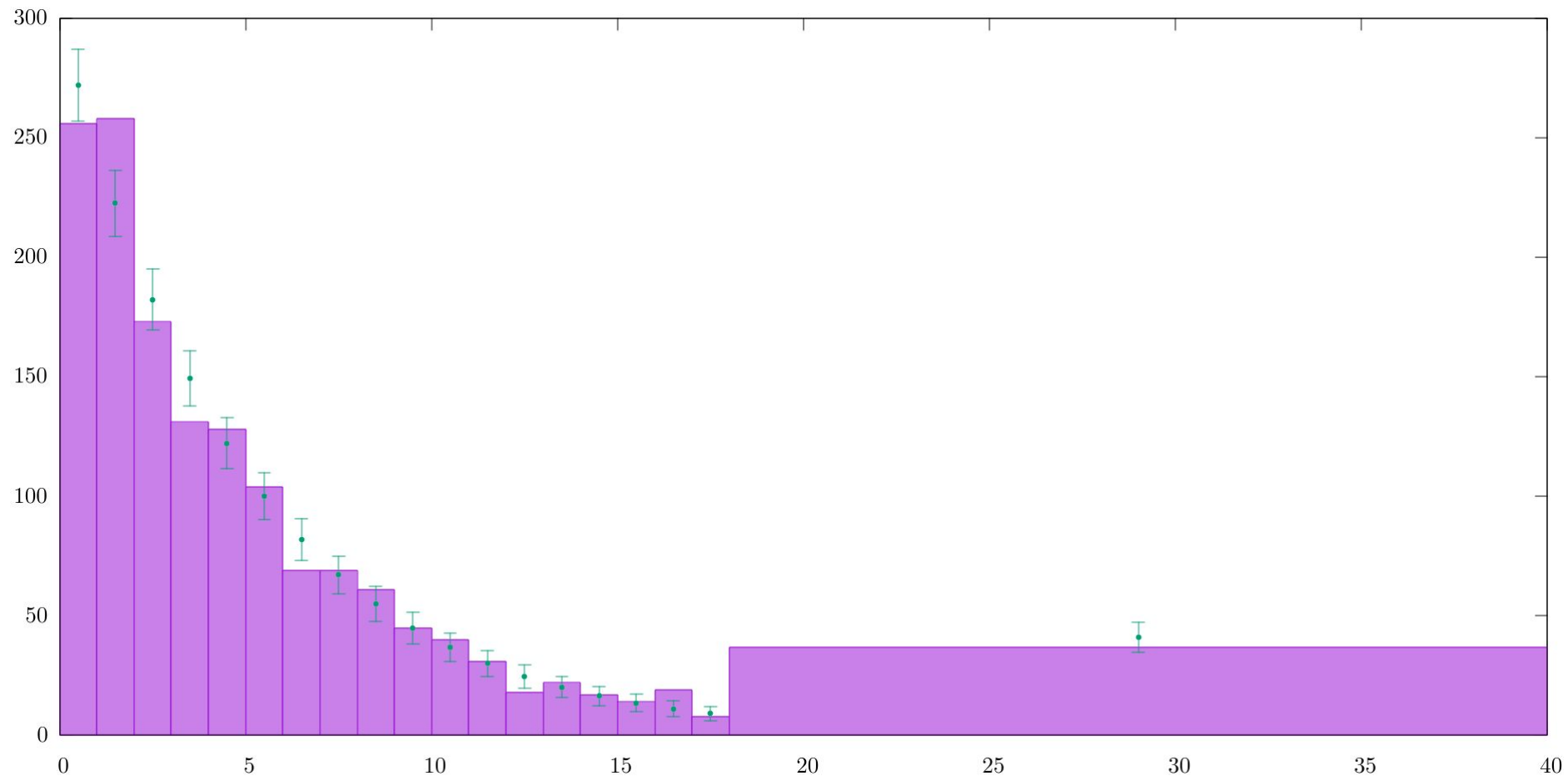


Analisi dati Radioattività ambientale

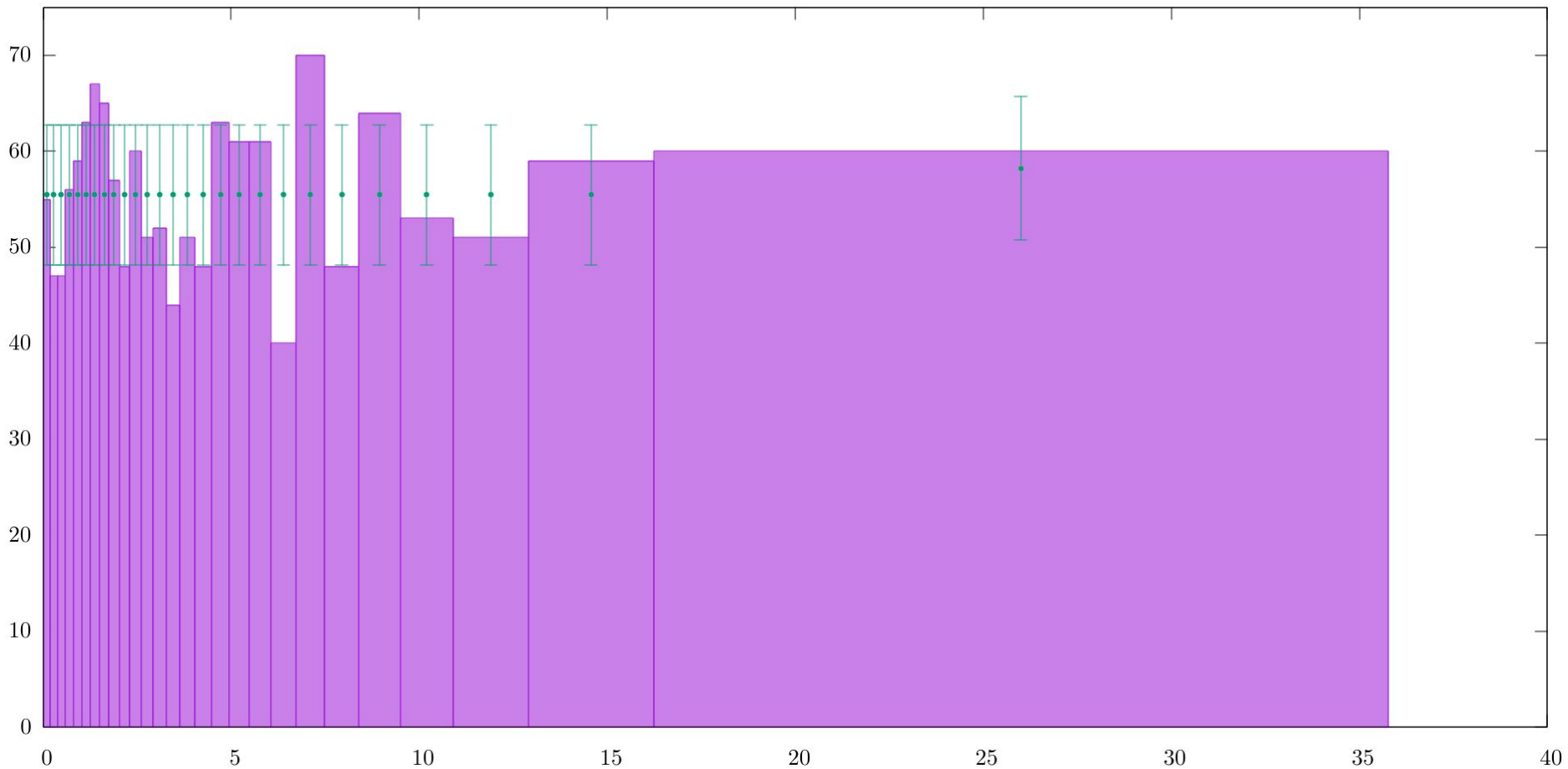
Giulio Ticli



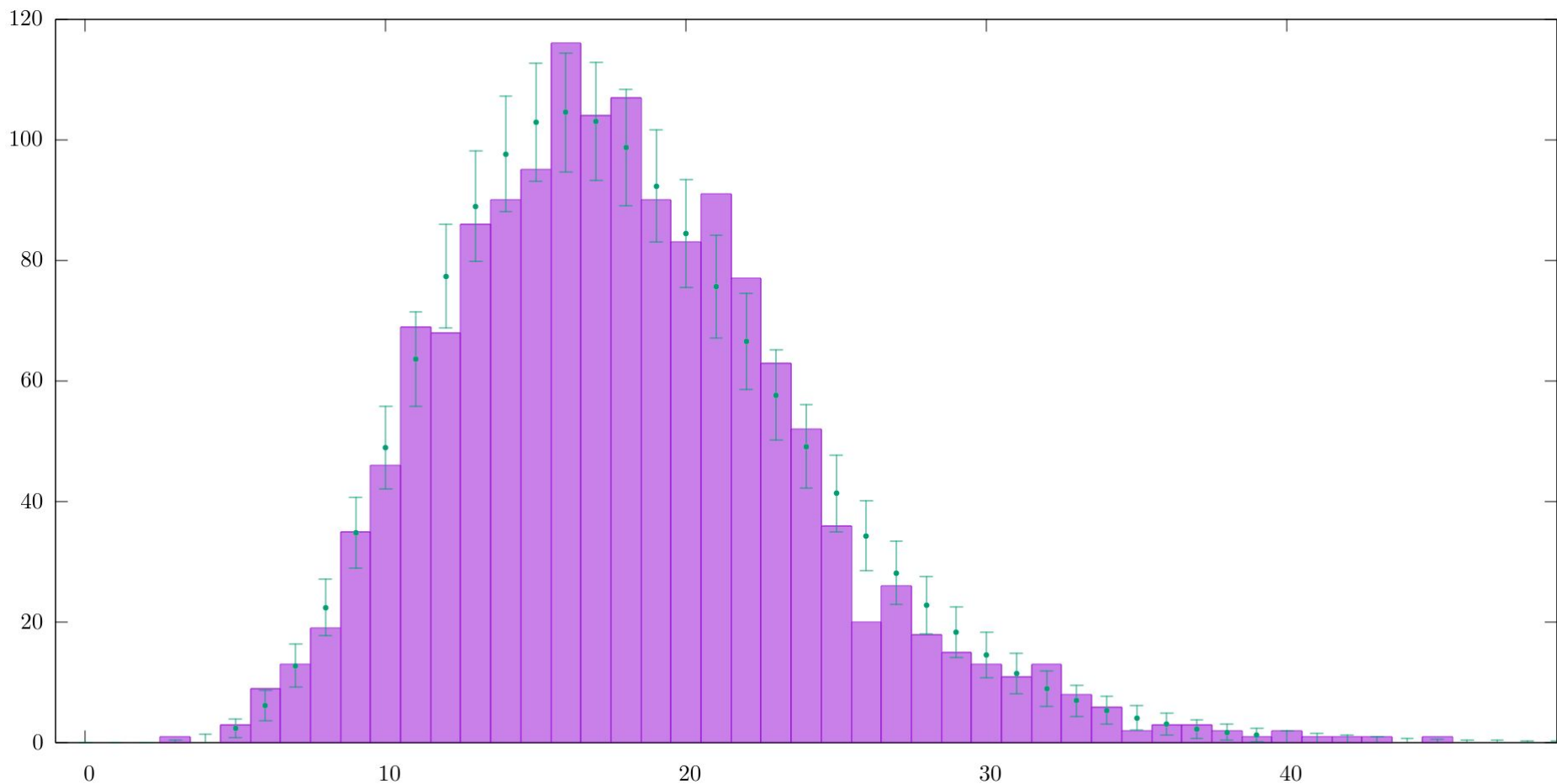
Bontà del test di ipotesi



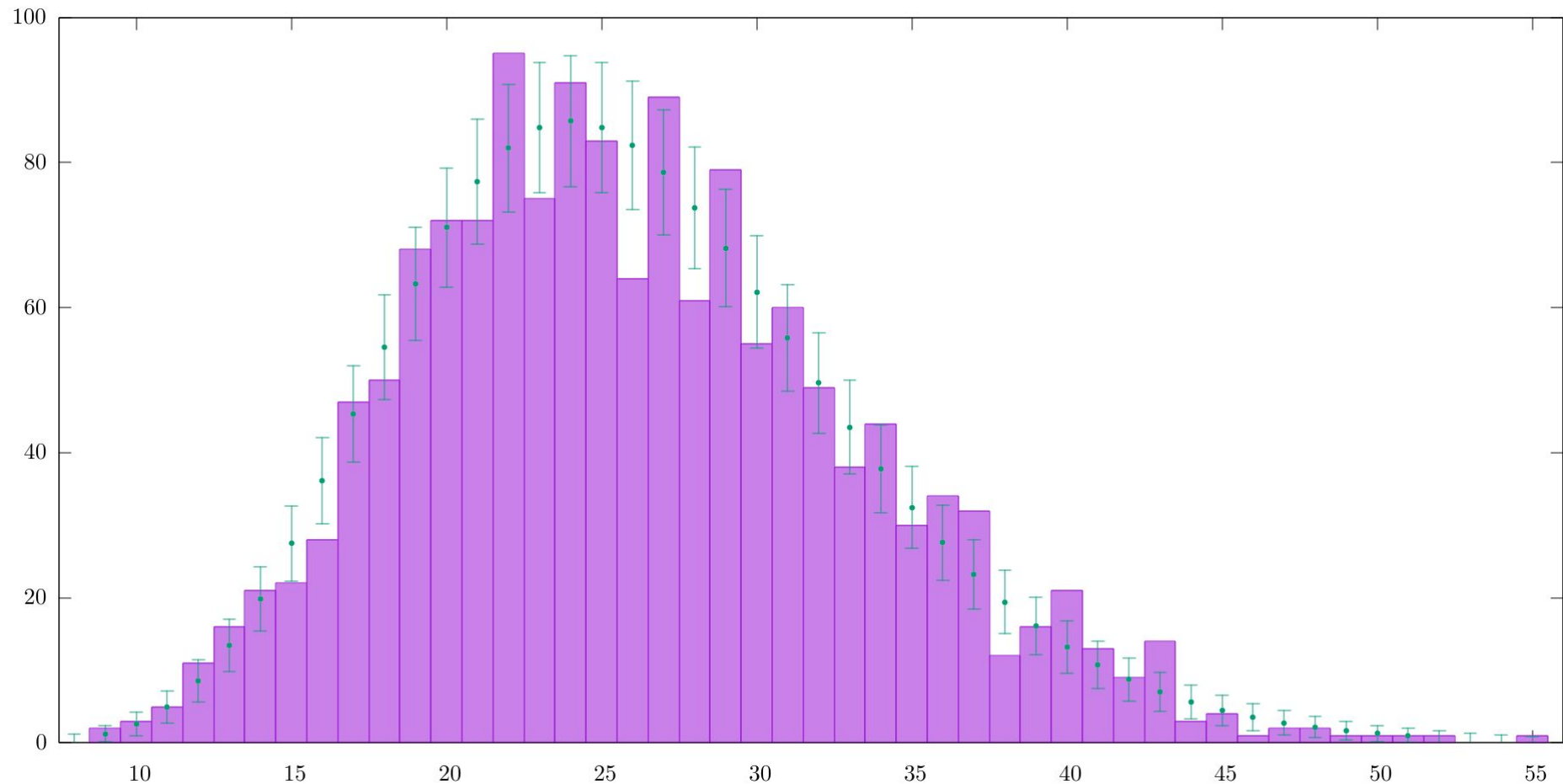
3a - B - Simulazione - Istogramma - Intervalli di ampiezza costante. $X^2 = 20.917 < 28.869$



3a - B - Simulazione - Istogramma - Intervalli di ampiezza non costante. $X^2 = 16.131 < 38.885$



3a - B - Istogramma Chi-quadro con intervalli di ampiezza costante (1500 simulazioni)



