Proposal to COSY
on

Do unpolarized electrons affect the polarization of a stored proton beam?

(ANKE and $\mathcal{PAX}$ Collaborations)

Jülich, September 2006

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Abstract

Understanding the interplay of the nuclear interaction with polarized (anti) protons and the electromagnetic interaction with polarized electrons in polarized atoms is crucial to progress towards the PAX goal to eventually produce stored polarized antiproton beams at FAIR. Presently, there exist two competing theoretical scenarios: one with substantial spin filtering of (anti)protons by atomic electrons, while the second one suggests an almost exact self-cancellation of the electron contribution to spin filtering. The existing experimental data from the FILTEX experiment allow neither an unambiguous discrimination between the two scenarios nor do they give a direct constraint on the rôle of the spin-flip scattering in spin filtering as discussed recently by Walcher et al. In order to clarify this issue, we suggest to study the depolarization effect of a proton beam stored at COSY injection energy of $T_p = 45$ MeV by electrons in a ^{4}He storage cell target, as in effect inverse to a polarization buildup by polarized electrons as predicted by Meyer and Horowitz. These studies can in be carried out at the ANKE IP.

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for the ANKE and $\mathcal{PAX}$ Collaborations

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1 Introduction

In this proposal for COSY, the ANKE and PAX collaborations suggest to study the depolarization in a proton beam at the COSY injection energy of $T_p = 45$ MeV. The main objectives of this experiment at COSY relate to a clarification of the rôle electrons play for the polarization of a stored beam. At present there exist two theoretical scenarios for spin filtering of stored (anti)protons. In the first scenario, suggested by H.O. Meyer [1], the stored beam becomes polarized by the QED process of spin transfer from polarized electrons in a Polarized Internal Target (PIT). On the other hand, the 2005 scrutiny of the spin-filtering process suggests [2] an almost exact cancellation of the electron contribution to the polarization of the transmitted stored beam and beam particles elastically scattered off electrons — the deflection of the latter is negligibly small and they all stay within the ring acceptance. In the second scenario only the nuclear interaction would contribute to spin filtering. Understanding which of these two scenarios is really at work is crucial to progress towards the goal to eventually produce stored polarized antiproton beams at FAIR. It is assumed that the answer must be obtained experimentally in a situation where one knows well the spin-dependent ingredients of the two scenarios.

In the theoretical treatment of spin filtering, the polarization buildup depends on the interplay of the polarization-dependent transmission of the beam through the PIT and the spin-flip of the beam particles. As Walcher et al. emphasized ([3], a similar discussion is found in ref. [4]), in a pure electron target the attenuation of a stored (anti)proton beam vanishes and the beam becomes polarized by the *ep* spin-flip interactions. The idea of using polarized electron coolers as a PIT has been discussed earlier in the PAX Technical Design Report [5, 6] and dismissed on the grounds of too low a target density. Walcher et al. noticed an interesting possibility of enhancing the spin-flip rate by a judicious choice of the relative velocity of comoving polarized-electron and stored-proton beams. The reported filtering rate has been evaluated attributing the electron-to-proton spin transfer to the proton spin-flip process. It turned out that with the polarized electron beam technology of today the expected spin filtering rate is too low for the purposes of PAX, thus prompting Walcher *et al.* to withdraw their proposal. Still this important discussion adds to the urgency of constraining experimentally the rôle of atomic electrons. The principal idea behind this proposal is to reverse the rôle of the beam and target polarizations: from the viewpoint of the kinetics of the spin-filtering, depolarization of polarized stored protons in an unpolarized electron target is equivalent to the buildup of the polarization of the initially unpolarized beam proton by multiple passage through the polarized electron target. This way one could circumvent the technical problems of operating a PIT with vanishing nuclear and pure electron polarization, which requires an additional effort with respect to the installation of a new experimental setup at TP1 of COSY, as discussed in the recent Letter–of–Intent to the COSY PAC [7]. Here one would use atomic electrons and in order to avoid nuclear spin-flip effects the best choice would be a ^{4}He target, in which the nuclei possess spin 0.

The PAX collaboration has recently suggested in a Letter–of–Intent to the SPS committee of CERN [8] to study the polarization buildup by spin filtering of stored antiprotons by multiple passage through a polarized internal hydrogen gas target. Through this investigation, one can obtain a direct access to the spin dependence of the antiproton–proton total cross section. Apart from the obvious interest for the general theory of $p\bar{p}$ interactions, the knowledge of these cross sections is necessary for the interpretation of unexpected features of the $p\bar{p}$, and other antibaryon–baryon pairs, contained in final states in J/Ψ and B–decays. Simultaneously, the confirmation of the polarization buildup of antiprotons would pave the way to high–luminosity double–polarized antiproton–proton colliders, which would provide the unique opportunity to study transverse spin physics in the hard QCD regime. Such a collider has been proposed recently by the PAX Collaboration [5] for the new Facility for Antiproton and Ion Research (FAIR) at GSI in Darmstadt, Germany, aiming at luminosities of 10^{31} cm^{-2}s^{-1}. An integral part of such a machine is a dedicated large–acceptance Antiproton Polarizer Ring (APR).

The QCD physics potential of experiments with high energy polarized antiprotons is enormous, yet hitherto high luminosity experiments with polarized antiprotons have been impossible. The situation could change dramatically with the realization of spin filtering and storing of polarized antiprotons, and the realization of a double–polarized high–luminosity antiproton–proton collider. The list of fundamental physics issues for such colliders includes the determination of transversity, the quark transverse polarization inside a transversely polarized proton, the last leading twist missing piece of the QCD description of the partonic structure of the nucleon, which can be directly measured only via double-polarized antiproton–proton Drell–Yan production. Without measurements of the transversity, the spin tomography of the proton would be ever incomplete. Other items of great importance for the perturbative QCD description of the proton include the phase of the timelike form factors of the proton and hard antiproton–proton scattering. Such an ambitious physics program has been formulated by the PAX collaboration (Polarized Antiproton eXperiment) and a Technical Proposal [5] has recently been submitted to the FAIR project. The uniqueness and the strong scientific merits of the PAX proposal have been well received [9], and there is an urgency to convincingly demonstrate experimentally that a high degree of antiproton beam polarization could be reached with a dedicated APR.

Here we recall, that for more than two decades, physicists have tried to produce beams of polarized antiprotons [10], generally without success. Conventional methods like atomic beam sources (ABS), appropriate for the production of polarized protons and heavy ions cannot be applied, since antiprotons annihilate with matter. Polarized antiprotons have been produced from the decay in flight of $\bar{\Lambda}$ hyperons at Fermilab. The intensities achieved with antiproton polarizations $P > 0.35$ never exceeded $1.5 \cdot 10^5$ s^{-1} [11]. Scattering of antiprotons off a liquid hydrogen target could yield polarizations of $P \approx 0.2$, with beam intensities of up to $2 \cdot 10^3$ s^{-1} [12]. Unfortunately, both approaches do not allow efficient accumulation in a storage ring, which would greatly enhance the luminosity. Spin splitting using the Stern–Gerlach separation of the given magnetic substates in a stored antiproton beam was proposed in 1985 [13]. Although the theoretical understanding has much improved since then [14], spin splitting using a stored beam has yet to be observed experimentally. In contrast to that, a convincing proof of the spin–filtering principle has been produced by the FILTEX experiment at the TSR–ring in Heidelberg [15].

The experimental basis for predicting the polarization buildup in a stored antiproton beam is practically non–existent. The AD–ring at CERN is a unique facility at which stored antiprotons in the appropriate energy range are available and whose characteristics meet the requirements for the first ever antiproton polarization buildup studies. Therefore, it is of highest priority for the PAX collaboration to perform subsequently to the COSY experiments spin filtering experiments using stored antiprotons at the AD–ring of CERN. Once this experimental data base will be available, the design of a dedicated APR can be approached.

