**THE ROLE OF MATHEMATICS IN PHYSICAL SCIENCES. INTERDISCIPLINARY AND PHILOSOPHICAL ASPECTS**

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**LECTURERS AND ABSTRACTS**

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**Mathematics and Physics: is the former only series of consequences of arbitrary axioms?**

The problem whether mathematics is a ``series of consequences of arbitrary axioms'' or a branch of natural sciences and of theoretical physics was a subject of discussion since Hilbert (who was a pursuer of Descartes and a precursor of Bourbaki) and Poincaré (the founder of modern mathematics, topology, chaos theory and dynamical systems). I will speak essentially about some examples, showing the cardinal differences of view points between the axiomaphiles and the naturalists already on some basic concepts, as derivatives and limits, theorems of existence and uniqueness, optimization and control theory, the unsolvability of certain problems and the measure of complexity of certain others. Despite the basic role played by mathematics in all sciences, there are still eminent mathematicians believing (or, perhaps, affirming to believe) that mathematics has nothing to share with our world. This stance is especially dangerous in our time, as a new obscurantism, mainly observed in the antiscientific reform of education (in most First World's countries) menaces the bases of our whole culture.

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**The Art of the Scientific Pursuit: Beauty, Truth and Mathematics**

Mathematical simplicity and beauty which arise from the philosophical background of the pursuit of science is the prime concern in arriving at truth. In the context of modern physics, we have encountered some apparent mathematical complexities, like symmetry braking and divergence, which eventually lead the inner beauty and helped us in realising truth. The apparent complex nature of mathematics as envisaged in noncommutative geometry, which is the outcome of the interplay between general relativity and quantum mechanics, leads to the beautiful inner visions of the world

of elementary particles and the structure of space-time. However, the poverty of philosophy in the pursuit of science often leads to mathematics complexities that are devoid of beauty and may miss certain aspect of truth.

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**A Semiotic Approach To Contemporary Physics And Dirac's Methodological Revolution**

In the first part of my talk, without assuming any metaphysics on the world, and without committing myself with any strong epistemology, but even without demeaning the importance of the problem by adopting an instrumentalistic approach, I will try to offer a descriptive account of the role of mathematics in contemporary physics. I will do this by taking advantage of what Ch. S. Peirce sustained regarding the structure of the physical theories which he analysed in semiotic terms.

In this way, I will propose some possible answers to the following questions:

1) Why mathematics in physics?
2) What is the role of mathematics in physics?
3) Why is mathematics effective in physical sciences?

In the second part of my talk, I will analyse what might be called Dirac's methodological revolution, according to which before doing physics we must work on mathematics. Unfortunately this methodological revolution concerning the heuristic role of mathematics is incredibly underestimated both in the philosophical analysis of the scientific method and in the historical analysis of the development of science

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**On The Interplay Between Mathematics And Physics: What Does History Teach Us?**

In the talk I will focus on the first three questions asked by the organizers:

1) How can we explain the success of mathematics in the physical sciences?
2) Is mathematics only a tool or something more?
3) Which is the interplay between physics and mathematics?

In order to provide a tentative answer to them I will consider a couple historical cases:

1. Riemann's 1854 paper on the foundations of geometry
2. Poincaré's work on the three-body problem.

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## From Spinor Geometry to Physics

E'. Cartan, the founder of spinor gemetry, formulated the conjecture that simple spinors might the elementary constituents of euclidean geometry, in so far with them one may construct (bilinearly) euclidean null vectors and sums (or integrals) of null vectors give generally ordinary euclidean vectors (or minimal surfaces and strings). From Cartan's conjecture one may draw several consequences which, while relevant for elementary particle physics, transparently illustrate some specific roles of pure mathematics in physics. We will mention, among others the following ones:

1) The Cartan's equations defining simple spinors may be interpreted as equations of motion, in momentum space for fermions; the elementary constituents of matter, and most of the equations, defined ad hoc in elementary particle theoretical physics, are then naturally obtained. In particular the so called internal symmetry groups transparently originate from the 3 complex division algebras: complex numbers, quaternions and octonions.
2) Momentum spaces, constructured with simple spinors, result compact and isomorphic to spheres imbedded in each other.
3) The euclidean concept of point-event has to be abandoned when dealing with quantum mechanics of fermions and must necessarily be sobstituted with that of string (or integral of null vectors) fundamentally non local.

