# Position determination and resolution of two-dimensional position-sensitive solid-state detectors 

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#### Abstract

The determination of the coordinates of the incidence position on large solid state detectors, where the position sensitiveness is due to resistive electrodes, is discussed. A method to extract the $\boldsymbol{x}$ and $\boldsymbol{y}$ values from the signals at the corners of the resistive layer is described and tested. The results have been checked with different detectors and a variety of energies and ion beams.


## 1. Introduction

In heavy ion experiments the use of detectors with a large angular coverige is becoming more and more requested [1,2]. Together with a good energy resolution and suitable $Z$ resolving power, a good position resolution is typically required to investigate, for instance, the reaction dynamics [3]. This is particularly true in reverse kinematics experiments where the emitted particles are forward focussed.

A new generation of large area two dimensional position sensitive solid state detectors has been recently developed. These detectors use either two series of crossed strips on both sides [4] or a resistive layer on one surface [5,6]. The first kind of detectors provides a precise position determination, essentially given by the strip size, and needs no position calibration [4], but a good energy resolution can hardly be obtained [7]. On the other hand detectors with a resistive layer give a good energy resolution (like non-position-sensitive detectors of the same area), but they need position calibrations. An accurate determination of the position is anyhow not straightforward and moreover the position resolution [5] is not constant on the whole detector.

In this article we report on the performances, par-
ticularly as far as the position determination and resolution are concerned, of a silicon detector of the second kind, to be used in the MULTICS experiment \#1.

Section 2 evidences the experimental dependence of signals on the incidence point of the ion on the detector. A method to determine the coordinates of this point is described in section 3 , where details of the experimental measurements, together with the results on the position resolution, are also given.

## 2. Dependence of the position signals on the incidence point coordinates

The detectors here used, $500 \mu \mathrm{~m}$ thick ${ }^{\# 2}$, have a square shape and an active area of $40.5 \mathrm{~mm} \times 40.5$ mm . They are n-type silicon detectors, with a resistivity in the range $8.5-14 \mathrm{k} \Omega \mathrm{cm}$. At each corner of the resistive face, which has a sheet resistance of $\approx 5.5$ $\mathrm{k} \Omega$, a contact is fixed (see fig. 1 of ref. [5]), with a side

[^0]length of 8 mm . The four signals from these contacts are used to extract the coordinates of the incidence point and in the following they will be referred to as position signals.

The signal taken from a contact on the rear face of the detector is proportional to the energy deposited by the incident particle. The main importance of this fifth contact consists in having directly the energy information, instead of reconstructing it summing the position signals, with the advantage of a better energy resolution. In addition one can use different shaping times for the amplification chains in order to optimize the energy and position information [7].

The knowledge of the position signals does not allow, in general, for a straightforward determination of the incidence position. One can face the problem with a purely experimental approach consistıng in bombarding the surface of the detector with a beam of particles, through a mask with a series of regular holes. Any other position is obtained by interpolating on the four-dimensional space of the signals. This procedure, however, requires an accurate calibration of all the detectors and the interpolation is a long procedure in particular for a large number of events.

A second approach to the problem can be based on analytical relationships giving the position signals as a function of the hitting point coordinates. The inversion of these functions, however, gives rise to several numerical problems: one has indeed to take into account that these relationships are in general not linear and that, due to experimental errors, real solutions not always do exist.

In the particular case of point electrodes at each corner, connected by resistive curved strips of suitable line resistivity, an exact relationship has been given [6] relating the incidence point coordinates $x, y$ to the position signals:
$x=\frac{q}{2} \frac{B+D-(A+C)}{A+B+C+D}$,
$y=\frac{q}{2} \frac{A+B-(C+D)}{A+B+C+D}$,
here $A$ and $B$ are the signals from the left and right upper contacts, $C$ and $D$ the ones from the left and right lower contacts and $q$ is the distance between two adjacent vertices. Eqs. (1) hold in a reference frame with the origin in the center of the detector and the $x$ and $y$ axes parallel to the sides of the detector.

Eqs. (1), however, are only a rough approximation [5] for the detector here considered: irradiating uniformely the detector with a radioactive source and using eqs. (1), the distribution of fig. 1 is obtained. It is evident that this distribution does not reflect the true geometry of the detector and that a better approximation is needed. To this aim a method was suggested in


Fig. 1. $x y$ distribution from eqs. (1); A, B, C, D are the detector vertices, P the point with $x=0$ and maximum $y$.
ref. [5]: by solving the Poisson equation for a discrete set of points, the position coordinates are obtained through eqs. (1) and compared with the values calculated from eqs. (1) with the position signals. The impact point coordinates are finally obtained by an interpolation procedure. Big cuts at the four corners are, however, reported [5] and the interpolation over $x$ and $y$ from eqs. (1) limits the accuracy in the position determination.

To investigate possible improvements, the dependence of the position signals on the coordinates of the hitting point was reconsidered. An empyrical method, giving a reasonable evaluation of the incidence point, was found and is hereafter described.