2 Physics case

2.1 FILTEX experiment: The reference case

The spin filtering in storage rings is based on the multiple passage of a stored beam through a PIT. When the interaction depends on the relative spin orientations of beam and target, the target polarization is transferred to the beam in precisely the same way as in the familiar polarization of light transmitted through an optically active medium [16, 17]. In the realm of strongly interacting particles, spin filtering works by removing (absorbing out) one of the spin states of the incident beam. The celebrated example is the extremely effective polarized ^{3}He spin filter for cold, thermal and hot neutrons: neutrons with spin component antiparallel to the nuclear spin have a gigantic cross section for capture into a broad resonance, $J^\pi = 0^+$, in the intermediate ^{4}He*, which decays to $t + p$, and the transmitted neutron beam becomes polarized parallel to the nuclear spin (see ref. [18] and references therein).

In the optical experiments, one usually deals with the polarization of the transmitted light which propagates at exactly zero angle. The above described ^{3}He also polarizes the transmitted neutron beam. In particle scattering experiments, one is after the polarization of scattered (recoil) particles, and the transmitted beam and the scattered particles are not mixed with each other.

Spin filtering of (anti)protons in storage rings is rich in subtleties noticed by H.O. Meyer [1]. Firstly, a unique geometrical feature of storage rings is that particles scattered off a PIT within the ring acceptance angle $\theta_{\rm acc}$ remain in the beam. Such a scattering–within–the–ring (SWR) mixes the polarization of transmitted beam and scattered particles. Secondly, polarized atoms of a PIT contain polarized electrons. The interaction of the spin of the electron with the spin of stored (anti)protons is a non–negligible one. At low energies, for instance, this interaction is responsible for the hyperfine splitting in atoms. At high energies, it describes the spin transfer from a polarized electron beam to the scattered protons — the recent polarimetry of scattered nucleons at MAMI, BATES and Jefferson Lab has led to major discoveries in the physics of electromagnetic form factors of nucleons (for a review, see ref. [19]). Under the conditions of the FILTEX experiment, the spin transfer from atomic electrons to the stored protons is comparable to that from the nuclear interaction of the stored protons with the polarized protons in the PIT [20].

Finally, at low to intermediate energies, proton–proton scattering at angles below and close to $\theta_{\rm acc}$ is strongly dominated by the Coulomb interaction, and an accurate evaluation of Coulomb–nuclear interference (CNI) effects is called upon. As a matter of fact, no direct experimental observations of *pp* scattering at angles $\theta \lesssim \theta_{\rm acc}$ are possible, such interactions are of relevance only to storage rings and their numerical evaluations require a careful small-angle extrapolations of CNI effects.

Meyer noticed that because of the very small mass of the electron, the deflection of the much heavier protons in *pe* interactions is so small, $\theta \leq m_e/m_p \ll \theta_{\rm acc}$, that all protons scattered off electrons stay within the beam. Meyer argued that with the $\uparrow\uparrow$ hyperfine state of the hydrogen atoms in the PIT of FILTEX, the polarization transfer from electrons to scattered protons is crucial for a quantitative interpretation of the filtering rate measured by the FILTEX collaboration. In the pure transmission picture, the FILTEX polarization rate as published in 1993, can be re–interpreted in terms of the effective polarization cross section as

$$\sigma_{\rm eff}({\rm FILTEX}) = 63 \pm 3 \quad ({\rm stat.})\ {\rm mb}.$$

The transmission effect from absorption by pure nuclear elastic scattering at all scattering angles, $\theta > 0$, based on the pre-93 SAID database [21], was

$$\sigma_1({\rm Nuclear}; \theta > 0) = 122\ {\rm mb}. \tag{1}$$

The factor of two disagreement between $\sigma_{\rm eff}$(FILTEX) and σ_1 called for an explanation. Meyer pointed out that scattering at angles $\theta \leq \theta_{\rm acc}$ does not contribute to the absorption of the stored beam. He also noticed the importance of CNI effects and, based on the pre-93 SAID database, he evaluated the CNI corrected value to be

$$\sigma_1({\rm CNI}; \theta > \theta_{\rm acc}) = 83\ {\rm mb}. \tag{2}$$

This substantial departure from 122 mb of Eq. (1) is entirely due to the interference of the Coulomb and double–spin dependent nuclear amplitudes.

The estimate in Eq. (2) was still about seven standard deviations from the above cited $\sigma_{\rm eff}$(FILTEX). The spin transfer from polarized target electrons to scattered protons, which in *ep* interactions all stay within the beam, amounts to a very large correction to Eq. (2) [1, 20]

$$\delta\sigma_1^{ep} = -70\ {\rm mb}. \tag{3}$$

Finally, Meyer added the polarization transfer from polarized protons in the PIT to stored protons scattered elastically within the acceptance angle,

$$\delta\sigma_1^{pp}({\rm CNI}; \theta_{\rm min} < \theta < \theta_{\rm acc}) = +52\ {\rm mb}, \tag{4}$$

which brought the theory to a perfect agreement with the experiment:

$$\begin{aligned} \sigma_{\rm eff} &= \sigma_1({\rm CNI}; \theta > \theta_{\rm acc}) + \delta\sigma_1^{pp}({\rm CNI}; \theta_{\rm min} < \theta < \theta_{\rm acc}) + \delta\sigma_1^{ep} \\ &= 135\ {\rm mb} + \delta\sigma_1^{ep} \\ &= (135 - 70)\ {\rm mb} = 65\ {\rm mb}. \end{aligned} \tag{5}$$

The experimental database on the double–spin dependence of the antiproton–proton interaction is basically nonexistent. For this reason, the success of Meyer's explanation of the FILTEX result, and the large value of $\delta\sigma_1^{ep} = -70$ mb, has prompted the idea to base the antiproton polarizer of the PAX experiment on the spin filtering by polarized electrons in a PIT [6]. In the context of the PAX proposal, the feasibility of the electron mechanism of spin filtering has thus become a major issue. During the past year, two groups of theorists from the Budker Institute [4] and the Institut für Kernphysik of Forschungszentrum Jülich [2] revisited the impact of SWR on the spin filtering process. Two very different formalisms have been used: the kinetic equation for the spin state population numbers by the Budker group, and the quantum evolution equation for the spin–density matrix of the stored beam in the Jülich approach. The final conclusions are identical, though. Roughly speaking, in the spin filtering by transmission one must divide the effect into the contribution from spin–dependent absorption by scattering of protons beyond the acceptance angle, $\theta \geq \theta_{\rm acc}$, and within the ring, *i.e.* $\theta \leq \theta_{\rm acc}$. The polarization brought into the stored beam by protons scattered within the beam, basically cancels the latter contribution of the transmission effect. For the pure electron target, both groups find an almost exact cancellation of the transmission and SWR effects — electromagnetic proton-spin flip interaction with electrons is entirely negligible and polarized electrons would not polarize stored protons. In the proton–proton interaction, one faces a similar cancellation of the transmission and SWR effects, so that to the first approximation one must start with Meyer's Eq. (2) — to this end, Meyer's Eq. (2) already accounts for SWR, and adding a correction [Eq. (4)] amounts to double counting. To the second approximation, the cancellation of the transmission and SWR effects is broken by spin–flip scattering (a full summary of formulas for the evolution of the beam polarization is given in Appendix A). However, in the theoretical calculations numerically the most important spin–flip cross section, $\Delta\sigma_1(\mathrm{SF})$, turns out to be negligibly small,

$$\Delta\sigma_1(\mathrm{SF}) \ll \delta\sigma_1^{ep}, \tag{6}$$

and for all practical purposes, the effective polarization cross section can be evaluated from Meyer's Eq. (2) corrected for the spin-flip:

$$\sigma_{\rm eff} = \sigma_1(\mathrm{CNI}; \theta > \theta_{\rm acc}) + \Delta\sigma_1(\mathrm{SF}) = \sigma_1(\mathrm{CNI}; \theta > \theta_{\rm acc}). \tag{7}$$