Adopting Cartan's conjecture one may also proove that Cartan's equations of motion may be conceived as representing general or "framing laws" which are indipendent from space and time, in particular they may be conceived atemporal, while their solutions are "evolutive laws" which describe the evolution of particular phaenomena in space and time. This distinction may render significant the role of mathematics in physics, specially after the Dirac's methodological revolution. In fact when a new framing law is discovered through mathematics - rather than, as in Galilei Newton mechanics, by induction on evolutive laws of known phaenomena - its consequent new evolutive laws may predict the existence of completely new phaenomena neither observed nor imagined before; as it happened for the discovery of antimatter, gravitational lenses and black holes. This role of mathematics may be compared to that of a telescope with which a new astronomical phaenomenon is discovered; however now it deals of an abstract, mental telescopes. This particular role of pure mathematics conceived as an instrument for the discovery of new natural phaenomena may be analised in the frame of those historycally proposed (platonic, Galilei, Berkeley, Kant, Wigner...). Some possible interpretations are proposed.

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**Different Types Of Holism And Their Mathematical Features**

Although holism in physics is usually associated with quantum theory, elements of holism can also be discerned elsewhere in physical theory. We will discuss several instances: the way local inertial frames are determined by global characteristics of the universe in a relational version of classical mechanics; the way in which global descriptions can go beyond local ones in special relativity (in the case of accelerated frames) and general relativity; and, finally, the holism relating to entanglement in quantum mechanics. The mathematical characterizations of these various types of holism are different. We will discuss these differences and ask whether it is appropriate to use the term 'holism' indiscriminately in all cases.

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**Why is Physics Mathematical?**

In my talk I try to evaluate what I regard as being the main attempts at answering the above ambitious as much as neglected question, first clearly posed by Kant and then rehearsed by Wigner in the last century, namely (1) Antinaturalism, (2) Kantianism, (3) Semanticism, (4) Algorithmic Complexity Theory and (5) Deflationism. The first position has been defended by Mark Steiner (1998), who claims that the "user friendliness" of nature for the applied mathematician is the best argument against a naturalistic explanation of the origin of the universe. The second is naturalistic and mixes the kantian tradition with evolutionary studies about our innate mathematical abilities. The third turns to the Fregean tradition and considers mathematics a particular kind of language, thus treating the effectiveness of mathematics as a particular instance of the effectiveness of natural languages. The fourth hypothesis, building on formal results by Kolmogorov, Solomonov and Chaitin, claims that mathematics is so useful in describing the natural world because it is the science of the abbreviation of sequences, and mathematically formulated laws of nature enable us to compress the information contained in the sequence of numbers in which we code our observations. In this tradition, laws are equivalent to the shortest algorithms capable of generating the lists of zeros and ones representing the empirical data. Finally, the "deflationary explanation" claims that in wondering about the applicability of so many mathematical structures to nature, we tend to forget the many cases in which no application is possible. Since by forgetting the negative instances, we can make any hypothesis look good, we must consider the import of this skeptical outlook. In the final part of the talk, I argue that a plausible solution to the problem should be looked in the second and third hypothesis.

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## Quantum Non-Locality And Mathematical Representation

The relation between sensorial intuition and mathematical representation could be interpreted in at least four different manners:

1. *Instrumentalism.* Sensorial intuition and mathematical representation are altogether inhomogeneous, so that the latter becomes a mere instrument for calculation .
2. *Platonism*. Sensorial intuition is devoid of any cognitive value, but mathematical representation allows to catch the true reality beyond appearances.
3. *Transcendentalism.* Sensorial intuition is produced by the contact between subjective categories and a reality not knowable in itself. Therefore it possesses in itself those structures, which make possible its mathematical representation.
4. *Phenomenology.* There is an analogy between mathematical and sensorial structures, that makes possible the mathematical representation of the latter.

Through the example of quantum non-locality, we attempt to show that among the above-mentioned perspectives the phenomenological is the most reasonable one.

From a transcendental point of view, quantum non-locality could be interpreted as a bad use of mathematical representation. From the Platonic point of view, quantum non-locality could be interpreted as a further confirmation of the deceptive character of appearances.

Let us consider two systems 1 and 2. Let us measure the observable spin **S** on the directions _ and _, whose result we denote with *s*. Then quantum mechanics violates the following equality:

$$p(s_{1\square}) = p(s_{1\square} / s_{2\square}, \square) \qquad (1)$$

Where $p$ is the probability. The violation of (1) expresses that the probability of finding the result $s_1$_ on system 1, measuring on the direction _, depends on the whole constituted by the direction _ of the apparatus that measures on system 2 and the result $s_2$_ of that measurement.