3a - B - Istogramma Chi-quadro con intervalli di ampiezza non costante (1500 simulazioni)



Stima dei parametri Binned vs unbinned M.L.



Esercitazione 3C - Output (A)



$$\tau_1 = 5.36739 \text{ s} \quad (\text{Analitico})$$

$$\sigma(\tau_1) = 0.145544 \text{ s}$$

$$\tau_2 = 5.367 \text{ s} \quad (\text{Unbinned})$$

$$\sigma(\tau_2) = 0.143 \text{ s (left)}$$

$$\sigma(\tau_2) = 0.149 \text{ s (right)}$$

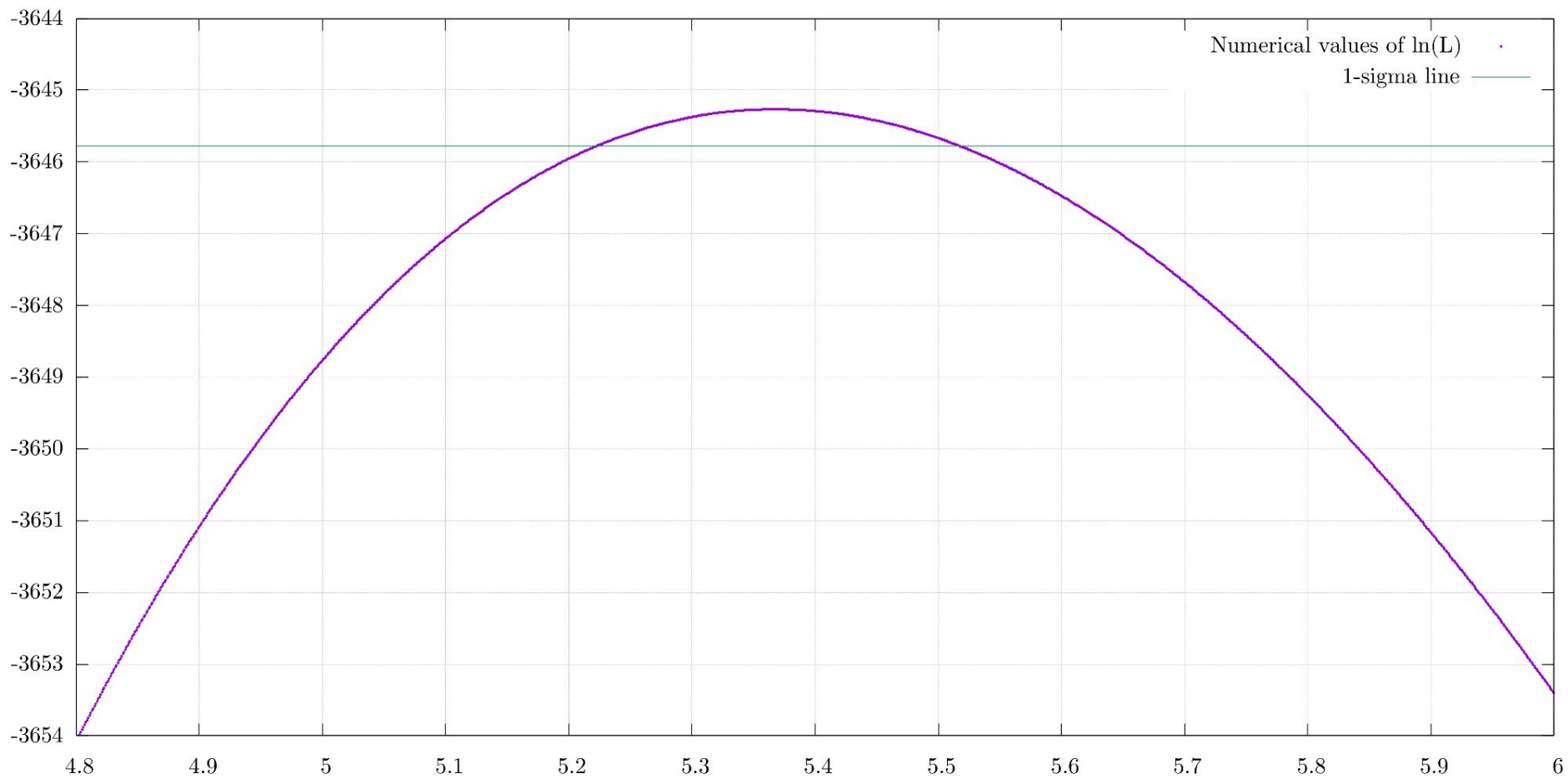
$$\tau_3 = 5.378 \text{ s} \quad (\text{Binned})$$