What then is the status of the Budker-Jülich interpretation of the FILTEX result? The conversion of the FILTEX polarization buildup rate, which by itself is the 20 (statistical) standard deviation measurement, into the polarization cross section $\sigma_{\rm eff}$ depends on the target polarization and the areal density of the PIT. The recent reanalysis [22] gave

$$\sigma_{\rm eff}(\mathrm{FILTEX} - 2005) = 72.5 \pm 5.8 \quad \mathrm{mb}, \tag{8}$$

where both the statistical and systematical errors are included. The theoretical calculation of $\sigma_1(\mathrm{CNI}; \theta > \theta_{\rm acc})$ requires a careful extrapolation of the SAID output to extremely small scattering angles $\theta \leq \theta_{\rm acc}$, way beyond the angular range SAID was ever supposed to be applied. The latest version of the SAID database, SAID-SP05 [21], gives $\sigma_{\rm eff} = 85.6$ mb, which is consistent with the FILTEX result within the quoted error bars. The above result is found upon the extrapolation of separate spin observables which enter in σ_1 (see Appendix A). If the whole integrand is extrapolated, which is advisable, one finds $\sigma_{\rm eff}$ = 83 mb, as shown in Fig. 1. Starting with the Nijmegen nuclear phase shifts [23], and adding in the Coulomb interaction effects, the Budker group finds for the same quantity 89 mb [4].

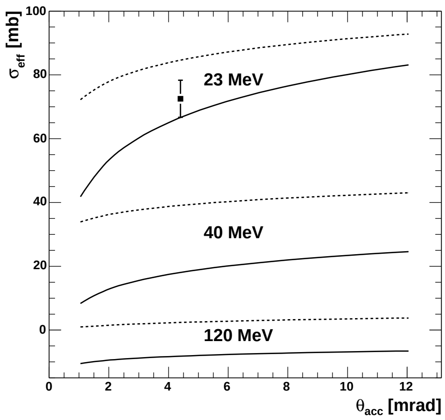

Figure 1: Experimentally observed polarization buildup cross section $\sigma_{\rm eff}$ for a 23 MeV proton beam in the TSR experiment [15] after re–analysis [22] as function of the acceptance angle $\theta_{\rm acc}$. The solid curves show the prediction from Meyer's approach which includes filtering on electrons of the polarized atoms [1, 20]. The dashed curves denote the prediction of the Budker-Jülich approach with self–cancellation of filtering on electrons [4, 2].

2.2 How to distinguish the two polarization buildup scenarios? Filtering and Depolarization

There is a fair agreement between the Budker-Jülich evaluation and the FILTEX result, perhaps, not as perfect as with Meyer's estimate (Fig. 1). The two competing approaches to the theoretical evaluation of $\sigma_{\rm eff}$ differ in their treatment of cancellations between the transmission and scattering–within–the–ring effects. One would reiterate that the double–spin QED interaction between electrons and antiprotons is well known, the hope of profiting from this knowledge is a quite natural one, and whether the self–cancellation of spin filtering on polarized electrons is correct or not, must be tested *experimentally* in a proton storage ring before proceeding to filtering experiments with antiprotons. The rôle of the spin-flip scattering in filtering also must be understood experimentally.

The question thus posed is: What is the electron contribution to the combined spin-filtering on an atom? Large Meyer-Horowitz cross section of the electron-to-proton spin transfer $\delta\sigma_1^{ep}$ as in Eq. (5) or negligibly small spin-flip cross section $\Delta\sigma_1({\rm SF})$ as in Eq. (7). There are several pathways to the discrimination of the two scenarios.

The ideal solution would be the null experiment with two hyperfine states in the PIT such that the net nuclear polarization of the target is zero, while the net electron polarization is large. This requires operating the PIT with longitudinal target polarization in a strong longitudinal guide field, where the stable beam spin direction must be aligned longitudinally at the target as well to preserve the longitudinal polarization of the stored protons. In a single hyperfine state mode, one could rely upon the different energy dependences of the electron and nuclear mechanisms. This point is made clear by the expected energy dependence of the effective polarization cross section, shown in Fig. 1. At COSY, an upper limit for the expected acceptance angle at the ANKE IP is ~ 3 mrad, and the Budker-Jülich and Meyer-Horowitz predictions for filtering at $T = 45$ MeV differ by a factor ≈ 3. One would conclude that a precision measurement of σ_{eff} at this energy would be sufficient to disprove or prove the presence of filtering on polarized electrons. The second null experiment — for the pure nuclear mechanism — can be performed by injecting two hyperfine states with identical proton polarizations and opposite electron polarizations. Such a pure nuclear polarization in the target can only be realized in a strong longitudinal holding field. That would require installation of a Siberian snake, but in the long run such an investment could well be worth the trouble, because the longitudinal filtering cross section is dramatically larger than the transverse one, as we discuss briefly below.

The Horowitz–Meyer *ep* contribution to σ_{eff} decreases with kinetic energy $\sim 1/T$. In contrast to that, the contribution from the nuclear *pp* interaction has a distinctly different energy dependence. In Fig. 3, we show predictions from the Budker-Jülich model for the energy dependence of the polarization of stored protons after filtering for 2 to 5 beam lifetimes τ_b using the ANKE and new PIT at TP1. The actual beam lifetime depends on the target density. The calculated beam lifetime

$$\tau_b(T) = \frac{1}{\sigma_{\text{tot}}(T)\, d_{\text{eff}}\, f_{\text{rev}}(T)} \tag{9}$$

is shown in Fig. 2. Here $d_{\text{eff}} = d_t + d_{rg}$, where d_t is the areal thickness of the PIT and d_{rg} is the areal density of the residual gas in the ring, evaluated assuming a residual gas pressure of 10^{-9} mbar, produced mainly by H_2, f_{rev} denotes the revolution frequency. The achievable target thickness with the ANKE and HERMES target [24] is discussed in more detail in Sec. 4.1 of ref. [7]. The total cross section including the Coulomb interaction is obtained using the SAID-SP05 solution by evaluation of

$$\sigma_{\text{tot}} = \int_{\theta_{\text{acc}}}^{\theta_{\text{max}}} \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} \mathrm{d}\Omega\ . \tag{10}$$

The energy dependence of the resulting beam polarization in COSY using the ANKE PIT and the new PIT at TP 1, shown in Fig. 3, closely follows, although because of the CNI effects it is not identical to the experimentally measured energy dependence of the transverse total cross section $\Delta\sigma_T$, shown in Fig. 5 of ref. [7]. The results for σ_{eff} from our calculations show that CNI makes σ_{eff} substantially smaller than $\Delta\sigma_T$ — the same trend as seen from a comparison of the results in Eq. (1) and Eq. (2).

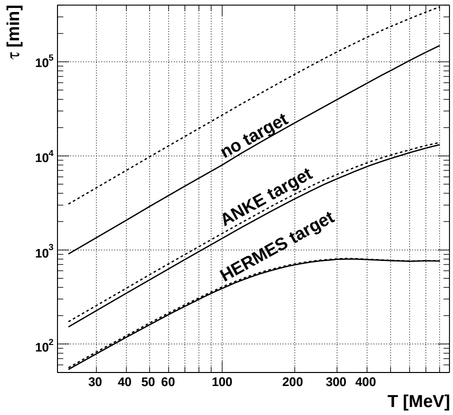

Figure 2: Lifetime of the COSY beam calculated using Eq. (9) for a target thickness $d_t^{\rm ANKE} = 2\times10^{13}$ atoms/cm^2 ($\theta_{\rm acc}^{\rm ANKE} = \sqrt{10}$ mrad) and $d_t^{\rm NEW} = 6\times10^{14}$ atoms/cm^2 ($\theta_{\rm acc}^{\rm NEW} = 10$ mrad) using the HERMES target, and a residual gas pressure of 10^{-9} mbar, produced mainly by H_2. The solid curves correspond to an average ring acceptance angle of $\theta_{\rm acc} = 1$ mrad, while the dashed ones to $\theta_{\rm acc} = 2$ mrad. (A discussion of the density achievable with the new low–β section for the HERMES target is given in Sec. 4.1 of ref. [7].)