On the other hand, quantum mechanics does not violate the two following equalities:

$$p(s_1) = p(s_1 / s_2) \qquad (2)$$

$$p(s_{1\square}) = p(s_{1\square} / \square) \quad (3)$$

The violation of (2) would imply that the probability of the result on system 1 would depend on the results on the second system *tout court*. On the other hand, the violation of (3) would imply that the probability of the result on system 1 would depend on the direction of the apparatus measuring on system 2. If (2) or (3) were violated, it would be possible to build a quantum telephone.
In our opinion, the violation of (2) would be a confirmation of Platonic perspective, since it would prove an actual action at distance, that is against our sensorial intuition.
If (3) were violated, then the violation would depend on the measure apparatus, which implements the kind of mathematical representation that has been chosen. This could be a confirmation of the transcendental perspective, according to which non-locality depends on a bad mathematical representation.
On the contrary (1) is violated; and this can be interpreted only in the light of instrumentalism or phenomenology. That is quantum non-locality could be considered either as a recording in the mathematical representation of a phenomenon that does not need further explanation or as a confirmation that the mathematical representation is rooted in the sensorial intuition, i.e. non-locality is neither altogether objective, neither due solely to the representation that has been chosen.
For reasons external to the problem of quantum non-locality, we incline to the phenomenological interpretation.

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## Some Mathematical Aspects Of Modern Science And Their Relevant Physical Implications: The Subtle Interplay Of Entanglement And Nonlocality

We stress how the mathematical framework which is characteristic of the modern conception of natural processes, i.e. the Hilbert space structure of the set of the allowed states, combined with the direct product rule for the description of composite systems, implies an extremely peculiar and typical aspect of the formalism, i.e. entanglement, the feature which Schrödinger has appropriately identified as *the characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought*. In turn, the physical counterpart of this formal aspect are the nonlocal aspects of natural processes, a feature that nobody had appropriately taken into account (or even contemplated) before the revolutionary investigations by J.S. Bell. We stress how these nonlocal aspects have a completely general conceptual status which is totally independent from the position one takes about the theory and its interpretation. Moreover, they have been proved to actually occur in the wonderful experiments of A. Aspect and nowadays they find repeated confirmations in everyday experimental practice in quantum optics and various other fields which have a particular relevance for quantum cryptography, quantum teleportation and quantum computation.

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## Geometric Representation And Algebric Generativity. Remarks On The Constitution Of Space

In our talk we are going to analyse different descriptions of space through different geometries, considered, each of them, as a study of invariant relations as regards a specific group of tranformations. This method enables a fruitful treatment of physical space and sets the condition for a riqualification of the methodological and epistemological problem of space representation.

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## A Debate Between Rationalism And Empiricism On The Role Of Mathematics In The Construction Of Physics

The official birth of the modern experimental method commonly referred to Galileo's work, and, considered at the beginning as a revolutionary procedure, is one of the two main instruments of the modern science. The other one is the mathematical method, employed to formulate the scientific explanations. This is mainly the merit of the ancient Greek philosophers, and its power to understand and describe the world phenomena, which the Greeks had fully grasped, was and is continuously confirmed in the development of the scientific thought. What has made so powerful the physics is the "hypothetical-deductive method", which explains via mathematical hypotheses experimentally observed facts. The mathematics provides modern physics of the predictive power and this is strictly related to the possibility of expressing the determinism in mathematical form via differential equations.

Moreover, this procedure in the development of physics, has been questioned in many historical moments by pure empiricists, who did not accept the assumption, fundamental for the rationalists, of the capability of the human mind to generalize, as "natural laws", what are simply experimentally observed regularities, and to overestimate the role of mathematics in building the science.

I will consider these two attitudes in the framework of the development of the history of the science.

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## The Logic Of Quantum Computation

Quantum computation has recently suggested new forms of quantum logic that have been called *quantum computational logics* (QCL) ([CDCGL01]). These logics are based on the following semantic idea: unlike orthodox quantum logic [DCG02], the *meaning* of a sentence is identified with a *qubit* or a *quregister* (a system of qubits). From a physical point of view, qubits represent possible *pure states* of quantum systems , whose associated Hilbert space is $C^2$ (based on the two-dimensional space consisting of all ordered pairs of complex numbers). Quregisters represent pure states of compound systems, whose associated Hilbert space is an *n*-fold tensor product $\_^{n}C=C^2\_\ldots\_C^2$.

