Reducibility and a new entropic term in multifragment charge distributions

A. Ferrero,* I. Iori,[1] A. Moroni,[1] F. Petruzzelli,[1] R. Scardaoni,[1] L. G. Moretto,[2] D. R. Bowman,[3] M. Bruno,[4] P. Buttazzo,[5] L. Celano,[6] N. Colonna,[6] M. D'Agostino,[4] J. D. Dinius,[7] M. L. Fiandri,[4] E. Fuschini,[4] C. K. Gelbke,[7] T. Glasmacher,[7] F. Gramegna,[8] D. O. Handzy,[7] D. Horn,[3] W. C. Hsi,[7] M. Huang,[7] G. J. Kunde,[7] M. A. Lisa,[7] W. G. Lynch,[7] P. F. Mastinu,[4] P. M. Milazzo,[4] G. V. Margagliotti,[5] C. P. Montoya,[7] G. F. Peaslee,[7] L. Phair[7] R. Rui,[5] C. Schwarz,[7] M. B. Tsang,[7] G. Vannini,[5] and C. Williams[3]

[1]*Istituto Nazionale di Fisica, Nucleare and Dipartimento di Fisica, Via Celoria 16, 20133 Milano, Italy*

[2]*Nuclear Science Division, Lawrence Berkeley Laboratory, Berkeley, California 94720*

[3]*AECL Research, Chalk River Laboratories, Chalk River, Ontario, Canada K0J 1J0*

[4]*Dipartimento di Fisica and Istituto Nazionale di Fisica Nucleare, Via Irnerio 46, 40126 Bologna, Italy*

[5]*Dipartimento di Fisica and Istituto Nazionale di Fisica Nucleare, Via A. Valerio 2, 34127 Trieste, Italy*

[6]*Istituto Nazionale di Fisica Nucleare, Via Amendola 173, 70126 Bari, Italy*

[7]*National Superconducting Cyclotron Laboratory, Michigan State University, East Lansing, Michigan 48824*

[8]*Istituto Nazionale di Fisica Nucleare, Laboratori Nazionali di Legnaro, Via Romea 4, 35020 Legnaro, Italy*

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The charge distributions and their dependence on fragment multiplicity have been studied for the multifragmentation of 30 MeV/nucleon Xe+Au, Cu. For both targets, the charge distributions are approximately independent of the fragment multiplicity n. However, a residual systematic dependence on n is detectable at the largest values of the total charge multiplicity N_c. Such n dependence obeys a simple scaling law and suggests the presence of an entropic term possibly related to the mechanism of multifragmentation. Thermal scaling between the different bins of N_c seems to occur.

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In the multifragmentation of Ar+Au at 110 MeV/nucleon [1] the fragment multiplicity distribution P_n, at any given event transverse energy $E_t=\Sigma E_i \sin^2\vartheta_i$, was shown to be reducible to an elementary one-fragment emission probability p by means of the well known binomial equation.

It was also noticed that the logarithm of the extracted p depends linearly on $1/\sqrt{E_t}$ (Arrhenius plot). These empirical observations were confirmed at other energies and for other systems [2].

Binomial reducibility permits the reconstruction of the probability of an n-fragment event in terms of a binomial combination of a fixed one-fragment probability, and suggests that the fragments are emitted independently with constant probability p.

The linearity of the Arrhenius plots in turn suggests that the probability p has a Boltzmann-like thermal dependence:

$$p=\exp(-B/T). \tag{1}$$

The implications of binomial reducibility and thermal scaling for the mechanism of multifragmentation have prompted a more attentive consideration of the associated charge distributions. It was shown in Ref. [3] that the simplest condition to be required of the charge distributions to satisfy reducibility is independence from the fragment ($3 \leqslant Z \leqslant 20$) multiplicity n, namely

$$P_1(Z)=P_n(Z)=P_{\text{singles}}(Z)=P(Z). \tag{2}$$

Furthermore, Eq. (1) suggests a thermal scaling of these charge distributions of the form

$$\sqrt{E_t}\ln P(Z)=-C(Z), \tag{3}$$

assuming that E_t is proportional to the excitation energy.