At present, electron cooling at COSY is available only up to kinetic energies around $T = 120$ MeV, and the interesting energy dependence at higher energies can not be exploited. However, the possibility of filtering at 800 MeV, where stochastic cooling becomes available, must be further explored.

The figure of Merit of the beam, defined by the beam polarization squared multiplied by the beam intensity I, $P^2 \times I$, shown in Fig. 4 as function of beam energy, shows that the optimium energy for spin filtering at COSY is around $T_p = 60$ MeV.

As emphased above, the conceptial difference is between the large cross section of the electron-to-proton spin transfer $\delta\sigma_1^{ep}$ in the Meyer-Horowitz scenario and the negligibly small spin-flip cross section $\Delta\sigma_1(\mathrm{SF})$ in the Budker-Jülich scenario. A verification that the spin–flip cross section is small would be an important cross check of the theory of filtering. As discussed in Sec. 1 and in Appendix A, one can conveniently evaluate spin-flip effects from the measurements of the rate of depolarization of the polarized proton beam in an unpolarized internal target. In order to minimize the nuclear spin effects, it is advisable to take atoms with spin-0 nuclei, ^{4}He emerges as a natural choice.

3 Depolarization measurements

3.1 Experimental setup

Proton–α scattering at COSY injection energy provides good counting rates and a large analyzing power. In Fig. 5, the differential cross section and the analyzing power, both

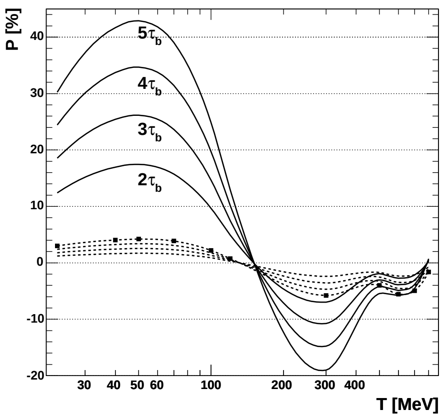

Figure 3: Polarization buildup in the COSY ring for a target thickness $d_t^{\mathrm{ANKE}} = 2 \times 10^{13}$ atoms/cm^2 as function of beam energy (dashed lines) for an average ring acceptance angle of $\theta_{\mathrm{acc}} = 2$ mrad using the Budker-Jülich approach [2], with an acceptance angle at the target of $\theta_{\mathrm{acc}}^{\mathrm{ANKE}} = \sqrt{\frac{\varepsilon}{\beta_{\mathrm{ANKE}}}} = \sqrt{\frac{30\pi\,\mathrm{mm\,mrad}}{3\,\mathrm{m}}} = \sqrt{10}$ mrad. The duration of the filtering process is given in units of the beam lifetime τ_b. Also shown is the polarization buildup in the COSY ring for a target thickness of $d_t^{\mathrm{NEW}} = 6 \times 10^{14}$ atoms/cm^2 as function of beam energy for the ring acceptance angle of $\theta_{\mathrm{acc}} = 2$ mrad using the Budker-Jülich approach [2]. The new target region acceptance angle is $\theta_{\mathrm{acc}}^{\mathrm{NEW}} = 10$ mrad.

taken from ref. [25], is shown together with the calculated $\mathrm{FOM}(\theta_{\mathrm{cm}}) = \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(\theta_{\mathrm{cm}}) \cdot A_y(\theta_{\mathrm{cm}})^2$. The FOM for the determination of the beam polarization reaches a maximum near $\theta_{\mathrm{cm}}^{\mathrm{opt}} = 150°$, which leads to the experimental setup of the storage cell with respect to the Silicon Tracking Telecsopes (STT's) [26], as shown in Fig. 6.

3.2 Limitation of the target thickness of ^{4}He

Multiple scattering of beam particles leads to emittance growth in the beam that has to be compensated by electron cooling. The requirement is to provide stable beam conditions during the experiment, thus the maximum target density that can be used in the experiment depends on the tolerable emittance growth. Using the BETACOOL code, a number of calculations were performed to determine the maximum target thickness the electron is able to compensate in terms of multiple scattering. In Fig. 7 the time-dependence of the emittance is shown for the highest possible target thickness of $d_t^{\mathrm{ANKE}} = 3 \cdot 10^{14}$ ^{4}He/cm^2. It turns out that when during beam storage the RF in COSY is switched on, one can compensate the mean energy loss in the ring due to the presence of the target and one can utilize a target thickness that is about 50% higher compared to when the RF is off. As illustrated in Fig. 7, it is possible to adjust the COSY beam emittance such that it is the same with and without target. This may become important as the amount of depolarization in the COSY beam may depend on the actual beam emittance.

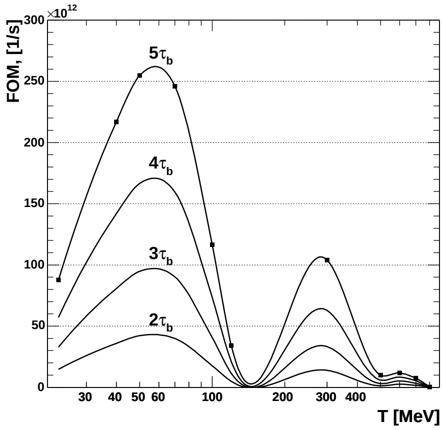

Figure 4: The figure-of-merit ($P^2 \times I$) for different beam lifetimes τ_b calculated for a target thickness $d_t^{\mathrm{NEW}} = 6 \times 10^{14}$ atoms/cm^2 ($\theta_{\mathrm{acc}}^{\mathrm{NEW}} = 10$ mrad) and an average ring acceptance angle of $\theta_{\mathrm{acc}} = 2$ mrad.

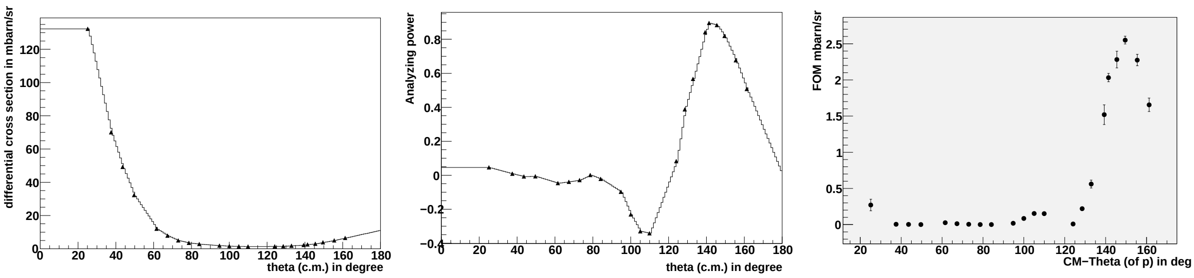

Figure 5: Differential cross section for $p\alpha$ scattering (left panel), analyzing power (middle), both from ref. [25], and FOM (right).