In order to investigate the response of the detector as a function of the incidence position at a fixed energy we measured the four position signals for a large number of incidence positions with a ${ }^{241} \mathrm{Am} \alpha$ source and different masks in front of the detector.

Assuming that the position signals are proportional to the energy signal $E$ of the incident particle [7] one can write:
$A=E f(x, y)$,
$B=E g(x, y)$,
$C=E h(x, y)$,
$D=E l(x, y)$.
From the conservation of the electric charge and for symmetry properties, one can state:

$$
\begin{align*}
& f(x, y)+g(x, y)+h(x, y)+l(x, y)=1,  \tag{3}\\
& f(0,0)=g(0,0)=h(0,0)=l(0,0)=\frac{1}{4} . \tag{4}
\end{align*}
$$



Fig. 2. Normalized position signals vs the normalized distances. (a) Experimental data measured with different masks and the continuous line is an exponential function; and (b), the position signals calculated from eqs. (5).

A reasonable assumption is that the limiting values for $f, g, h, l$ are 0 and 1 , based on the fact that, when a particle hits a point near a corner, the position signal for that vertex tends to the energy signal $E$ while all the other signals tend to zero.

We would like to emphasize the importance of eq. (4) which requires to equalize the position signals corresponding to the central point of the detector. Experimentally one has to make a measurement with a shield with a central hole and to software adjust the relative normalization of the four position signals. In this way the obtained figure is symmetric (see fig. 1).

To determine the relationship between signals and coordinates, we started with the assumption that $f,{ }_{s}$, $h, l$ of eqs. (2) are a unique function of the distance between the impact point and the vertex where the signal is collected. Plotting the position signals $S$ (normalized to their maximum value, i.e. to the energy signal) as a function of the distance $\eta$ (normalized to its maximum value, i.e. to the length of the diagonal of the detector), a roughly exponential dependence is found $S=\mathrm{e}^{-k \pi}$ (see fig. 2a) in the range $0 \leq \eta \leq 0.5$. The constant $k$ can be calculated from the eq. (4). For larger distances big deviations from that behaviour are evident: for a fixed distance $\eta$, different $S$ values are found.

Due to these considerations and mainly to the fact that a pure exponential behaviour does not satisfy eq. (3), a dependence of the signals on all the distances from the four vertices has been introduced: the exponential dependence on the distance to the vertex where
the signal is collected is combined to a weaker dependence on the other distances:

$$
\begin{align*}
& S_{\mathrm{A}}=\frac{\beta \gamma \delta \mathrm{e}^{-k \alpha}}{\beta \gamma \delta \mathrm{e}^{-k \alpha}+\alpha \gamma \delta \mathrm{e}^{-k \beta}+\alpha \beta \delta \mathrm{e}^{-k \gamma}+\alpha \beta \gamma \mathrm{e}^{-k \delta}} \\
& S_{\mathrm{B}}=\frac{\alpha \gamma \delta \mathrm{e}^{-k \beta}}{\beta \gamma \delta \mathrm{e}^{-k \alpha}+\alpha \gamma \delta \mathrm{e}^{-k \beta}+\alpha \beta \delta \mathrm{e}^{-k \gamma}+\alpha \beta \gamma \mathrm{e}^{-k \delta}} \\
& S_{\mathrm{C}}=\frac{\alpha \beta \delta \mathrm{e}^{-k \gamma}}{\beta \gamma \delta \mathrm{e}^{-k \alpha}+\alpha \gamma \delta \mathrm{e}^{-k \beta}+\alpha \beta \delta \mathrm{e}^{-k \gamma}+\alpha \beta \gamma \mathrm{e}^{-k \delta}} \\
& S_{\mathrm{D}}=\frac{\alpha \beta \gamma \mathrm{e}^{-k \delta}}{\beta \gamma \delta \mathrm{e}^{-k \alpha}+\alpha \gamma \delta \mathrm{e}^{-k \beta}+\alpha \beta \delta \mathrm{e}^{-k \gamma}+\alpha \beta \gamma \mathrm{e}^{-k \delta}} \tag{5}
\end{align*}
$$

here $S_{\mathrm{A}}, S_{\mathrm{B}}, S_{\mathrm{C}}, S_{\mathrm{D}}$ are the normalized signals and $\alpha$, $\beta, \gamma, \delta$ are the normalized distances of the incidence point from the vertices $A, B, C$, and $D$ respectively.

The value of $k=0.579$, which for symmetry reasons can be assumed equal for the four signals, is obtained by fitting the experimental points of fig. 2a to eqs. (5). The signals calculated with cqs. (5) for equally spaced points on the detector are reported in fig. $2 b$; the agreement with the experimental data of fig. $2 a$ is satisfactory.

In figs. 3 a and 3 c the calculated and experimental values of the signals for two adjacent vertices are reported, while in figs. 3 b and 3 d the same is reported for two opposite vertices; the correlation between the experimental signals is very well reproduced by eqs. (5).