The analysis of the charge distributions in Ar+Au at 110 MeV/nucleon, which, for each fragment multiplicity n, have the exponential form

$$P_n(Z)=e^{-\alpha_n Z}, \tag{4}$$

led to a surprising result [3]. Rather than observing $\alpha_1=\alpha_2=\alpha_n=\alpha=k/\sqrt{E_t}$ as expected, it was found that a more general reducibility equation holds, of the form

$$\alpha_n=\frac{k}{\sqrt{E_t}}+nc \tag{5}$$

(k,c being two constants), suggesting a more general reduced form for a charge distribution of arbitrary shape

$$[\ln P_n(Z)+ncZ]/\sqrt{E_t}=F(Z), \tag{6}$$

for all n, E_t. This implies for $P_n(Z)$ the form

$$P_n(Z)=\exp\left[-\frac{B(Z)}{T}-ncZ\right]=\exp\left[-\frac{\Delta E(Z)}{T}-\Delta S(Z)\right]. \tag{7}$$

The temperature independent entropy term $\Delta S=ncZ$ was claimed [3] to point to an asymptotic combinatorial structure

*On leave from CNEA, Buenos Aires, Argentina.

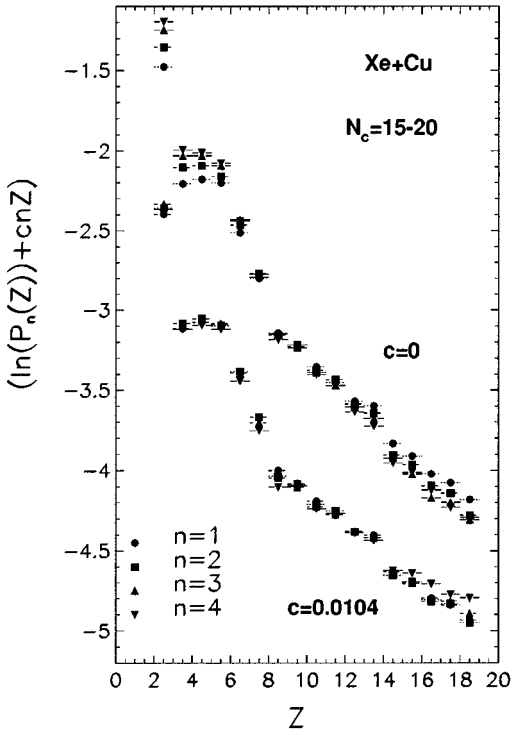

FIG. 1. Charge distributions and their dependence on fragment multiplicity n (upper part) and charge distributions in their reduced form [see Eq. (6)] (lower part; data are shifted by -0.95 on the y axis) for the reaction Xe+Cu at 30 MeV/nucleon and total charge multiplicity N_c bin as shown.

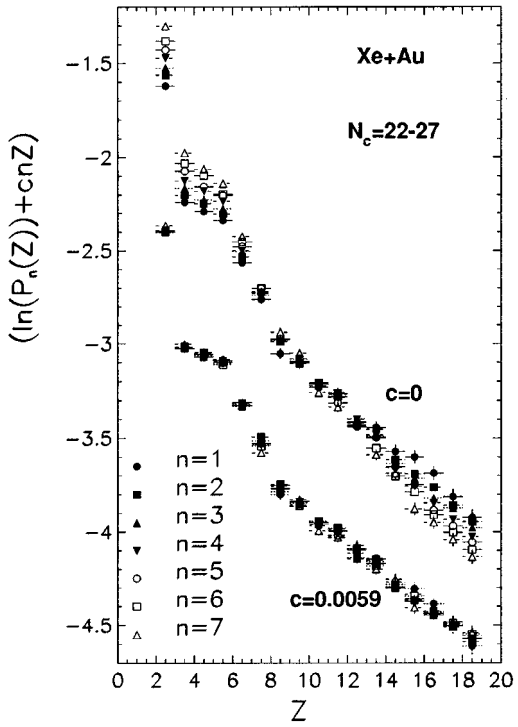

FIG. 2. Same as Fig. 1, for the reaction Xe+Au at 30 MeV/nucleon (data of the lower part are shifted by -0.8 on the y axis).