The two basis-elements |0>:={1,0} and |1>:={0,1} of $C^2$ are usually taken as encoding the classical bit-values 0 and 1, respectively. From a semantic point of view, they can be also regarded as the classical truth-values *Falsity* and *Truth*. Recalling the Born rule, any qubit $|□>= c_0|0>+c_1|1>$ (with $|c_0|^2+|c_1|^2=1$) can be regarded as an *uncertain piece of information*, where the answer *NO* has probability $|c_0|^2$, while the answer *YES* has probability $|c_1|^2$.

In quantum computation, *quantum logical gates* (shortly *gates*) are unitary operators that transform quregisters into quregisters. Being unitary, gates represent characteristic *reversible transformations* [TO80]. Generally, quantum gates correspond to some basic *logical operations* that admit a reversible behaviour. We will consider the following gates: *negation* (NOT), *conjunction* (AND) and the *square root of negation* ($\sqrt{NOT}$) [DEL00]. The first two gates are *semiclassical*. In other words, their quantum logical behaviour only emerges when they are applied to superpositions. When restricted to classical registers, they turn out to behave as classical (reversible) truth-functions. The gate $\sqrt{NOT}$, instead, is a *genuine quantum gate* that transforms classical registers into quregisters that are superpositions.

QCL is based on a sentential language *L* with the following connectives: *negation* (□), *square root of not* ($\sqrt{□}$), *conjunction* (□). The basic concept of our semantics is represented by the notion of *quantum computational model*, i.e., an interpretation of the language *L* that associates to any sentence _ ?a quregister. Since the meaning associated to a given sentence reflects the logical form of the sentence under consideration, we can say that our semantics has a typical *intensional* character. We define in the expected way the notion of *truth*, *logical truth*, *consequence* and *logical consequence* in QCL. Interestingly enough, QCL turns out to be *unsharp*, because the non–contradiction principle can be violated: the negation of a contradiction (_?□□ _) is not necessarily true. Further, QCL does not have any logical truth. Finally, we will contrast QCL and orthodox quantum logic and we will show that these two logics are incomparable.

REFERENCES
[CDCGL01] G. Cattaneo, M. L. Dalla Chiara, R. Giuntini and R. Leporini, "An unsharp logic from quantum computation", to be published in *International Journal of Theoretical Physics*; e-print: *quant-ph/0201013*.
[DCG02] M. L. Dalla Chiara and R. Giuntini, "Quantum logics", in G. Gabbay and F. Guenthner (eds.), *Handbook of Philosophical Logic*, vol. VI, Kluwer, Dordrecht, 2002, pp. 129–228.
[CDCGL02] M. L. Dalla Chiara, R. Giuntini, A. Leporati and R. Leporini, "Qubit semantics and quantum trees", to be published in *International Journal of Theoretical* Physics; e-print: quant-*ph/0211190*.
[CDCGL02] M. L. Dalla Chiara, R. Giuntini, A. Leporati and R. Leporini, "Quantum Computational Structures", to be published in *Tatra Mountains Journal of Mathematics*.
[DEL00] D. Deutsch, A. Ekert, and R. Lupacchini, "Machines, logic and quantum physics", *Bulletin of Symbolic Logic*, **3**, 2000, pp. 265–283.
[TO80] T. Toffoli, "Reversible computing", in J. W. de Bakker and J. van Leeuwen (eds.), *Automata, Languages and Programming*, Springer, 1980, pp. 632–644. Also available as TechnicalMemo MIT/LCS/TM-151, MIT Laboratory for Computer Science, February 1980.

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## Quantum Physics And The Mathematical Debates Concerning The Problem Of The Ontological Priority Between Continuous Quantity And Discrete Quantity

In his book about the Categories (that is about the ultimate element of classification and order), in the chapter concerning the quantity (IV, 20) Aristotle says that this concept recovers two kinds of modalities: the discrete quantity and the continuous quantity and he gives as examples the number for the first one; line, surface, solid, times and space for the second one. The main philosophical problem raised by this text is to determine which of the two modalities of the quantity has the ontological priority over the other (given two concepts A B we assume that A has ontological priority over B if every entity that possesses the quality B possesses necessarily the quality A). The problem is magnified by the fact that space, which in some part of Aristotle's Physics is mentioned not only as a category properly speaking but even as the main category whose power can be amazing, is in the evoked text of the Categories's Book reduced to expression of the continuum, and sharing this condition with time. In this matter the controversy is constant through the common history of Science and Philosophy. For example, the Cantor's theory of transfinite numbers implies the ontological subordination of the continuum to the concept of cardinality, which can be considered as intrinsically discrete; on the other side, some contemporary mathematicians claim for the continuum as the intrinsic support of the infinite, rejecting therefore the philosophical foundations of modern mathematic set theory. But in this debate there is room for a third argument to be raised: that of the philosophical theories which have strongly revindicated the mutual implication between the concepts of continuous quantity and discrete quantity, tied together in an dialectic knot, without possibility to give ontological priority to one of them.