$$\sigma(\tau_3) = 0.144 \text{ s (left)}$$

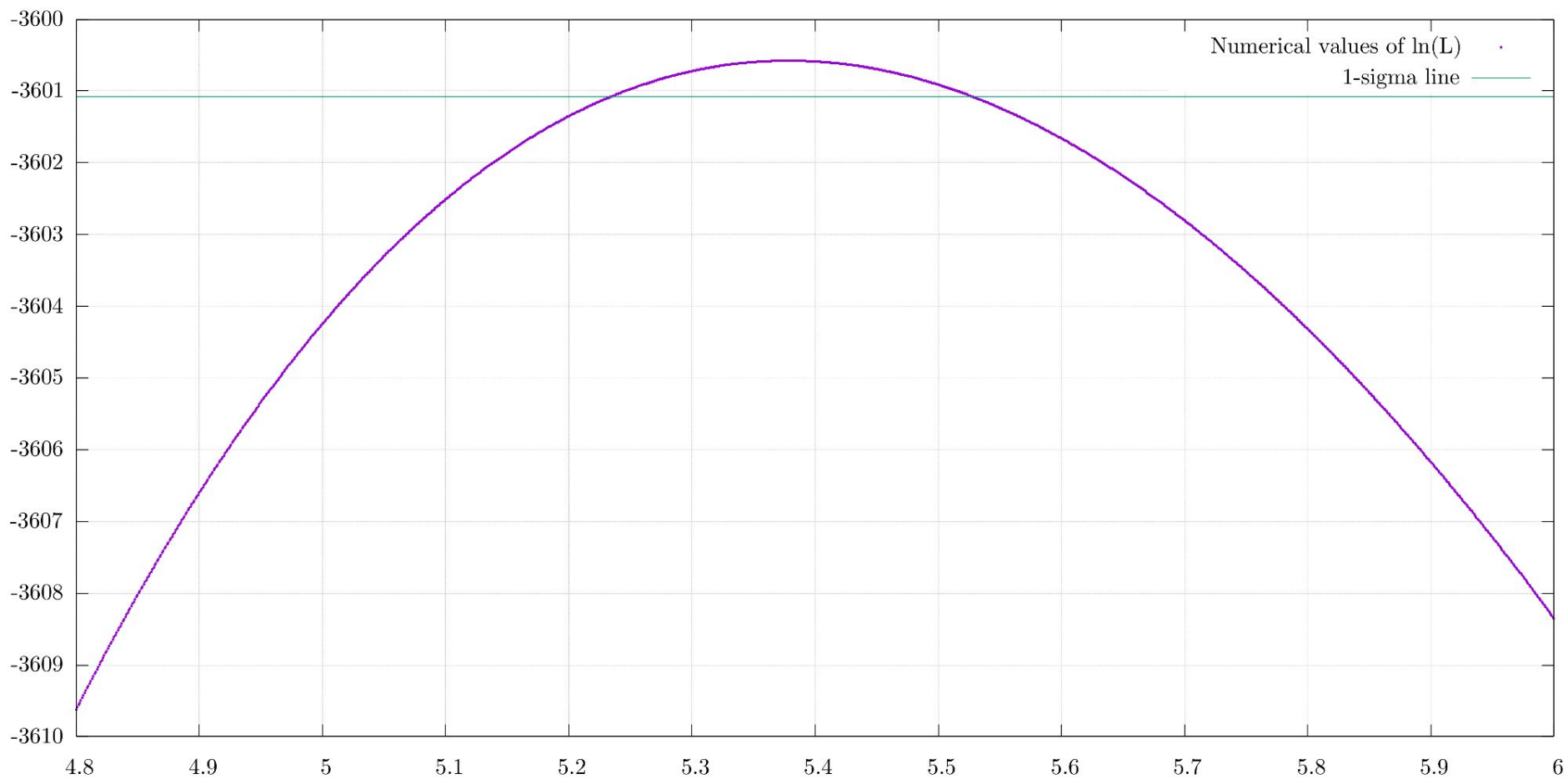
$$\sigma(\tau_3) = 0.150 \text{ s (right)}$$

Compatibilità A1-A3: $x = 0.721622$

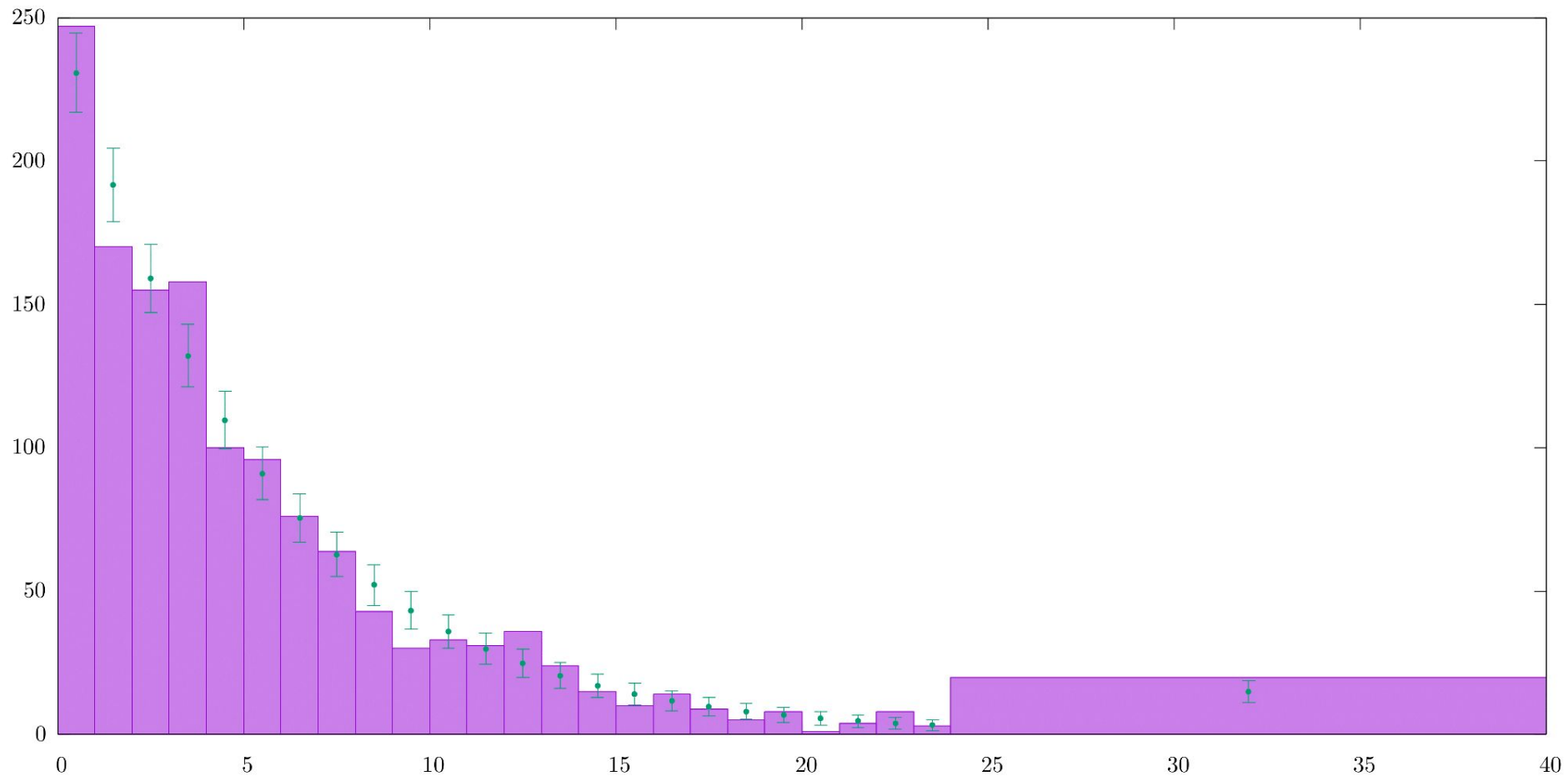
(usando $p = 0.995$). Compatibile se $x < 1.96$



A2 Unbinned numerical M.L.



A3 Binned numerical M.L.



A3 Binned numerical M.L. - Istogramma



Aggiungiamo rumore...



Esercitazione 3C - Output (B)

