

Exercises in Statistical Data Analysis

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Exercise 7.1: Galileo's studies of motion included experiments with a ball and an inclined ramp. The ball's trajectory is made horizontal before it falls over the edge, as shown in Fig. 7.1. The horizontal distance d from the edge to the point of impact is measured for different values of the initial height of the ball h . Five data points obtained by Galileo in 1608 are shown in Table 7.1.¹

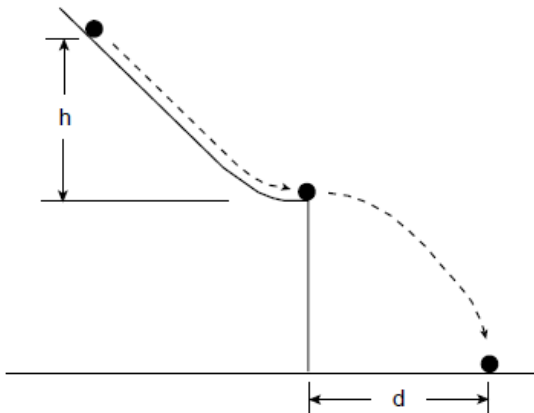


Figure 7.1: The configuration of the ball and ramp experiment performed by Galileo.

Table 7.1: Galileo's data on horizontal distance before impact d for five values of the starting height h . The units are punti (points); one punto is slightly less than 1 mm.

h	d
1000	1500
828	1340
800	1328
600	1172
300	800

Assume the heights h are known with negligible error, and that the horizontal distances d can be regarded as independent Gaussian random variables with standard deviations of $\sigma = 15$ punti.

¹See Stillman Drake and James MacLachlan, Galileo's discovery of the parabolic trajectory, *Scientific American* 232 (March 1975) 102; Stillman Drake, *Galileo at Work*, University of Chicago Press, Chicago (1978).

(It is not actually known how Galileo estimated the measurement uncertainties, but 1–2% is plausible.) In addition, we know that if $h = 0$, then the horizontal distance d will be zero, i.e. if the ball is started at the very edge of the ramp, it will fall straight down to the floor.

(a) Consider relations between h and d of the form

$$d = \alpha h \tag{7.1}$$

and

$$d = \alpha h + \beta h^2. \tag{7.2}$$

Find the least-squares estimators for the parameters α and β . What are the values of the minimized χ^2 and the P -values for the two hypotheses?

(b) Assume a relation of the form

$$d = \alpha h^\beta. \tag{7.3}$$

Write a computer program to perform a least squares fit of α and β . Note that this is a nonlinear function of the parameters and must be solved numerically. ~~A solution using the MINUIT minimization routines from the CERN library is given in `fit_galileo.f`, `fcn_galileo.f`.~~

(c) Galileo regarded the motion as the superposition of horizontal and vertical components, where the horizontal motion is of constant speed, and the vertical speed is zero at the lower edge of the ramp, but then increases in direct proportion to the time. Show that this leads to a relation of the form

$$d = \alpha \sqrt{h}. \tag{7.4}$$

Find the least squares estimate for α and the value of the minimized χ^2 . What is the P -value for this hypothesis?

Simulare l'esperimento 1000 volte per verificare quanto ottenuto nell'ipotesi (7.3).

Quanto minori dovrebbero essere le incertezze statistiche per rigettare le ipotesi non corrette?