3.3 Measurement cycle

The experimental task is to determine the polarization lifetime of COSY together with the depolarizing effect predicted by Meyer-Horowitz (MH) due to the unpolarized electrons in the ^{4}He target. The total polarization lifetime can be written as

$$\frac{1}{\tau_p^{\mathrm{total}}} = \frac{1}{\tau_p^{\mathrm{COSY}}} + \frac{1}{\tau_p^{\mathrm{MH}}} , \tag{11}$$

where τ_p^{COSY} denotes the polarization lifetime of COSY alone, while τ_p^{MH} is attributed to the depolarizing effect due to the target electrons. One could argue that the two contributions can be determined during separate measurements. It is however possible that there are systematic variations τ_p^{COSY}, which should be traced simultaneously with the measurements. Thus, the measuring cycle we propose extends in total over a time period $T = 2 \cdot \tau_b = 1340$ s with maximum target thickness of $d_t^{\mathrm{ANKE}} = 3 \cdot 10^{14}$ ^{4}He/cm^2. The

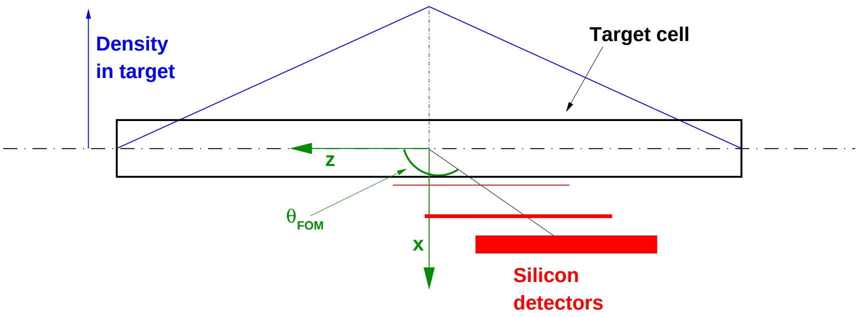

Figure 6: Experimental setup of the detector system using two STT's left and right of the storage cell (shown is only the detector on the left). Both STT'S are placed at an angle near the maximium FOM for a point-like target.

cycle is composed of three parts of duration $T = T_1 + T_2 + T_3$, during T_1, the ^{4}He target is switched on, during T_2, the target is switched off, and during T_3 the target is again switched on, as indicated in Fig. 9. From cycle to cycle the beam spin will be alternated from $\uparrow$ to $\downarrow$ to zero.

3.4 Count rate estimates

In order to arrive at a count rate estimate, we have taken for the COSY beam intensity $2 \cdot 10^{10}$ protons, that have been already accumulated after injection and electron cooling into COSY in a previous ANKE experiment. The storage cell is of cross section 20×20 mm^2 and of 220 mm length. The detector response using the density distribution of the target (Fig. 6) has been simulated using a Monte-Carlo program. Shown in Fig. 8 are the count rates vs scattering angle and vs azimuth in the left and right STT for a beam polarization of $P = 0.8$.

3.5 Determination of the effect

After interaction with the ^{4}He target the beam polarization has either developed an additional depolarization due to the target electrons or not. This situation is indicated in Fig. 9, where the beam polarization is measured every ten seconds. Subsequently the combined statistics of a 4 week measurement is analyzed using the fitting function

$$\ln[P(t)] = \begin{cases} p_0 - t \cdot (p_1 + p_2) & : \quad t \le t_1 \\ p_0 - t_1 \cdot (p_1 + p_2) - (t - t_1) \cdot p_2 & : \quad t_1 < t \le t_2 \\ p_0 - t_1 \cdot (p_1 + p_2) - (t_2 - t_1) \cdot p_2 - (t - t_2) \cdot (p_1 + p_2) & : \quad t_2 < t \end{cases},$$

with the definitions $p_0 = \ln[P(t=0)]$, $p_1 = 1/\tau_p^{\rm MH} = 1/\tau_p^{\rm total} - 1/\tau_p^{\rm COSY}$, and $p_2 = 1/\tau_p^{\rm COSY}$. The result of this fit is indicated in Table 1 below.

This result corresponds to a significance of 5.5 standard deviations of the measurement of $\tau_p^{\rm MH}$ for $2 \cdot 10^{10}$ protons injected into COSY. If 3, 4, or even $5 \cdot 10^{10}$ protons

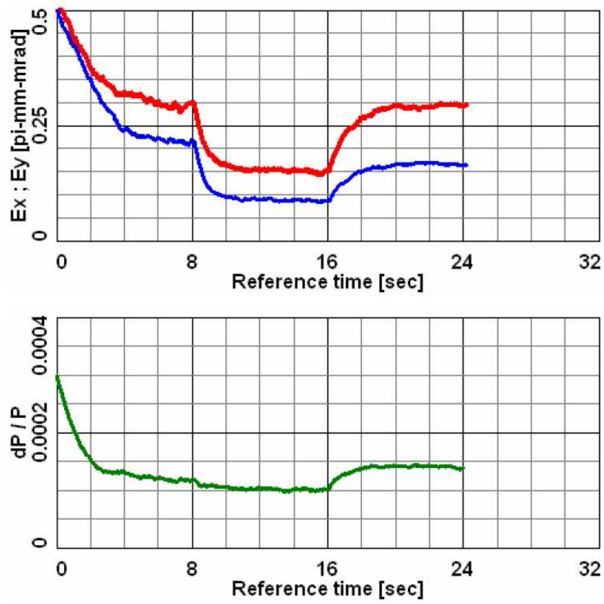

Figure 7: Dependence of the horizontal and vertical beam emittance in the presence of a target thickness of $d_t^{\rm ANKE} = 3 \cdot 10^{14}$ ^{4}He/cm^2 using a current of 250 mA in the electron cooler. At $t = 8$ s, the target is switched off and the beam emittances are reduced to about half the equilibrium value with target. At $t = 16$ s, the electron current is reduced to 40 mA.

	Input	Output	Error
$\ln[P(t=0)]$	-0.2231	-0.2294	2e-4
$1/\tau_p^{\rm MH}$	8.82e-6	8.5e-6	1.6e-6
$1/\tau_p^{\rm COSY}$	1.11e-5	1.14e-6	6.7e-7

Table 1: Result of the fit to the time-dependent polarization of Fig. 9 using the fit function given above.

could be injected, the significance would improve to 6.8, 7.9, and 8.8 standard deviations, respectively.

4 Additional requirements

4.1 Beam lifetime in COSY

The present beam lifetimes at COSY injection energy are roughly two orders of magnitude shorter than one would expect theoretically (Fig. 2), see ref. [27, 28]. Careful tuning of the machine parameters, and a reduction of closed orbit distortions should be carried out.

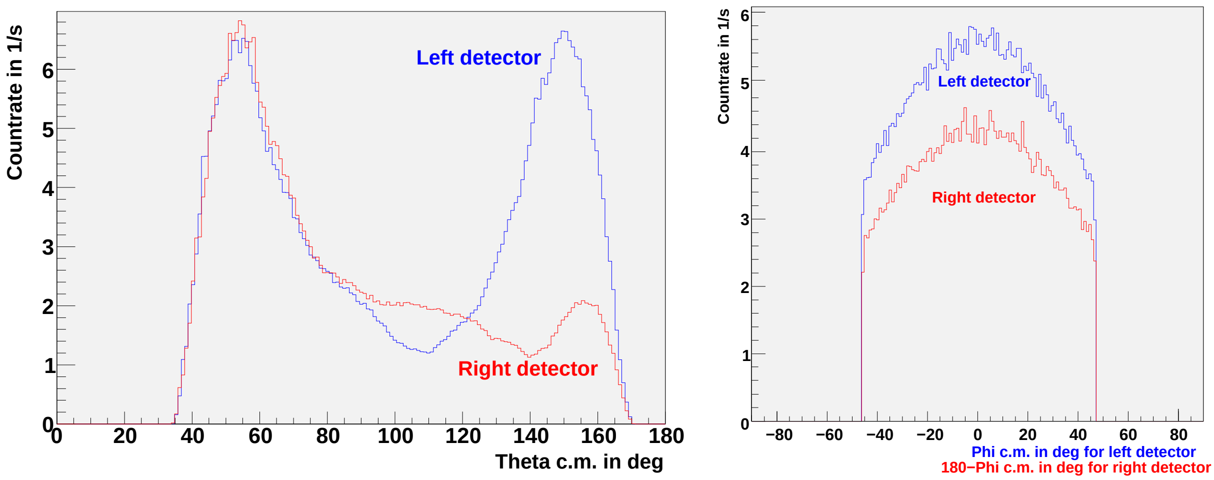

Figure 8: Count rate vs scattering angle (left panel) and vs azimuth in the left and right STT's and a beam polarization of $P = 0.8$. For the actual detector setup, the average of the trigonometric function $< \cos\phi >= 0.9054$.