Unbinned

$$\tau_2 = 5.18 \text{ s}$$

$$\sigma(\tau_2) = 0.316 \text{ s (left)}$$

$$\sigma(\tau_2) = 0.344 \text{ s (right)}$$

$$a_2 = 0.49$$

$$\sigma(a_2) = 0.021 \text{ (left)}$$

$$\sigma(a_2) = 0.0215 \text{ (right)}$$

Binned

$$\tau_3 = 5.176 \text{ s}$$

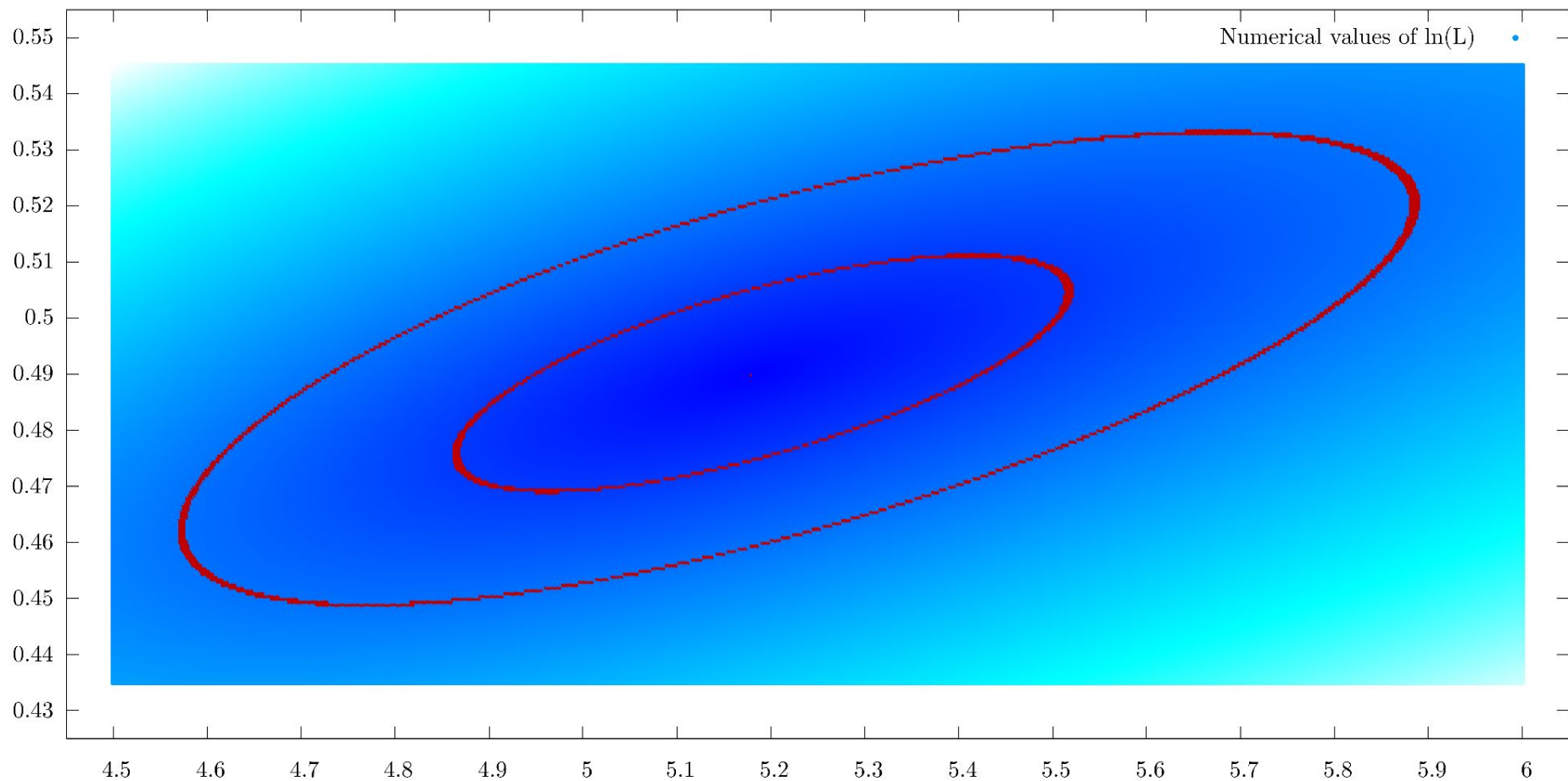
$$\sigma(\tau_3) = 0.320 \text{ s (left)}$$

$$\sigma(\tau_3) = 0.344 \text{ s (right)}$$

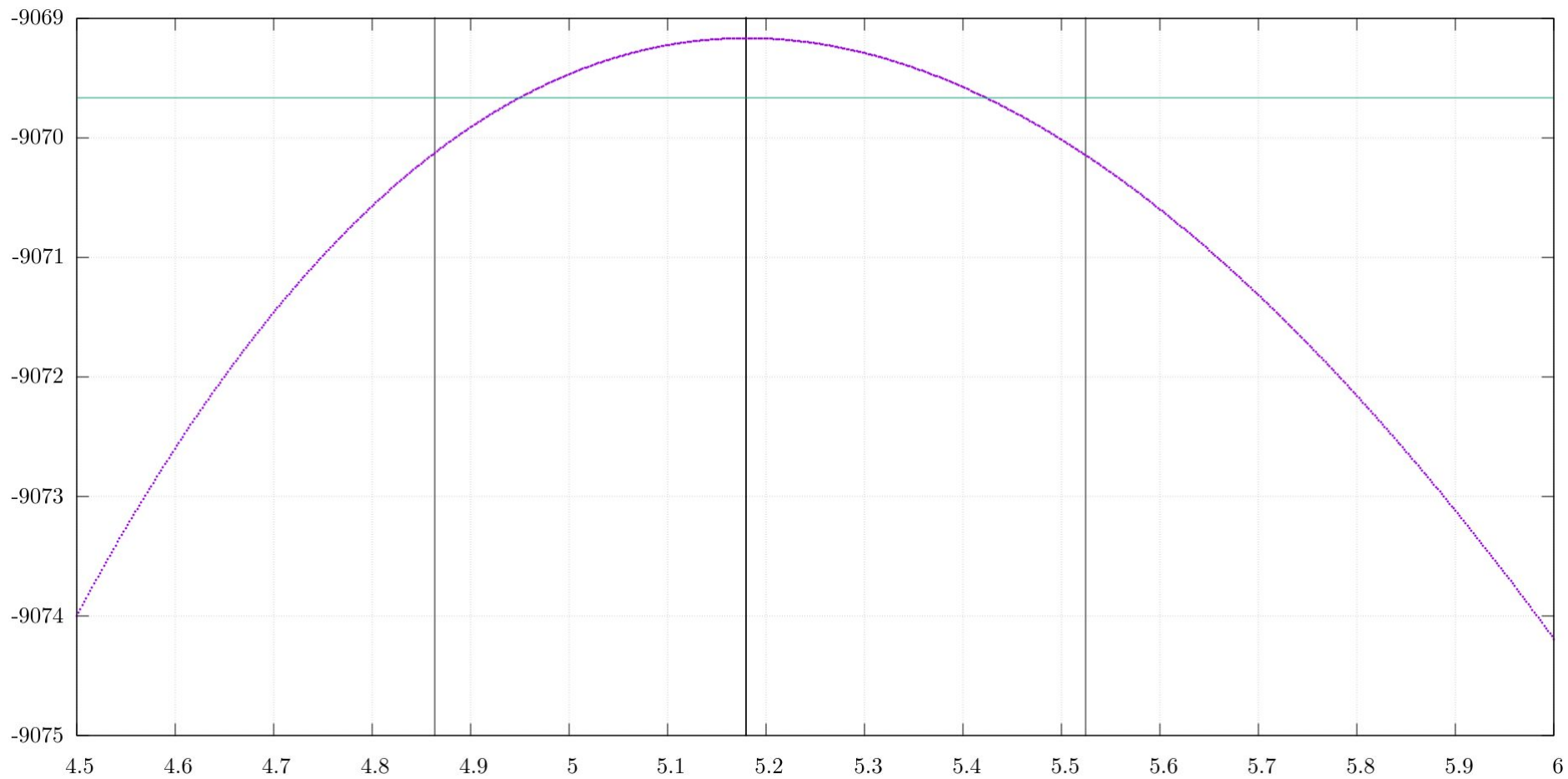
$$a_3 = 0.4905$$

$$\sigma(a_3) = 0.021 \text{ (left)}$$

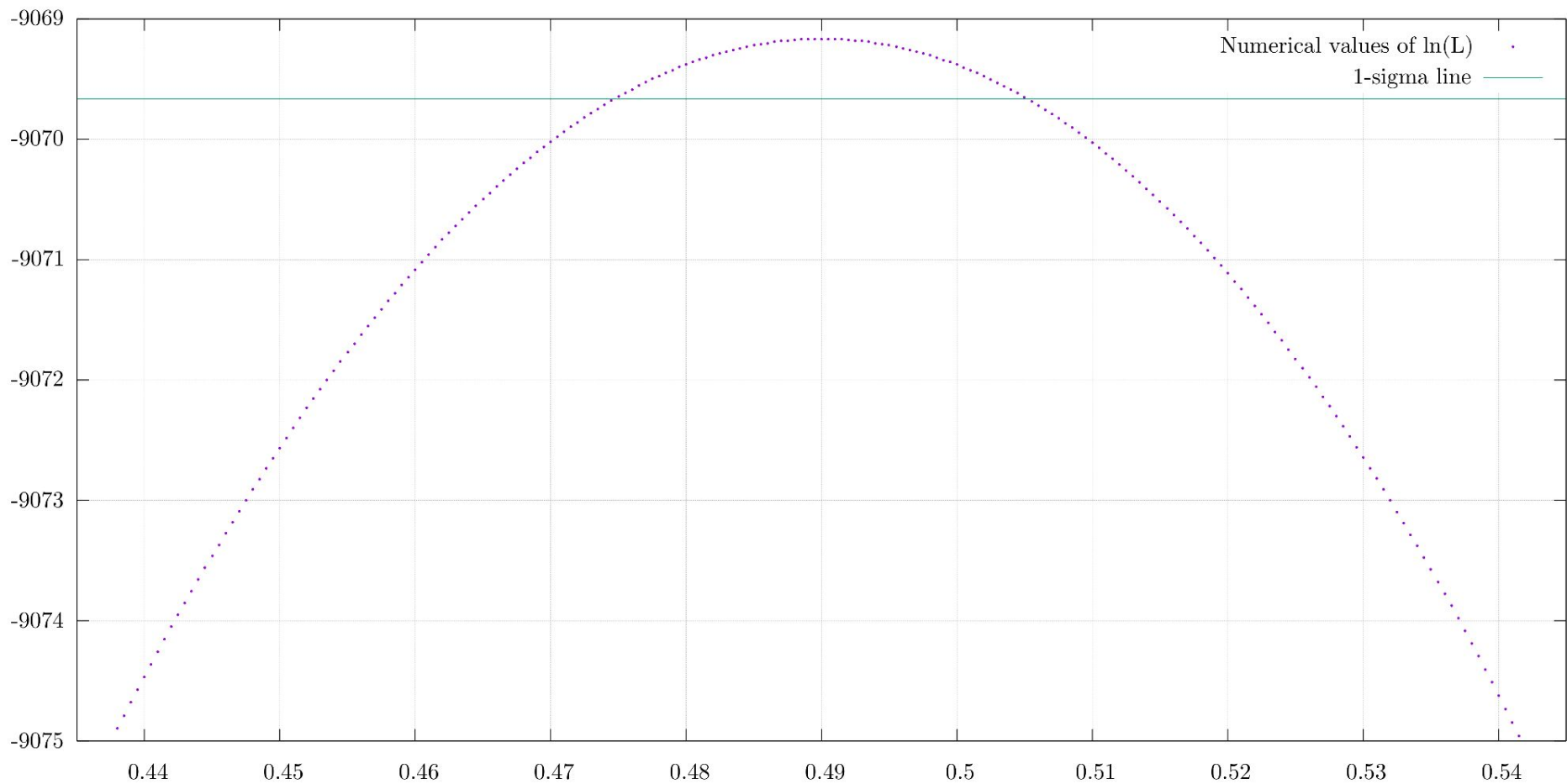
$$\sigma(a_3) = 0.0215 \text{ (right)}$$