In this paper we will recall the main points of projection of the controversy through the history of though, from Zeno's apories (and the mathematic attempts of solution) to the contemporary Non Standard Analysis. To summarize: in order to display the ontological weight of quantum physics we will replace in its philosophical background the dramatic moment when Einstein suggested that Max

Planck's theory was faraway of being merely an speculative mathematical construction, and that energy in nature actually comes in indivisible packets, instead of infinitely divisible streams. We will ask ourselves what different answers to the question have been brought forward by the ulterior developments of the discipline. In a second part of the paper we will try to establish the link between the problem raised up, the controversies about quantum non locality and the contemporary philosophical objections concerning a lack of rational explanation in the quantum theory, in spite of being largely successful at predicting the results of atomic processes. For, as the Newton's hypothesis *non fingo* displays, description and prevision doesn't necessarily means explanation.

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### Mathematics And Its Application In Plato's And Aristotle's physics

In the contribution is discussed the different views on mathematics in Plato's and Aristotle's natural philosophy. This difference arises from the specific approaches in those two natural philosophies. The world of physical objects is not the world of ideas in Plato's natural philosophy. It is questionable if mathematics could be applied on this physical world.

Aristotle's sublunar and supralunar physical world are also two different types of physical world and two types of physics exist. One could be mathematical physics, the other one could not. In the article is stressed how and way mathematics is or is not applied to the physical world. It also means that the application of mathematics generally depends on the ontological and cosmological point of view.

Plato's and Aristotle's approaches have become two different paradigms in the explanation of physical world and the possibility of applied mathematics to the physical world.

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### Einstein, Picasso: Mathematics, Relativity and Cubist Art

I will explore similarities in the research of Einstein and Picasso during their most creative years, 1905-1915. I focus on Einstein's and Picasso's discovery of new aesthetics and of the effect of non-

Euclidean geometry and x-rays on Picasso's thought. Their dazzling works that will serve as examples are Einstein's 1905 relativity theory and Picasso's Les Demoiselles d'Avignon.

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### Uncertainty

Every measurement in science is affected by uncertainties and errors. The *Heisenberg uncertainty relation* in quantum physics is perhaps best known and of particular philosophical importance, because it shows that the measuring subject may in principle affect the outcome of the measurement, which thus depends not only on the measured object. *Classical measuring errors*, from reading errors in astronomy and geodesy (as recognizes already by Boscovich, Legendre and, above all, by Gauss) are more commonplace but ever-present and thus more influential and significant in everyday scientific practice than even Heisenberg's uncertainties. The latter, however, are again interesting by the fact that the mutual influence between object and subject occurs as a matter of fact in biological, psychological and medical sciences: a dog "observed" by you may react by barking or even biting you, a girl who feels "observed" by a man may react by blushing or instinctively arraigning her hair, and the "placebo effect" in medicine shows how difficult it is to separate the "objective" effect of a medicament from the patient's belief. We may thus speak of "*Heisenberg-type uncertainties in life-sciences*". Uncertainties occur even in logic, as the inevitable *logical paradoxes* show, and in mathematics according to *Gödel's theorem*.

Thus physical measurements etc. are inaccurate, but there arises a related problem: *How accurate are physical theories*? A simple counterexample shows that even the simplest theory cannot be absolutely accurate. Consider Euclid's geometry as a physical theory: even the distance between two points cannot be measured with absolute accuracy because a physical point simply does not exist in reality as quantum theory shows. Uncertainty everywhere ...

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### Mathematics/Physics Interaction From The Perspective Of A Working Mathematician

In this talk, I will deal with some problems facing a working mathematician in trying to keep in touch with current research and advances in areas of Physics which are related to one's own field of research in Mathematics. One problem is to (understand and) interpret the insights and results of physicists in a language familiar to mathematicians. The talk will be illustrated with examples in Differential and Algebraic Geometry arising from my personal experience.

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## The Role Of Mathematics In Micro-Physics. The Case Of The Mathematical Language Of Nuclear Physics

When Galileo said that "nature speaks the language of mathematics", he was naturally making a reference to classical mechanics and elementary mathematics.

The relationship between physics and mathematics has changed somewhat since those times, owing to the great development of both the mathematical and the physical sciences. Within this framework, the discovery of the physical world at the microscopic level, which goes back to the beginning of the 20th Century, was of crucial importance, owing to the emergence of a new, problematic relationship between the human mind and the ``objects" within the microcosm. And the new physics led to the selection of some mathematical language, able to describe discontinuous dynamics.