## 3. Determination of the incidence position and experimental results

As previously mentioned, the problem of inverting eqs. (5) is a very difficult task: even a simple exponential dependence gives rise to several numerical problems, since one has to deal with logarithms and small experimental signals.

We tried therefore to combine the algorithm of eqs. (5) with an interpolation procedure. First the position signals from eqs. (5) are calculated for a large number
of points equally spaced on the detector, then the experimental signals are compared with the calculated ones and finally the position coordinates are linearly interpolated over the four dimensional space of the signals. This procedure is quite accurate, but it is very time consuming.

Thus a different and much faster approach has been followed.

Let us consider for each event the weighted four dimensional distance $D(x, y)$ between the experimental position signals $S_{i}^{\text {exp }}$ and the generic point $S_{i}^{\text {th }}$ on


Fig. 3. Correlation between normalized position signals calculated (a) and (b) and experimentally measured for uniform irradiation.
(a) and (c) refer to adjacent vertices, (b) and (d) to opposite ones.


Fig. 4. $x y$ distribution reconstructed with the method based on eqs. (5).
the surface given by eqs. (5):
$D^{2}(x, y)=\sum_{i}\left(\frac{S_{i}^{\mathrm{exp}}-S_{i}^{\mathrm{th}}}{\Delta S_{i}^{\mathrm{exp}}}\right)^{2}$.
The $x$ and $y$ values we are looking for are the ones that minimize $D(x, y)$ given by eq. (6). This task has been accomplished numerically calculating the first and second partial derivatives, using the Migrad subroutine oí the CERN Library.

We note that, analyzing the $D(x, y)$ distribution, a criterion can be choosen to select spurious events. We found that, for our data, random coincidences are discarded if only events with distances $D(x, y)$ within three standard Jeviations are accepted.

With this procedure the positions corresponding to the uniform $\alpha$-source irradiation are determined. The results (fig. 4) show that the cut near the vertices is $\approx 1 \%$ of the total surface.

To study the accuracy in the position determination and the resolution, measurements have been made using a mask with holes space from 0.25 to 5.25 mm (see fig. 5a). In fig. 5 b the $x y$ distribution, reconstructed with the previously outlined method, is reported and in fig. 6 the projection of experimental data on the $x$ axis and on the diagonal are reported for selected regions of the detector.

From the data analysis the following conclusions can be drawn: the central region ( $30 \mathrm{~mm} \times 30 \mathrm{~mm}$ ) shows a resolution better than 1 mm both for $x$ and $y$; near the detector boundaries the resolution in $x(y)$ direction is better than 1 mm close to the upper and lower (right and left) side of the detector and it is about 3 mm close to the left and right (upper and lower) border, as already noticed in ref. [5]. The differences between the true and the calculated positions range from 0.5 to 3 mm . The regions of poorest accuracy correspond to the regions of poorest resolution. It has to be noticed that a shift of $10 \%$ in the $x$ direction near the border at $y=0$, produces a difference in the signal smaller than $3 \%$. This fact, which does not depend on the used approach, limits the possibility to have a good accuracy in these regions.


Fig. 5. (a) Shape of the mask: the slits have a height of 5 mm (except the ones $n \mathrm{n}$ the diagonal $y=x$ which have a height of 4 mm ) and a width of 1.5 mm , (b) $x y$ distribution reconstructed using eqs. (5).


Fig. 6. (a) Projection on the $x$ axis of the experimental data from the slits in the upper part of fig. 5a. The arrows show the true incidence $x$ coordinates. (b) Projection on the diagonal ( $y=-x$ ) of the data from the slits on the same diagonal. The arrows show the true incidence positions.

All the previous results have been obtained for a particular detector and in principle can be generalized to other detectors of the same kind. Tests on many detectors have given similar results except for the value of $k$ of eqs. (5) which ceriainily depends on the detector thickness.

In order to avoid the long calibrating procedure previously outlined to determine the $k$ value for each detector, the following method can be used: given the point P of coordinates $(0, q / 2)$ at the boundary of the detector, eqs. (5) contain $k$ as the only unknown parameter. Using the position signals $S$ of eqs. (5) in eqs. (1) the coordinate $y_{\mathrm{P}}$ is obtained as a function of $k$.

Then $k$ is:
$k=\frac{2 \sqrt{2}}{\sqrt{5}-1} \ln \left[\frac{1}{\sqrt{5}} \frac{1+\frac{y_{\mathrm{P}}}{q / 2}}{1-\frac{y_{\mathrm{P}}}{q / 2}}\right]$,
and can be calculated experimentally measuring $y_{P}$ (see fig. 1). For the detector previously examined the $k$ value from eq. (7) is 0.576 , to be compared with the value 0.579 reported in section 3 .

In the case of $300 \mu \mathrm{~m}$ thick detectors the $k$ value is about 0.207.

Tests have been made at the XTU Tandem of the Laboratori Nazionali di Legnaro and at the Ganil Laboratories with different ion beams at different energies. We found the procedure reported in this article valid for all detectors and beams considered.

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