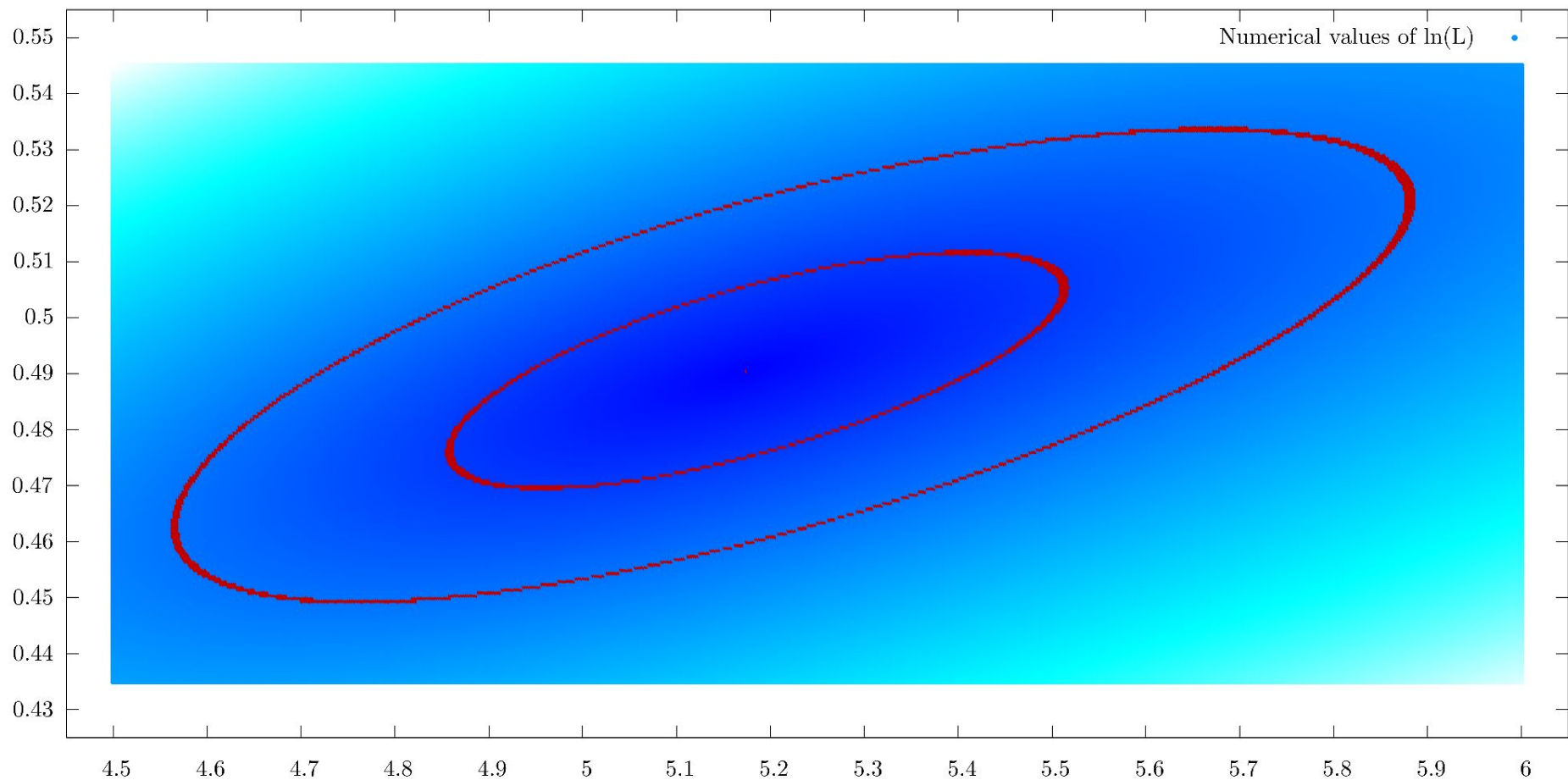
B2 Unbinned numerical M.L. - Regioni di confidenza per $\ln(L)$



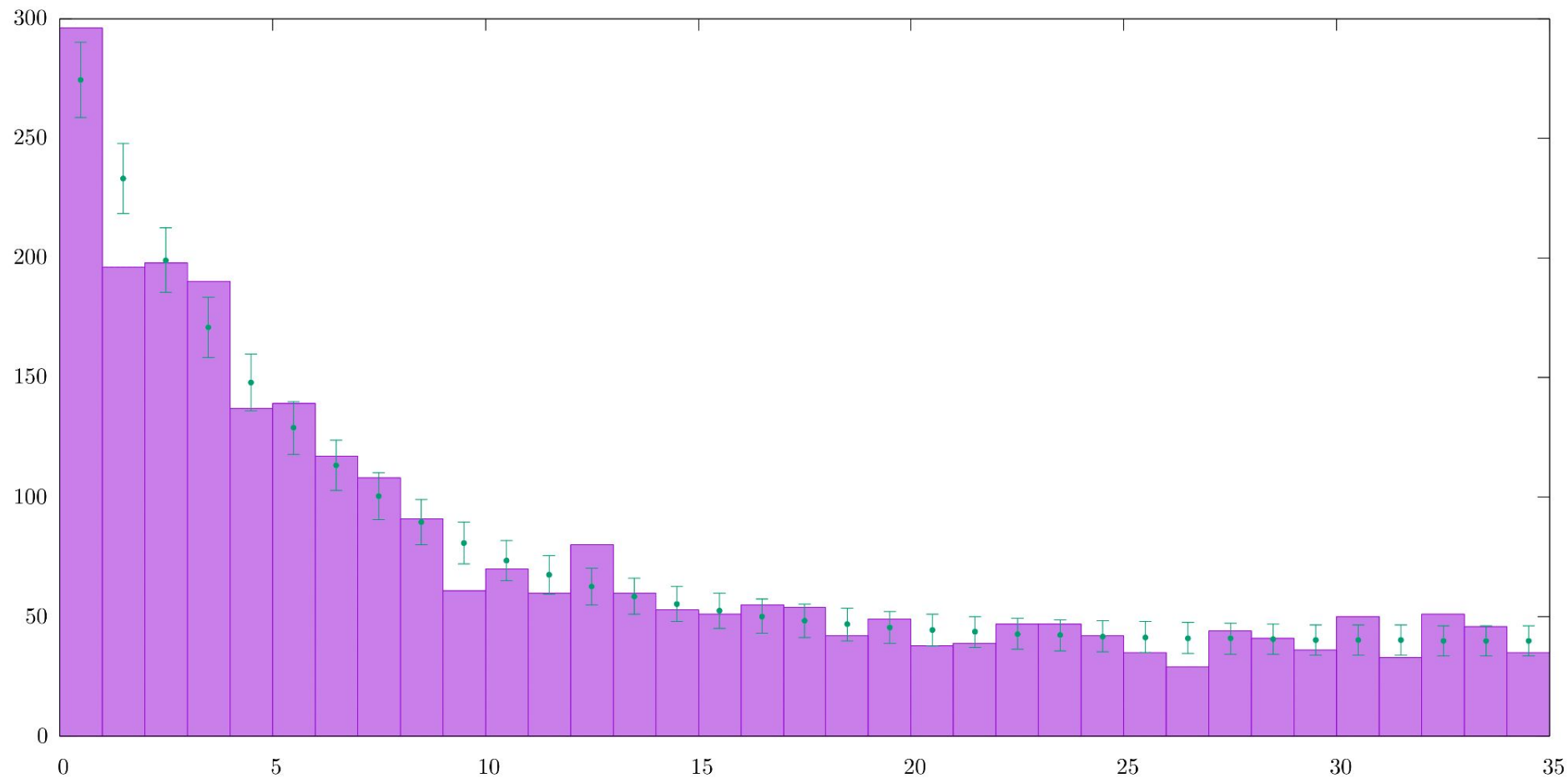
Valori di $\ln(L)$ in funzione di τ , fissata la migliore stima di α . In nero τ migliore $\pm \sigma(\tau)$.



Valori di $\ln(L)$ in funzione di α , fissata la migliore stima di τ .