I will endeavour to deal with these very general problems, making particular reference to the mathematical language used in nuclear physics. Most of the behaviour of the atomic nucleus, both with respect to spectroscopic and dynamic phenomenology, can be described using the algebra of irreducible spherical tensors. Nuclear phenomenology consists of a large collection of data which appears to be difficult to enclose within a single, reductionist theory. Nevertheless, it represents a mathematical logic which does obey a complicated mathematical formalism, that is, the quantum angular momentum theory which is substantially reducible to the rotation group logic.

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## J. von Neumann's Views On Mathematical And Axiomatic Physics

The physics community is split over the role and significance of mathematical rigor in physics: the protagonists demand mathematical exactness, the antagonists view mathematical precision misplaced in the context of physics. Accordingly, one can discern two attitudes towards von Neumann's achievements in mathematical physics: the appreciative and the ambivalent. While both camps view

J. von Neumann's work in physics as the typical representative of the modern mathematical-axiomatic approach to physics, the appreciative group regards von Neumann's axiomatic approach as a useful one yielding deep insights, the ambivalent attitude ignores von Neumann's achievements. The talk recalls the main characteristics of von Neumann's views on mathematical physics and on the role of the axiomatic method in physics. It will be argued that striving for mathematical precision was for von Neumann never a goal in itself and that he opted for the method of what will be called in the talk an "opportunistic soft axiomatism" in mathematical physics. The practice of this method will be illustrated on von Neumann's work in quantum theory.

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## Theoretical Explanations In Mathematical Physics

> The theoretical scientist is compelled in an increasing degree to be guided by purely mathematical, formal considerations in his search for a theory, because the physical experience of the experimenter cannot lead him up to the regions of highest abstraction.
> (A. Einstein, *Mein Weltbild*, 1934)

Many physicists wonder at the usefulness of mathematics in physics. According to Einstein mathematics is admirably appropriate to the objects of reality. Wigner asserts that mathematics plays an unreasonable important role in physics. James Jeans affirms that God is a mathematician, and that the first aim of physics is to discover the laws of nature, which are written in mathematical language. Dirac suggests that God may have used very advanced mathematics in constructing the universe. And Barrow adheres himself to Wigner's claim about the unreasonable effectiveness of mathematics for the workings of the physical world. Finally among the philosophers of science Scheibe considers that physics is overloaded or overdetermined with mathematics.

Wondering at the usefulness of mathematics in the physical description of reality is understandable indeed, if we assume that the laws, hypotheses, and theories of mathematical physics do describe, represent, or mirror Nature. But the fact that these physical constructs sometimes are empirically adequate is no compelling logical reason for claiming that they do this job. The inference from empirical success to truth is logically illegitimate.

Theoretical models of physics use to be thought to represent reality. It is sometimes claimed that mathematical physics attempts to 'simulate' reality by means of models. But as Popper has argued, models are vast and schematic oversimplifications, so that they simply cannot be true. Moreover, as the history of modern physics shows, it is perfectly possible to have different models of the same domain of phenomena, both empirically successful and based on entirely different assumptions. Thus theoretical models cannot be supposed to represent or simulate reality either.

If instrumentalism about theories and theoretical models is adopted instead of realism in the philosophy of physics, the alleged unreasonable usefulness of mathematics is less alarming. Mathematics then becomes an appropriate language, an useful instrument, in order to deal with Nature. But this has also consequences for the doctrine of theoretical explanations. Deprived of metaphysical connotations any physical construct, like facts, laws, hypotheses, etc., is considered to receive a theoretical explanation only when it has been deduced mathematically in the framework of another physical construct of higher level. Thus not only facts, but also empirical generalizations, abstract laws and even theories themselves admit of explanations in this sense. Now, explanation is, as well as prediction, the most important instance of realization of the hypothetic-deductive method. Since the methodology of physics is unthinkable without mathematics, mathematics becomes the possibility condition for theoretical explanations in physics.

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**What Mathematics Is About**

We discus the role of abstract objects in mathematics. Are they dispensable or indispensable? Do they exist or not? Are there any factual criteria for answering these questions? We offer some arguments that there is a fact of the matter as to whether abstract objects are dispensable or not, but that there is no fact of the matter as to whether abstract object exist or not.