This is urgently needed for the present proposal and for all subsequent spin filtering studies at COSY.

4.2 Beam polarization lifetime in COSY

An analysis of the beam polarization lifetime of data taken in 2003 with polarized protons revealed that there are indeed large differences observed between different energies. In Fig. 10, the result of our analysis is shown. Our analysis $\tau_p^{\mathrm{COSY}}(500\ \mathrm{MeV}) = (29.1 \pm 1.3)$ min and $\tau_p^{\mathrm{COSY}}(800\ \mathrm{MeV}) = (381.6 \pm 138.3)$ min. It is therefore crucial, to improve the polarization lifetime in COSY.

4.3 Beam profile monitor

Since the depolarization of COSY may depend on the emittance of the stored beam, it is important that during the measurements, the beam emittance is measured. One way to accomplish this is to observe the H^0's produced by recombination in the electron cooler, which is possible, since we will leave the ANKE spectrometer magnet in 0° position. From the hitpattern in the installed detector system, one could determine the horizontal and vertical widths of the beam, and together with the horizontal and vertical β functions in the electron cooler, the horizontal and vertical beam emittances can be determined. The β functions in the electron are of course not exactly known, but we just have to make sure that the beam emittances with and without target are the same, in order to avoid a possible emittance dependent depolarization.

Another option is to use the new beam profile monitor of COSY, which should be operational by the time we perform our measurements.

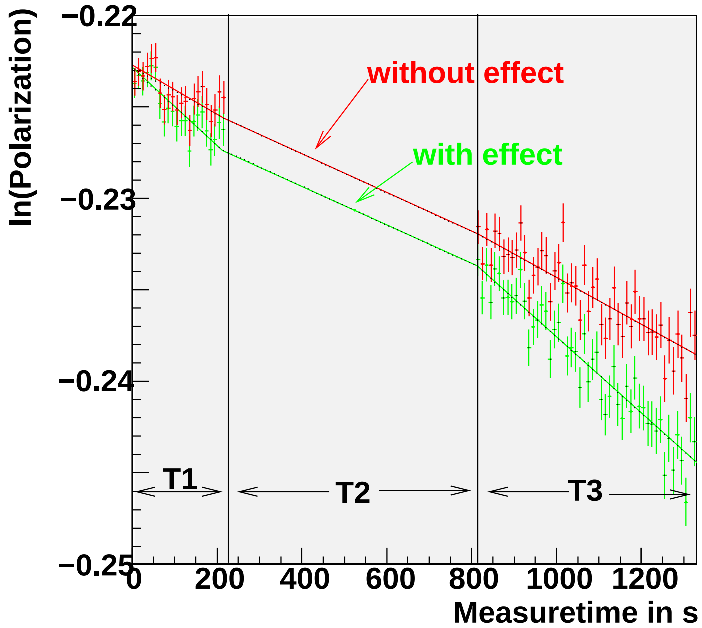

Figure 9: Plot of the resulting accuracies of the time dependent beam polarization vs measuring time. The cycle is composed of three parts of duration $T = T_1 + T_2 + T_3 = 1340$ s, target-off times are 214 s $< t <$ 814 s.

5 Beam request

1. **We request two weeks of beam time for machine development in the first half of 2007** to carry out the necessary preparatory investigations with respect to improvements of the beam lifetimes and the beam polarization lifetimes, and to gain operating experience with the new beam profile monitor. The H^0 detection to obtain a beam emittance measurement should be also exercised.

2. **We request a total beam time of four weeks for data taking to carry out the proposed measurements.** The beam time should be preceded by a machine development week in order to set up electron cooling and stacking of the beam through the storage cell at ANKE, and to provide a high beam polarization and beam polarization lifetime at injection.

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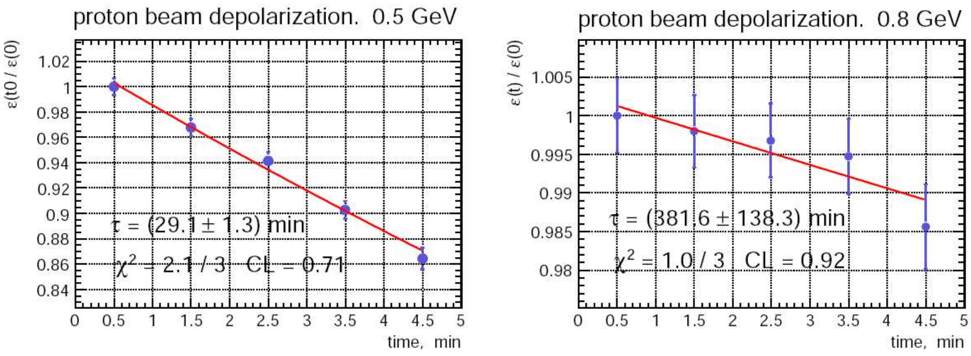

Figure 10: Measurement of the polarization lifetime of COSY using ANKE data from 2003 for two energies 500 and 800 MeV.

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A Evolution of the spin-density matrix of the stored beam

A.1 Spin-filtering in the transmission process

In fully quantum-mechanical approach, the beam of stored antiprotons must be described by the spin-density matrix

$$\hat{\rho}(\boldsymbol{p}) = \frac{1}{2}[I_0(\boldsymbol{p}) + \boldsymbol{\sigma s}(\boldsymbol{p})], \tag{12}$$

where $I_0(\boldsymbol{p})$ is the density of particles with the transverse momentum $\boldsymbol{p}$ and $\boldsymbol{s}(\boldsymbol{p})$ is the corresponding spin density. In optical experiments one often deals with the pure transmitted beam which propagates at exactly zero scattering angle. In storage rings the regime of pure transmission would correspond to a ring with vanishing acceptance angle, i.e., when particle which scattered in PIT at any non-vanishing angle will be thrown out of the ring.

As far as the pure transmission is concerned, it can be described by the polarization dependent refraction index for the hadronic wave, given by the Fermi-Akhiezer-Pomeranchuk-Lax formula [16]:

$$\hat{n} = 1 + \frac{1}{2p} N\hat{\mathsf{F}}(0). \tag{13}$$

The forward NN scattering amplitude $\hat{\mathsf{F}}(0)$ depends on the beam and target spins, and the polarized target acts as an optically active medium. It is convenient to use instead the Fermi Hamiltonian (with the distance z traversed in the medium playing the rôle of time)

$$\hat{H} = \frac{1}{2}N\hat{F}(0) = \frac{1}{2}N[\hat{R}(0) + i\hat{\sigma}_{tot}], \tag{14}$$

where $\hat{R}(0)$ is the real part of the forward scattering amplitude and N is the volume density of atoms in the target. The anti-hermitian part of the Fermi hamiltonian, $\propto \hat{\sigma}_{tot}$, describes the absorption (attenuation) in the medium.