B3 Binned numerical M.L. - Regioni di confidenza per $\ln(L)$



B3 Binned numerical M.L. - Istogramma considerato con la migliore stima di τ e α

Minimi quadrati



Esercitazione 3C - Output (C)

$$\tau_4 = 5.158 \text{ s}$$

$$\sigma(\tau_4) = 0.312 \text{ s (left)}$$

$$\sigma(\tau_4) = 0.342 \text{ s (right)}$$

$$a_4 = 0.4875$$

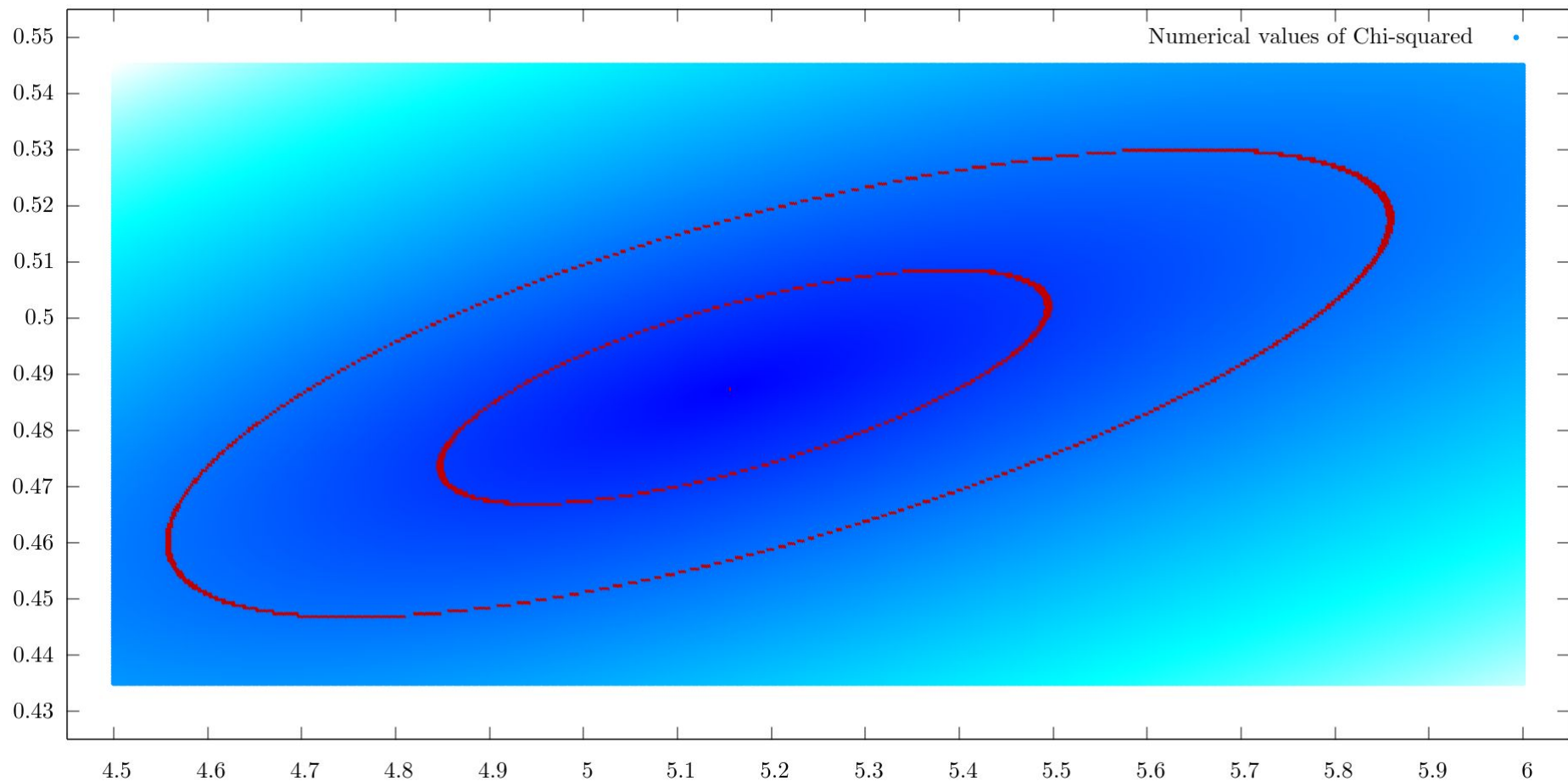
$$\sigma(a_4) = 0.0205 \text{ (left)}$$

$$\sigma(a_4) = 0.021 \text{ (right)}$$

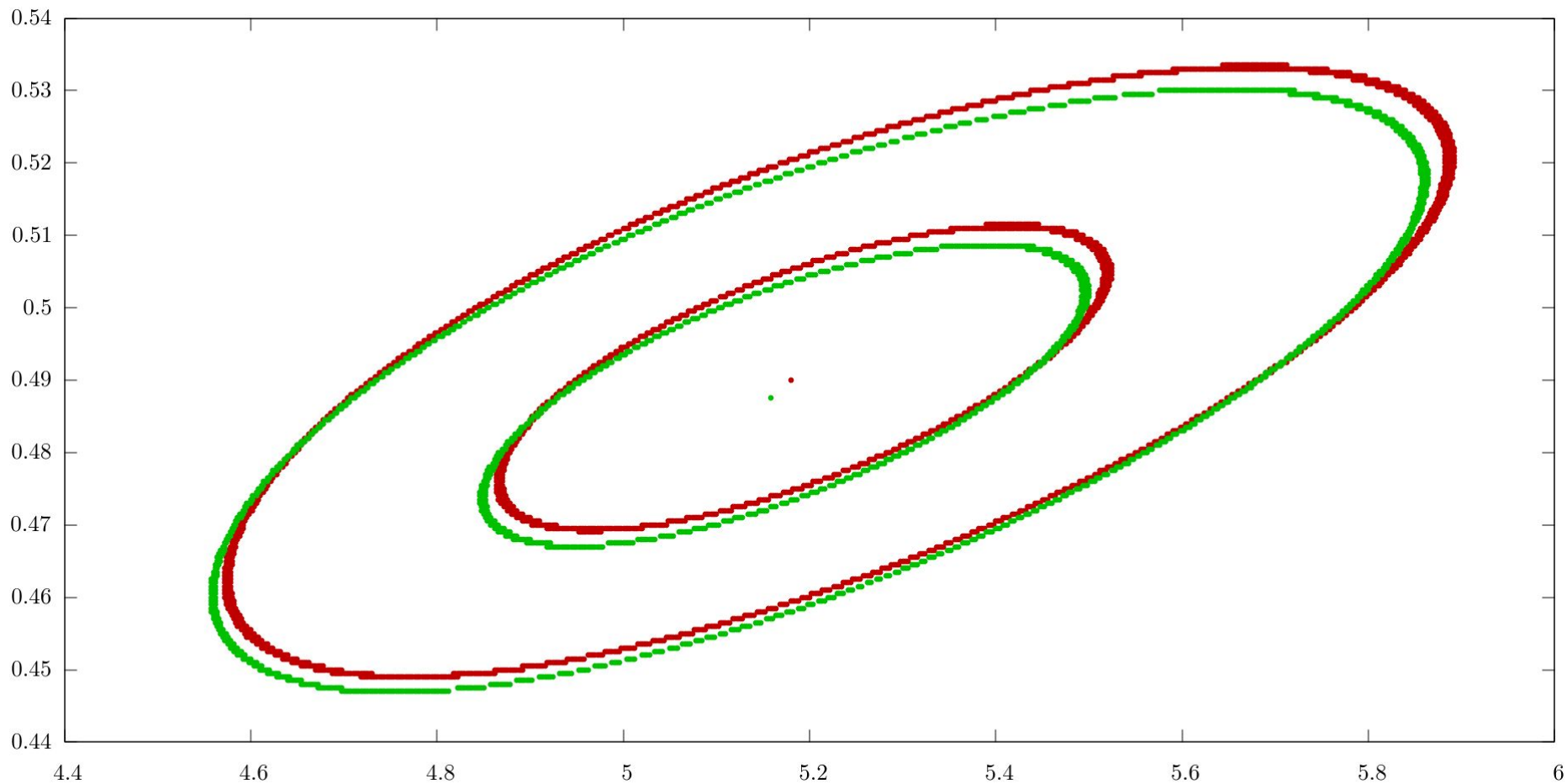
Compatibilità 2-4: $x(\tau) = 0.54568$ ($p = 0.995$).

Compatibilità 2-4: $x(a) = 1.38982$ ($p = 0.995$).

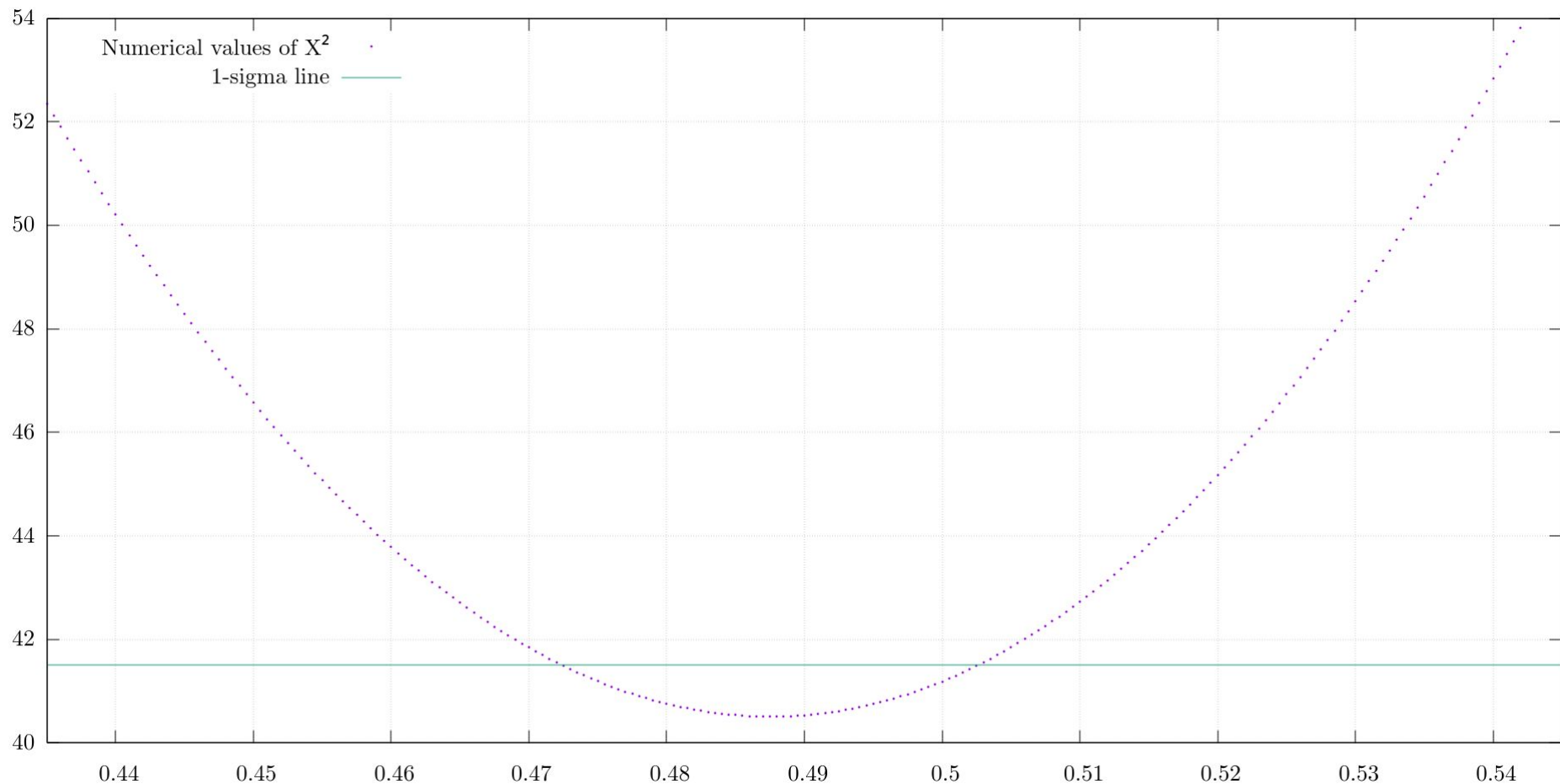
Compatibile se $x < 1.96$.



