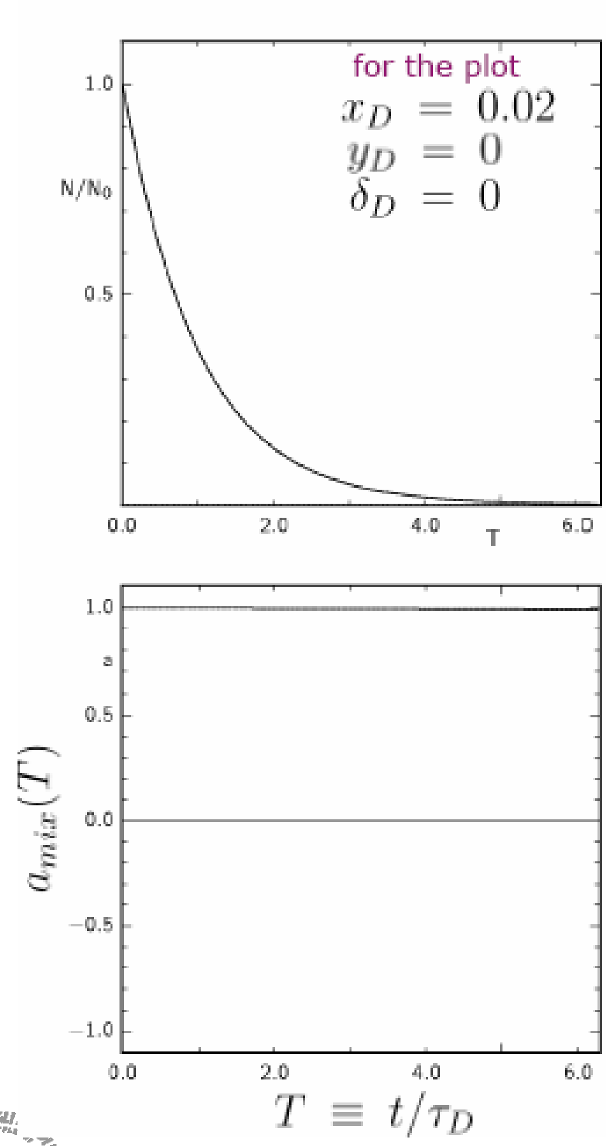


$$x_s = 15, \quad y_s = 0.10$$

← Demo plots with unrealistic values! →

$$x_s = 8, \quad y_s = -0.2$$
$$\mathrm{Im}\,\lambda = 0.2$$

The D System



In the D^0-$\overline{D}^0$ system, both are very small

- y_D very small: only few common states

$$CP = +1 \quad \pi\pi,\ K\overline{K},\ K^0_L\pi^0$$
$$CP = -1 \quad K^0_S\pi^0,\ K^0_S\omega$$

- x_D very small: strongly CKM suppressed

Mixing probability extremely small $\chi = \dfrac{x_D^2}{2(1+x_D^2)}$

interesting system to look for new physics

$$\boxed{a_{mix}(t) \approx 1 - \frac{x_D^2 + y_D^2}{2}(t/\tau)^2}$$

Present experimental limits will be summarized in the lecture on D mesons

Lecture 1 - Summary

- CPV tests probe fundamental symmetry properties of nature, with links to cosmology

- CPV seen in K *and* B mesons!

- Neutral pseudoscalar mesons (P= K, B, D) in particular offer a very rich and subtle phenomenology for stringent tests of theoretical predictions

- We will discuss in more detail (in the given order):
  - B mesons, K mesons, D mesons
  - “CPV without strangeness”: Electric Dipole Moments

Discovery potential in B mesons

- B mesons: specially suited for stringent experimental tests of a detailed pattern of theoretical expectations
  - "direct" CP violation in charged B decays
    - from the interference of different decay amplitudes
    - CP *asymmetries* can be large (O(10%))
  - CP violation in mixing: should be small
  - CP violation in the interference of neutral B decays with and without mixing
    - Several "clean" time-dependent CP asymmetries
    - The three Unitarity Angles: $\alpha(\phi_2)$, $\beta(\phi_1)$, $\gamma(\phi_3)$, can be determined by observables related to V_{td} and V_{ub}
    - The validity of the CKM model can be tested overconstraining the Unitarity Triangle
  - B_s mixing still to be determined (important for $|V_{ts}|$)!

Back-up slides

SI and natural units

- Preferred units in particle physics: “natural units”, just one unit for all physical quantities…

$$E:\ 1MeV = 10^6\,eV,\ \ 1GeV = 10^9\,eV;\ \ (L:\ 1fm = 10^{-15}m)$$

$$\hbar = c = 1\ \ (\text{a-dimensional})\ \ \Rightarrow [M] = [\mathrm{E}] = [T^{-1}] = [L^{-1}]$$

examples:

$$\text{Compton wavelength:}\ \ \lambda_C = \frac{\hbar}{mc}\ \ \rightarrow\ \ \frac{1}{m}\ \ \text{measured in}\ eV^{-1}\ \text{or}\ fm$$

$$\text{Lifetime:}\ \ \tau = \frac{\hbar}{\Gamma}\ \ \rightarrow\ \ \frac{1}{\Gamma}\ \ \text{measured in}\ eV^{-1}\ \text{or}\ s$$

SI and natural units

- SI units can be recovered in final results, by inserting appropriate powers of ℏ and c via dimensional analysis and using:

$$\boxed{\begin{array}{l} \hbar = 6.582\times10^{-22}\ MeV\,s \\ c = 3\times10^{23}\ fm\,s^{-1} \\ \hbar c = 197.33\,MeV\,fm \end{array}} \longrightarrow \boxed{\begin{array}{l} 1\,MeV = 1.52\times10^{21}\ s^{-1} \\ 1\,s = 3\times10^{23}\ fm \\ 1\,fm = 5.07\times10^{-3}\ MeV^{-1} \end{array}}$$

- Examples:

ω resonance: width and lifetime

$$\Gamma = 8.43\,MeV \quad \Rightarrow \quad 1/\tau = 8.43\times1.52\times10^{21}\ s^{-1} = 1.28\times10^{22}\,s^{-1} \quad \Rightarrow \quad \tau = 0.78\times10^{-22}\,s$$

π meson: mass and Compton wavelength

$$m = 140\,MeV/c^2 \quad \Rightarrow \quad \lambda = 1/m = \frac{1}{140}\,MeV^{-1} = \frac{1}{140\times5.07\times10^{-3}}\,fm = 1.41\,fm$$

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Acknowledgements

I am indebted to many colleagues whose seminars and lectures on B physics and CPV were a very helpful source of ideas and material, in particular:

Pat Burchat, Gautier Hamel de Monchenault, Achille Stocchi, …

For the most recent updates I relied mainly on review talks at the LP2005 conference (Uppsala, June 30 – July 5, 2005), by:

Kazuo Abe, Francesco Forti, Rick Jesick, Ulrich Nierste, Luca Silvestrini, Iain Stewart

The experimental data shown in these lectures are mainly from the B-factories and Tevatron collaborations:

CDF, D0, BaBar, Belle