In terms of the Fermi hamiltonian, the quantum-mechanical evolution equation for the spin-density matrix of the transmitted beam reads

$$\begin{aligned} &\frac{d}{dz}\hat{\rho}(\boldsymbol{p}) = i\Big(\hat{H}\hat{\rho}(\boldsymbol{p}) - \hat{\rho}(\boldsymbol{p})\hat{H}^{\dagger}\Big) \\ &= \underbrace{i\frac{1}{2}N\Big(\hat{R}\hat{\rho}(\boldsymbol{p}) - \hat{\rho}(\boldsymbol{p})\hat{R}\Big)}_{\text{Pure refraction}} - \underbrace{\frac{1}{2}N\Big(\hat{\sigma}_{tot}\hat{\rho}(\boldsymbol{p}) + \hat{\rho}(\boldsymbol{p})\hat{\sigma}_{tot}\Big)}_{\text{(Pure attenuation)}} \end{aligned} \tag{15}$$

In the specific case of spin-$\frac{1}{2}$ protons interacting with the spin-$\frac{1}{2}$ protons (and electrons) the total cross section and real part of the forward scattering amplitude are parameterized as

$$\hat{\sigma}_{tot} \;=\; \sigma_0 + \underbrace{\sigma_1(\boldsymbol{\sigma}\cdot\boldsymbol{Q}) + \sigma_2(\boldsymbol{\sigma}\cdot\boldsymbol{k})(\boldsymbol{Q}\cdot\boldsymbol{k})}_{spin-sensitive\ \ loss},$$

$$\hat{R} \;=\; R_0 + \underbrace{R_1(\boldsymbol{\sigma}\cdot\boldsymbol{Q}) + R_2(\boldsymbol{\sigma}\cdot\boldsymbol{k})(\boldsymbol{Q}\cdot\boldsymbol{k})}_{\boldsymbol{\sigma}\cdot\text{Pseudomagnetic field}} \tag{16}$$

Then, upon some algebra, one finds the evolution equation for the beam polarization $\boldsymbol{P} = \boldsymbol{s}/I_0$

$$\begin{aligned} d\boldsymbol{P}/dz \;&=\; \underbrace{-N\sigma_1(\boldsymbol{Q} - (\boldsymbol{P}\cdot\boldsymbol{Q})\boldsymbol{P}) - N\sigma_2(\boldsymbol{Q}\boldsymbol{k})(\boldsymbol{k} - (\boldsymbol{P}\cdot\boldsymbol{k})\boldsymbol{P})}_{\text{(Polarization buildup by spin−sensitive loss)}} \\ &+\; \underbrace{NR_1(\boldsymbol{P}\times\boldsymbol{Q}) + nR_2(\boldsymbol{Q}\boldsymbol{k})(\boldsymbol{P}\times\boldsymbol{k})}_{\text{(Spin precession in pseudomagnetic field)}}, \end{aligned} \tag{17}$$

where we indicated the rôle of the anti-hermitian – attenuation – and hermitian – pseudomagnetic field – parts of the Fermi Hamiltonian. It is absolutely important that the cross sections $\sigma_{0,1,2}$ in the evolution equation for the transmitted beam describe all-angle scattering.

Although the effects of precession of the spin of the stored beam in the psedomagnetic field of the PIT are missed in kinetic equation approach [4], upon the averaging over these precessions the density matrix approach [2] simplifies to the kinetic equation for spin population numbers.

A.2 Spin filtering from particles scattered elastically within the ring

Stored particles which scatter elastically in PIT at angles smaller than the ring acceptance angle, $\theta_{\rm acc}$, stay within the beam. A polarization of the elastically scattered particle is different from that of the incident particle. A mixing of spins of transmitted beam and of particles scattered within the ring is described as follows [2]. The quasielastic proton-atom collsiions can well be approximated by an incoherent sum of *ep* and *pp* differential cross sections:

$$\frac{d\hat{\sigma}_E}{d^2\boldsymbol{q}} = \frac{1}{(4\pi)^2}\hat{\mathsf{F}}(\boldsymbol{q})\hat{\rho}\hat{\mathsf{F}}^\dagger(\boldsymbol{q}) = \frac{1}{(4\pi)^2}\hat{\mathsf{F}}_e(\boldsymbol{q})\hat{\rho}\hat{\mathsf{F}}_e^\dagger(\boldsymbol{q}) + \frac{1}{(4\pi)^2}\hat{\mathsf{F}}_p(\boldsymbol{q})\hat{\rho}\hat{\mathsf{F}}_p^\dagger(\boldsymbol{q}) \tag{18}$$

The evolution equation for the spin-density matrix, corrected for SWR, takes the form

$$\begin{aligned} \frac{d}{dz}\hat{\rho} = i[\hat{H},\hat{\rho}] \;&=\; \underbrace{i\frac{1}{2}N\Big(\hat{R}\hat{\rho}(\boldsymbol{p}) - \hat{\rho}(\boldsymbol{p})\hat{R}\Big)}_{\textit{Pure precession \& refraction}} \\ &-\; \underbrace{\frac{1}{2}N\Big(\hat{\sigma}_{tot}\hat{\rho}(\boldsymbol{p}) + \hat{\rho}(\boldsymbol{p})\hat{\sigma}_{tot}\Big)}_{\textit{Evolution by loss}} \\ &+\; \underbrace{N\int^{\Omega_{acc}} \frac{d^2\boldsymbol{q}}{(4\pi)^2}\hat{\mathsf{F}}(\boldsymbol{q})\hat{\rho}(\boldsymbol{p}-\boldsymbol{q})\hat{\mathsf{F}}^\dagger(\boldsymbol{q})}_{\text{Lost \& found: scattering within the beam}} \end{aligned} \tag{19}$$

Notice the convolution of the transverse momentum distribution in the beam with the differential cross section of quasielastic scattering. This broadening of the momentum distribution is compensated for by the focusing and the beam cooling in a storage ring.

The ep scattering is pure SWR and the ep contribution to the transmission effect is exactly cancelled by the ep contribution to elastic SWR - electrons in the target are invisible. Upon some algebra, one finds the SWR-corrected coupled evolution equations

$$\frac{d}{dz}\begin{pmatrix} I_0 \\ s \end{pmatrix} = -N \begin{pmatrix} \sigma_0(>\theta_{\rm acc}) & Q\sigma_1(>\theta_{\rm acc}) \\ Q(\sigma_1(>\theta_{\rm acc}) + \Delta\sigma_1) & \sigma_0(>\theta_{\rm acc}) + 2\Delta\sigma_0 \end{pmatrix} \cdot \begin{pmatrix} I_0 \\ s \end{pmatrix}, \tag{20}$$

Here the spin-flip (SF) cross sections $\Delta\sigma_{0,1}(SF, \theta < \theta_{\rm acc})$ describe the imperfect cancellation between the transmission and SWR effects from proton-proton scattering within the acceptance angle. In terms of the standard observables as defined by Bystricky et al. (our θ is the scattering angle in the laboratory frame) [29]

$$\begin{aligned}
&\sigma_0^{el}(>\theta_{\rm acc}) = \frac{1}{2}\int_{\theta_{\rm acc}} d\Omega \frac{d\sigma}{d\Omega}, \\
&\sigma_1^{el}(>\theta_{\rm acc}) = \frac{1}{2}\int_{\theta_{\rm acc}} d\Omega \Big(d\sigma/d\Omega\Big)\Big(A_{00nn} + A_{00ss}\Big) \\
&\Delta\sigma_0(SF, \theta < \theta_{\rm acc}) = \frac{1}{2}[\sigma_0^{el}(\leq \theta_{\rm acc}) - \sigma_0^{E}(\leq \theta_{\rm acc})] \\
&= \frac{1}{2}\int_{\theta_{\rm min}}^{\theta_{\rm acc}} d\Omega \frac{d\sigma}{d\Omega}\Big(1 - \frac{1}{2}D_{n0n0} - \frac{1}{2}D_{s'0s0}\cos(\theta) - \frac{1}{2}D_{k'0s0}\sin(\theta)\Big) \\
&\Delta\sigma_1(SF, \theta < \theta_{\rm acc}) = \sigma_1^{el}(\leq \theta_{\rm acc}) - \sigma_1^{E}(\leq \theta_{\rm acc})\frac{1}{2} = \int_{\theta_{\rm min}}^{\theta_{\rm acc}} d\Omega \frac{d\sigma}{d\Omega} \\
&\times\Big(A_{00nn} + A_{00ss} - K_{n00n} - K_{s'00s}\cos(\theta) - K_{k'00s}\sin(\theta)\Big)
\end{aligned} \tag{21}$$