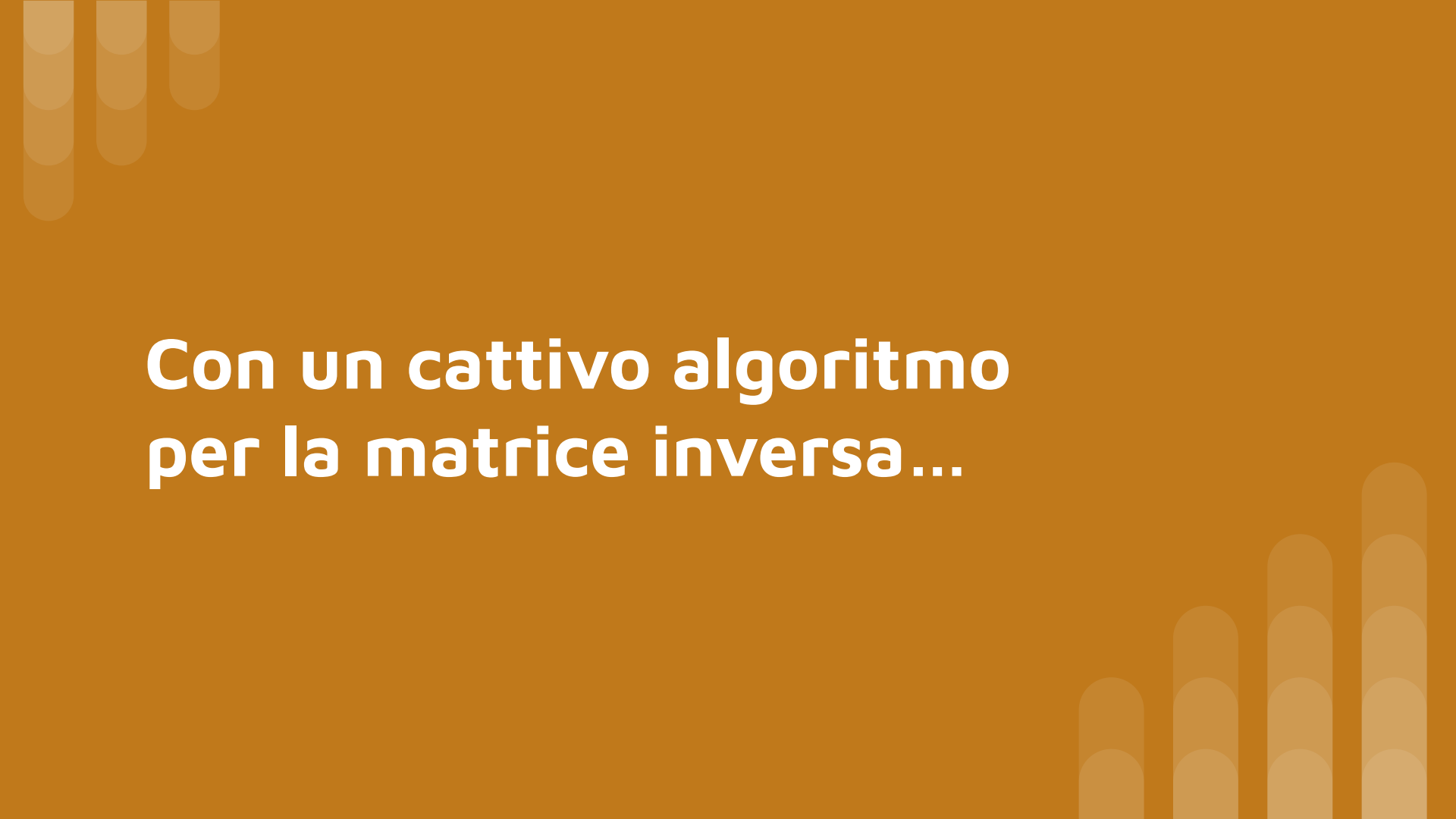
C - Minimi quadrati - Regioni di confidenza per χ^2



Ellissi di confidenza sovrapposte (**massima verosimiglianza** e **minimi quadrati**).



Valori di X^2 in funzione di α , fissata la migliore stima di τ .
Il minimo è $X^2 = 40.5 < 46.194$ ($\nu = 32$ d.f., significatività 0.05)



**Con un cattivo algoritmo
per la matrice inversa...**

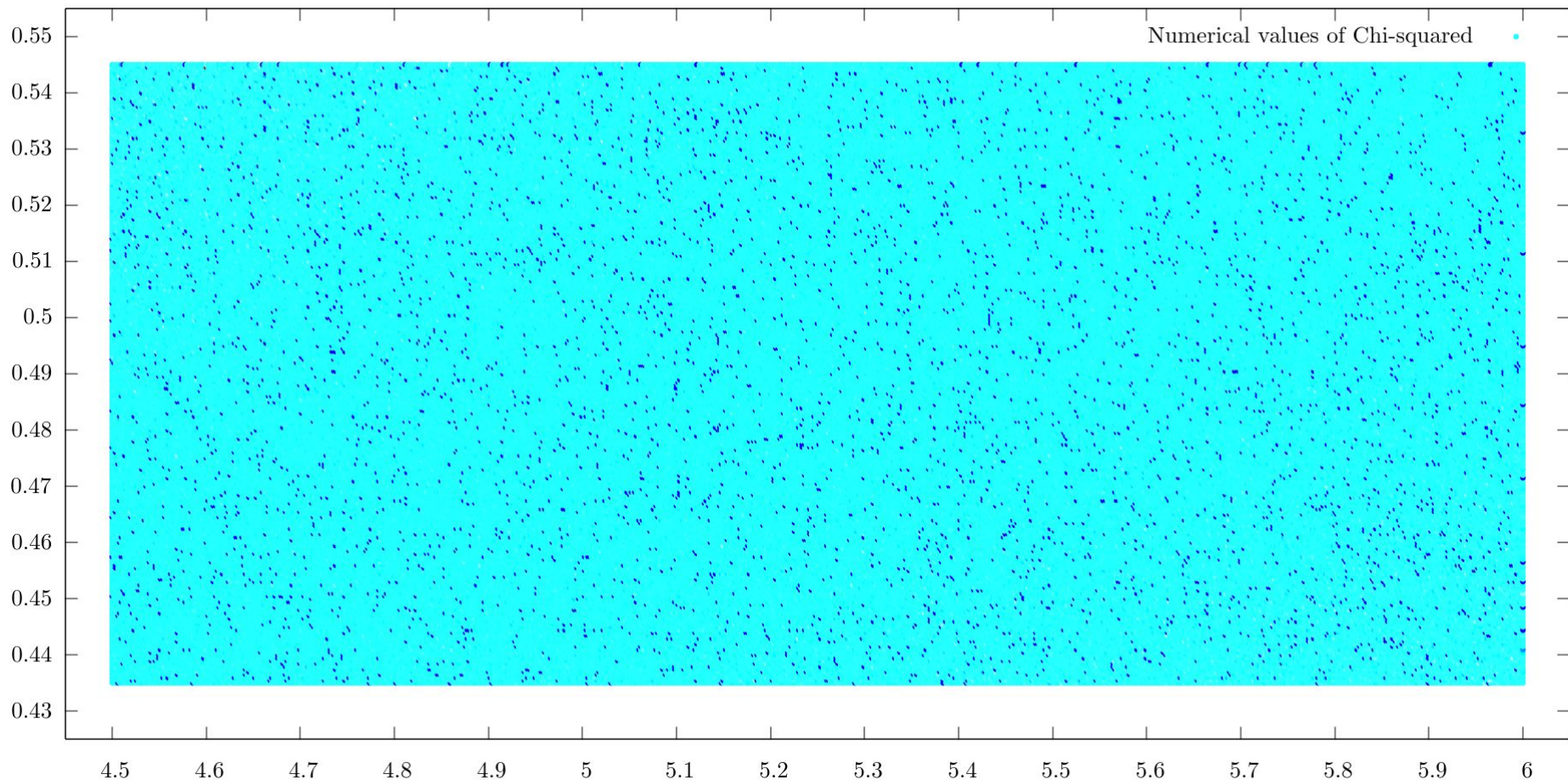


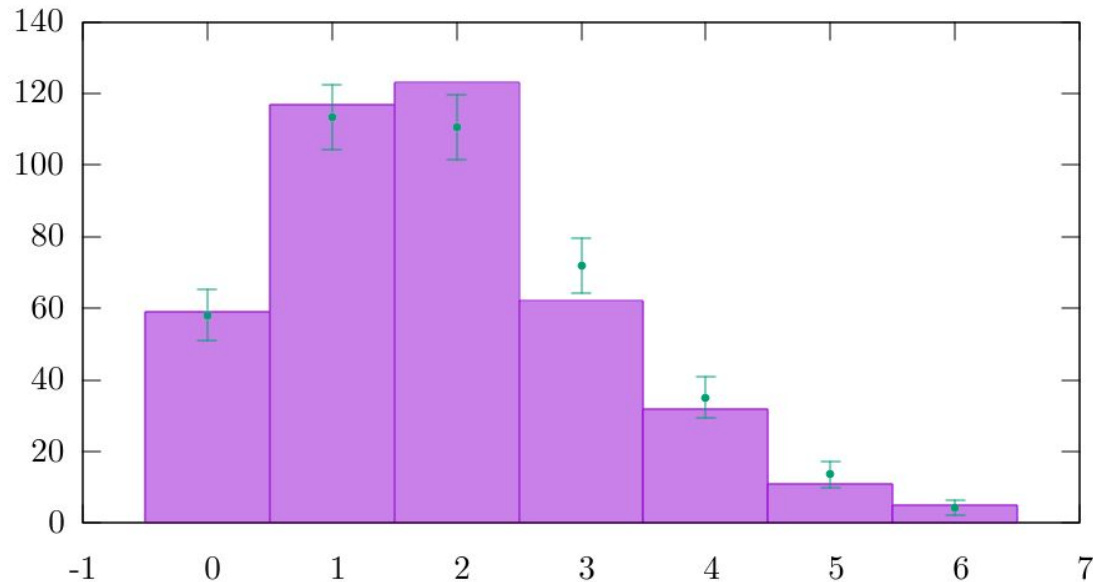
Grafico completamente privo di senso per i valori di X^2 se si usa un algoritmo poco solido per V^{-1} .
Le stime (totalmente insensate) sono $\tau_4 = 4.6$ s, $\alpha_4 = 0.5445$. Le σ sono addirittura negative!