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**Indian Tradition Of Mathematics With Respect To Theories Of Matter And Mind**

It is said that if in Greece mathematics was considered as queen of sciences then in India linguistics was considered as queen of sciences. Panini’s grammar is woven as a web of algorithmic procedures and much of early Indian mathematical thought drew from its overarching presence. It is noteworthy that in the Indian mathematical tradition the idea of dimensioned quantities; which can be physically interpreted and arranged in law-like formula (as in PV=RT or f=ma) never got articulated. On the other hand quite interestingly mathematical thought was employed for understanding 'working of mind' by different Indian philosophical schools. In particular there were intense debates on cognitive process of counting, nature of number and implication for working of cognition as such. We will introduce and explore reason for this strangeness by focusing on the nexus between the ideas of causation and mathematics in the Indian intellectual context.

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## On Theoretical Mathematics

The spectacular success of string theory in geometry has sparked some scepticism among mathematicians. Many heuristic proofs advanced by string theorists could be made rigorous by geometers. Many heuristically devised structures could be turned into well-entrenched mathematical concepts. But there were illusions and failures as well. Apart from sociological problems, among them how credit was shared, mathematicians wondered about the ontological status of non-rigorous results. Were they merely conjectures? The eminent mathematicians Arthur Jaffe and Frank Quinn have proposed to subsume them under theoretical mathematics, because theoretical results still require independent corroboration. This terminology suggests to consider rigorous mathematicians as the experimental counterparts of theoretical mathematicians; and, as a matter of fact, there are hardly other experimental partners for sting theorists.

After classifying the various positions in the Jaffe-Quinn debate I shall outline three approaches to make sense of theoretical mathematics. Imre Lakatos regards heuristics and conjectures as the real driving force of mathematical progress. Proofs themselves correspond to constantly refined thought experiments. But Lakatos's requirement of informal ancestors for any rigorous result proves problematic for the axiomatically introduced concepts prevailing in modern mathematical physics. In this respect, John von Neumann's opportunistic axiomatics performs better. Insisting that the best inspirations of mathematics stem from the physical sciences, he set out a series of aesthetic criteria of success. In virtue of its great reliability, mathematics occupies a special position among the sciences; but its complete freedom in concept formation is counterbalanced by the interaction with the empirical sciences. Within von Neumann's pragmatist ontology, axiomatisation is thus both exploratory and justificationist. Still one may hope that opportunistic strategies can be given a sound mathematical meaning. As recent works of Mark Wilson have shown it is quite difficult to disprove such a mathematical optimism even in the domain of applied mathematics. Combining this perspective with the earlier approaches teaches us that the problem might not be just one way. Setting up mathematical structures is, on pain of aestheticism, not fully independent of how these structures are applied to real-world problems. Mathematics might thus constantly oscillate between an opportunistic stand, driven by the quest for application, and an optimist stand, in which one tries to give mathematical meaning to successful application strategies.

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## Quest For Values And Meaning In The Quantum Universe

Cartesian problem of the mind - matter orthogonality persists for more than three centuries. Realm of classical physics has no natural place for human mind, not even for the very life. No any set of values can be attributed and humans are scientifically justified automata obeying mathematically expressed cold laws of physics. They are responsible for nothing and their ethics consists of own interests and survival. The only ethical act in this world view based on classical physics is the choice of initial conditions that fully fixed the destiny of spacetime. This is left for God.

At the other hand quantum theory creates a fundamental bridge between the matterlike and the idealike things in nature. A great deal of physicists now tries to look beyond the orthodoxy of the standard interpretation of quantum theory, which is pure epistemology, and eventually ask "what is really happening there?". At the level of actual quantum events (sometimes called the Heisenberg events) one may recognize that a profound quantum choice takes place everywhere and forever and injects the meaning into the physical universe. Mental universe is subject to the same mechanism once we accept the idea that human thoughts are just the actual quantum events over the entire brain.

Thus the mental and material universes are brought together on a deeper level of reality beyond our experience. Such a realm can accommodate concepts of choice, meaning, value, responsibility, ethics, etc.