Here certain spin-transfer and depolarization parameters enter with the coefficients $\sin(\theta), \cos(\theta)$ which describe the projection of the polarization of the scattered particle onto the normal to the storage ring plane. One can argue that [4]

$$\Delta\sigma_1(SF, \theta < \theta_{\rm acc}) \leq 2\Delta\sigma_1(SF, \theta < \theta_{\rm acc}). \tag{22}$$

Similar equations follow also for the pure longitudinal polarization maintained by Siberian Snakes.

A.3 Different regimes of filtering and depolarization

A.3.1 A generic solution

The SWR-corrected coupled evolution equations have the solutions $\propto \exp(-\lambda_{1,2}Nz)$ with the eigenvalues

$$\begin{aligned}
\lambda_{1,2} &= \sigma_0 + \Delta\sigma_0 \pm Q\sigma_3 \\
Q\sigma_3 &= \sqrt{Q^2\sigma_1(\sigma_1 + \Delta\sigma_1) + (\Delta\sigma_0)^2},
\end{aligned} \tag{23}$$

The polarization buildup follows the law [4, 2]

$$P(z) = -\frac{Q(\sigma_1 + \Delta\sigma_1)\tanh(Q\sigma_3 Nz)}{Q\sigma_3 + \Delta\sigma_0 \tanh(Q\sigma_3 Nz)}. \tag{24}$$

We recall that the integrated depth in the target equals

$$z = d\nu t \tag{25}$$

where t is the filtering time, ν is the beam revolution frequency and d is thickness of the target. For targets of nonuniform density one makes a substitution $Nd = d_t$, where d_t is the areal density of PIT.

A.3.2 A regime of pure transmission

In this regime $\theta_{\rm acc} \to 0$ and $\Delta\sigma_{0,1}(SF, \theta < \theta_{\rm acc}) = 0$. Coupled evolution equations for the spin-density matrix take the form

$$\frac{d}{dz}\begin{pmatrix} I_0 \\ s \end{pmatrix} = -N \begin{pmatrix} \sigma_0(> \theta_{\rm min}) & Q\sigma_1(> \theta_{\rm min}) \\ Q\sigma_1(> \theta_{\rm min}) & \sigma_0(> \theta_{\rm acc}) \end{pmatrix} \cdot \begin{pmatrix} I_0 \\ s \end{pmatrix}, \tag{26}$$

The two eigen-solutions $\propto \exp(-\lambda_{1,2} Nz)$ have the eigenvalues

$$\lambda_{1,2} = \sigma_0 \pm Q\sigma_1 \tag{27}$$

One can reduce Eq. (eq:Transmission) to Meyer's equation

$$\frac{dP}{dz} = -N\sigma_1 Q(1 - \boldsymbol{P}^2) \tag{28}$$

which has the solution

$$P(z) = -\tanh(Q\sigma_1 Nz). \tag{29}$$

In the regime of pure transmission any spin-dependent loss of the stored beam leads to a 100 per cent polarization of the stored beam irrespective of the target polarization.

A.3.3 A regime of weak spin-flip

As discussed in the main body of this proposal, the theoretical calculations exhibit a very strong CNI effect in the double-spin cross section σ_1, see a difference between the results (1) and (2). The same theoretical calculations suggest a weak interference between the pure Coulomb and spin-orbit interactions, which entails small values of $\Delta\sigma_{0,1}(SF, \theta < \theta_{\rm acc}) = 0$, much smaller than $\sigma_0^{el}(> \theta_{\rm acc})$. Roughly speaking, in the interaction with the zero-charge and the spin-0 target, an order of magnitude estimate for the contribution of small-angle scattereing to the spin-flip cross section is

$$\Delta\sigma_0 \lesssim \sigma_{tot}\theta_{\rm acc}^2 \lesssim 10^-4\sigma_{tot}. \tag{30}$$

If this is the case, then the effect of spin-flip $\Delta\sigma_0$ on both the beam lifetime and the filtering cross section will be negligible small. Simultaneously, the effective small-time polarization cross section would equal

$$\sigma_{eff} \approx -Q(\sigma_1 + \Delta\sigma_1). \tag{31}$$

For all the practical purposes, it is entirely dominated by σ_1: the theoretical evaluation at the FILTEX energy gives $\Delta\sigma_1 \approx 6 \cdot 10^{-3}$mb vs the results (2).

This theoretical point that

$$|\Delta\sigma_1| \ll \sigma_1 \tag{32}$$

must be verified experimentally, though.

A.3.4 Pure electron target and spin-flip: evolution equation

Elastic scattering of protons in the pure electron PIT is entirely within the ring acceptance,

$$\theta \leq m_e/m_p \ll \theta_{\rm acc}. \tag{33}$$

The corollary is that such a small-angle scattering does not remove protons from the beam,

$$\begin{aligned} \sigma_0(>\theta_{\rm acc}) &= 0, \\ \sigma_1(>\theta_{\rm acc}) &= 0, \end{aligned} \tag{34}$$

and

$$Q\sigma_3 = \Delta\sigma_0. \tag{35}$$

The evolution equation from the beam spin-density matrix takes the form

$$\frac{d}{dz}\left(\begin{array}{c} I_0 \\ s \end{array}\right) = -N \left(\begin{array}{cc} 0 & 0 \\ Q\sigma_1(>\theta_{\rm acc}) & 2\Delta\sigma_0 \end{array}\right) \cdot \left(\begin{array}{c} I_0 \\ s \end{array}\right),$$

Its solutions satisfy an expected conservation of the number of stored particles, $I_0(z) = I_0(0)$ — a nice property emphasized by Milstein-Strakhovenko and Walcher et al.

The evolution of the beam polarization depends on the relationship between two spin-flip cross sections

$$P(z) = P(0)\exp(-2N\Delta\sigma_0 z) + Q\frac{\Delta\sigma_1}{2\Delta\sigma_0}\Big\{1 - \exp(-2N\Delta\sigma_0 z)\Big\}. \tag{36}$$

There are two cases of the practical interest:

A.3.5 Spin-filtering of unpolarized protons

Here $P(0) = 0$. At small filtering times, $2N\Delta\sigma_0 z \lesssim 1$, one has

$$P(z) = Q\frac{\Delta\sigma_1}{2\Delta\sigma_0}\Big\{1 - \exp(-2N\Delta\sigma_0 z)\Big\} = QNz\Delta\sigma_1. \tag{37}$$

At large filtering times

$$P(z) = Q\frac{\Delta\sigma_1}{2\Delta\sigma_0} \tag{38}$$

and the limiting beam polarization would not exceed the target electron polarization, see inequlaity (22).

A.3.6 Depolarization of polarized protons by unpolarized electrons

Here $Q = 0$ and

$$P(z) = P(0) \exp(-2N\Delta\sigma_0 z) \tag{39}$$

Depolarization of stored polarized protons in unpolarized target measures $\Delta\sigma_0$. In view of the inequlaity (22), the experimental determination of $\Delta\sigma_0$ sets simultaneously an upper bound on $\Delta\sigma_1$.