Legame Erlang-Poisson

Esercitazione 3B - B

Poisson (10 s)

Stima di $\tau = 1/\mu_0 = 5.12563$ s
(stima indep. ottenuta da 240 s)
Chi-squared (cov = 0) = 4.64969
Chi-squared (cov $\neq$ 0) = 5.37811
Max chi-squared ($\alpha = 0.05$) = 12.5916





Altro (se serve)



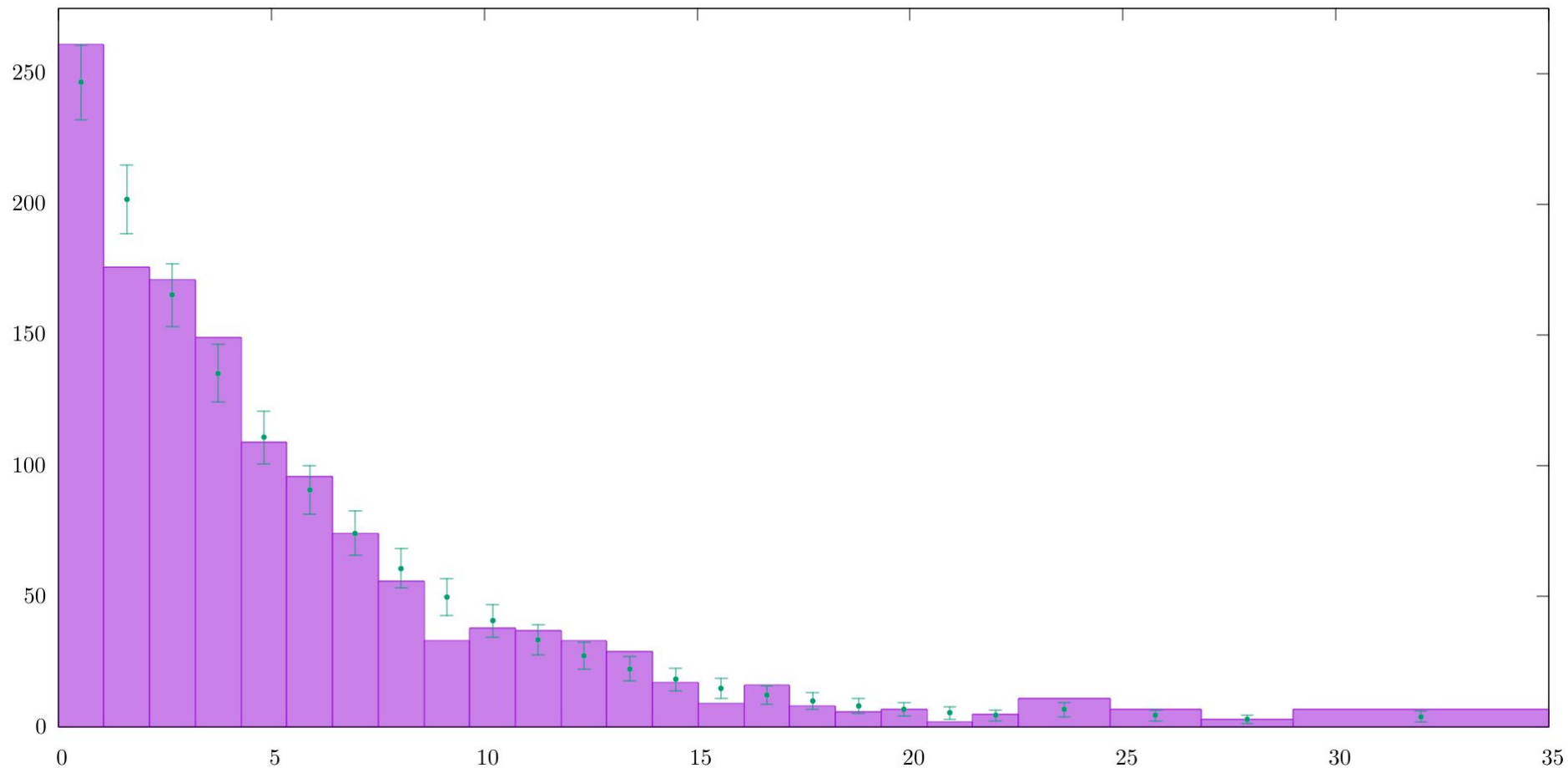
Esercitazione 3B - A (Output)

Erlang K =1

τ (stima) = (5.36739 $\pm$ 0.145544) s

Chi-squared = 30.8693

Max chi-squared (α = 0.05, v = 23) = 35.1725



3b - A - Erlang $K = 1$. τ (stima) = (5.36739 ± 0.145544) s. $X^2 = 30.869 < 35.1725$ ($\alpha = 0.05$, $v = 23$)

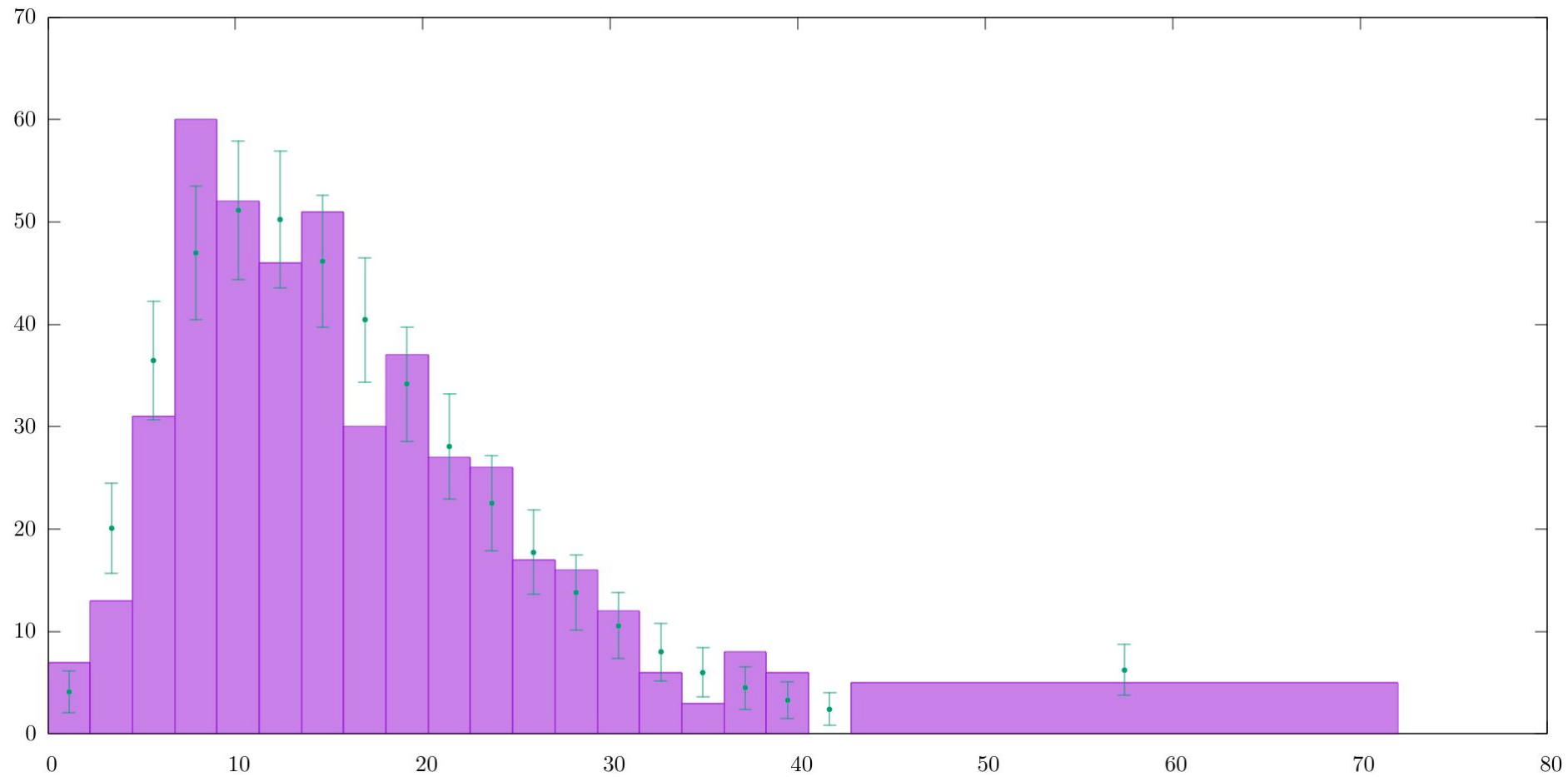


Esercitazione 3B - A (Output)

Erlang K =3

Chi-squared = 23.6407

Max chi-squared ($\alpha = 0.05$) = 28.8693



3b - A - Erlang $K = 3$. $X^2 = 23.641 < 30.144$ ($\alpha = 0.05$, $v = 18$)



Esercitazione 3B - C

r atteso = 0.958958
Chi-squared = 16.1924
 $16.1924 < 31.4104$