**Julius Wess**

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## Provisional Time Table

**Mo 25 August**
Opening
First session: *Mathematics and physics: reflecting on the historical role of mathematics*
Chair: **TBD**

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|---|---|---|
| 9,00-9,30 | **Opening ceremony Participants TBD** | |
| 9,30-10,10 +10' discussion | **Umberto Bottazzini** | *On The Interplay Between Mathematics And Physics: What Does History Teach Us?* |
| 10,20-11,00 +10' discussion | **Stipe Kutlesa** | *Mathematics And Its Application In Plato's And Aristotle's Physics* |
| 11,10-11,20 | **Break** | |
| 11,20-12,00 +10' discussion | **Marcello Giorgi** | *A Debate Between Rationalism And Empiricism On The Role Of Mathematics In The Construction Of Physics* |
| 12,10-12,50 +10' discussion | **Victor Gomez Pin** | *Quantum Physics And The Mathematical Debates Concerning The Problem Of The Ontological Priority Between Continuous Quantity And Discrete Quantity* |
| 13,00-18,45 | **Break** | |
| 18,45-19,00 | **Short communication** | *To be announced* |
| 19,00-19,40 +10' discussion | **Navijoti Singh** | *Indian Tradition Of Mathematics With Respect To Theories Of Matter And Mind* |

**Tu 26 August**
Second session: *Mathematics and physics: searching for the right mathematics*

Chair: **TBD**

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|---|---|---|
| 9,00-9,40 +10' discussion | **Giovanni Boniolo** | *A Semiotic Approach To Contemporary Physics And Dirac's Methodological Revolution* |
| 9,50-10,30 +10' discussion | **Paolo Budinich** | *From Spinor Geometry to Physics* |
| 10,40-10,50 | **Break** | |
| 10,50-11,30 +10' discussion | **Walter Pisent** | *The Role Of Mathematics In Micro-Physics. The Case Of The Mathematical Language Of Nuclear Physics* |
| 11,40-12,20 +10' discussion | **Mudumbai Narasimhan** | *Mathematics/Physics Interaction From The Perspective Of A Working Mathematician* |
| 12,30-18,45 | **Break** | |
| 18,45-19,00 | **Short communication** | *To be announced* |
| 19,00-19,40 +10' discussion | **Giulio Giorello & Corrrado Sinigaglia** | *Geometric Representation And Algebric Generativity. Remarks On The Constitution Of* |

**We 27 August**
Third session: *Mathematics and physics: philosophical analyses*
Chair: **TBD**

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|---|---|---|
| 9,00-9,40 +10' discussion | **Andrés Rivadulla** | *Theoretical Explanations In Mathematical Physics* |
| 9,50-10,30 +10' discussion | **Michael Stöltzner** | *On Theoretical Mathematics* |
| 10,40-10,50 | **Break** | |
| 10,50-11,30 +10' discussion | **Dennis Dieks** | *Different Types Of Holism And Their Mathematical Features* |
| 11,40-12,20 +10' discussion | **Mauro Dorato** | *Why is Physics Mathematical?* |
| 12,30-18,45 | **Break** | |
| 18,45-19,00 | **Short communication** | *To be announced* |
| 19,00-19,40 +10' discussion | **Arthur Miller** | *Einstein, Picasso: Mathematics, Relativity and Cubist Art* |

**Th 28 August**
Fourth session: *Mathematics and physics: the case of quantum mechanics*
Chair: **TBD**

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|---|---|---|
| 9,00-9,40 +10' discussion | **Giancarlo Ghirardi** | *Some Mathematical Aspects Of Modern Science And Their Relevant Physical Implications: The Subtle Interplay Of Entanglement And Nonlocality* |
| 9,50-10,30 +10' discussion | **Vincenzo Fano** | *Quantum Non-Locality And Mathematical Representation* |
| 10,40-10,50 | **Break** | |
| 10,50-11,30 +10' discussion | **Federico Laudisa** | *Is Information The Primary Object Of Quantum Theory?* |
| 11,40-12,20 +10' discussion | **Roberto Giuntini** | *The Logic Of Quantum Computation* |
| 12,30-18,45 | **Break** | |
| 18,45-19,00 | **Short communication** | *To be announced* |
| 19,00-19,40 +10' discussion | **Miklos Redei** | *J. Von Neumann's Views On Mathematical And Axiomatic Physics* |

**Fr 29 August**

Fifth session: *Mathematics and physics: insights*

Closing

Chair: **TBD**

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|---|---|---|
| 9,00-9,40 +10' discussion | **Helmut Moritz** | *Uncertainty* |
| 9,50-10,30 +10' discussion | **Zvonimir Sikic** | *What Mathematics Is About* |
| 10,40-11,20 +10' discussion | **Pratul Bandyopadhyay** | *The Art of the Scientific Pursuit: Beauty, Truth and Mathematics* |
| 11,30-11,40 | **Break** | |
| 11,40-12,20 +10' discussion | **Julius Wess** | *TBG* |
| 12,30-13,10 +10' discussion | **Nikola Zovko** | *Quest For Values And Meaning In The Quantum Universe* |
| 13,20 | **Closing** | |