of multifragmentation, and to be consistent with the Euler partition of an integer Z_0 into n pieces, which gives a fragment "Z" distribution:

$$P(Z)=\frac{n^2}{Z_0}\exp(-nZ/Z_0)=cn^2\exp(-cnZ). \qquad (8)$$

This interesting possibility has led us to investigate whether (a) the above features extend to different systems and different energies; (b) the features observed in Ref. [3] are specific to the choice of E_t as a quantity possibly proportional to the excitation energy; (c) the most general reducibility equation, Eq. (6), could be tested for systems for which the Z distributions are not exponential.

To this purpose, we have studied the reaction Xe+Cu, Au at 30 MeV/nucleon. This bombarding energy places the reaction near the lower limit of the energy range where multifragmentation is studied, while the two targets introduce an interesting range in the size of the systems.

The experiments were performed at the National Superconducting Cyclotron Laboratory of the Michigan State University, combining the Miniball [4] and the Multics [5] detection systems in order to cover with good efficiency the emission phase space of the emitted charged particles and fragments. The solid angle coverage was larger than 87% of 4π. The high resolution gas-Si-Si(Li)-CsI telescopes of the Multics array covered the forward cone, between 3° and 23°, with an angular resolution of $\approx$0.2° and identification threshold $E/A\approx 1.5$ MeV for fragments of all atomic numbers. The angles between 23° and 160° were covered by 158 elements of the Miniball, with identification thresholds of $E/A \approx 2,3,4$ MeV for $Z=3,10,18$ fragments, respectively.

In the present analysis all particles detected in Multics and those up to $Z=20$ in the Miniball are used. Multifragment emission in the reactions considered here has been reported in Ref. [6].

As an alternative to E_t as a measure of the excitation energy, we have chosen the total charge multiplicity N_c. We have verified that binomial reducibility occurs for the fragment multiplicities of both reactions for all values of N_c.

The charge distributions were obtained for four different bins of N_c. The accessible range of fragment multiplicities extends up to 4–5 for Cu and up to 7 for Au. These charge distributions deviate substantially from the exponential form observed in Ref. [3], so that an analysis similar to that performed in Ref. [3] cannot be attempted.

For each target, the distributions change little as a function of n, as shown for the Cu target in Fig. 1 and for the Au target in Fig. 2 (upper curves). On the other hand, at closer inspection, the charge distributions appear to become progressively and regularly steeper with increasing n. This is especially visible in the highest N_c bins presented in Figs. 1 and 2.

This n dependence can be eliminated by using Eq. (6). The lower curves in Figs. 1 and 2 show that by using $c=0.0104$ for Cu and $c=0.0059$ for Au, the charge distributions for all fragment multiplicities can be made to overlap quite precisely.

The analysis of all the charge distributions leads to the values of c given in Table I. It is rather interesting to observe that for both targets the value of c increases substantially with increasing N_c.

TABLE I. Values of c from Eq. (6) for different N_c windows.

Xe+Cu		Xe+Au	
N_c	$c\times10^3$	N_c	$c\times10^3$
2–6	0	3–9	0
7–10	0	10–15	1.6±0.1
11–14	8.5±0.1	16–21	5.8±0.1
15–20	10.4±0.2	22–27	5.9±0.2

A similar increase of c with E_t has been observed in [7].

Equation (6) suggests that charge distribution corresponding to different values of E_t can be compared to each other. It might be interesting to verify that the same kind of comparison can be made for different values of N_c.

On general grounds one expects the average number of charged particles $\langle N_c\rangle$ to be given by $\langle N_c\rangle \propto E/\Delta E$ where E is the excitation energy, and ΔE is the mean total energy per charged particle. ΔE is approximately given by $\Delta E = 2T+B$ where B is an average barrier for the emission. Therefore $\langle N_c\rangle \propto E/(2T+B)$. For $2T \ll B$, $\langle N_c\rangle \propto E$. This case should hbold for moderate excitation energies and for large Z systems where B is Coulomb dominated. On the other hand, at high excitation energies and for lighter systems one is moving towards a regime $2T \gg B$ which gives

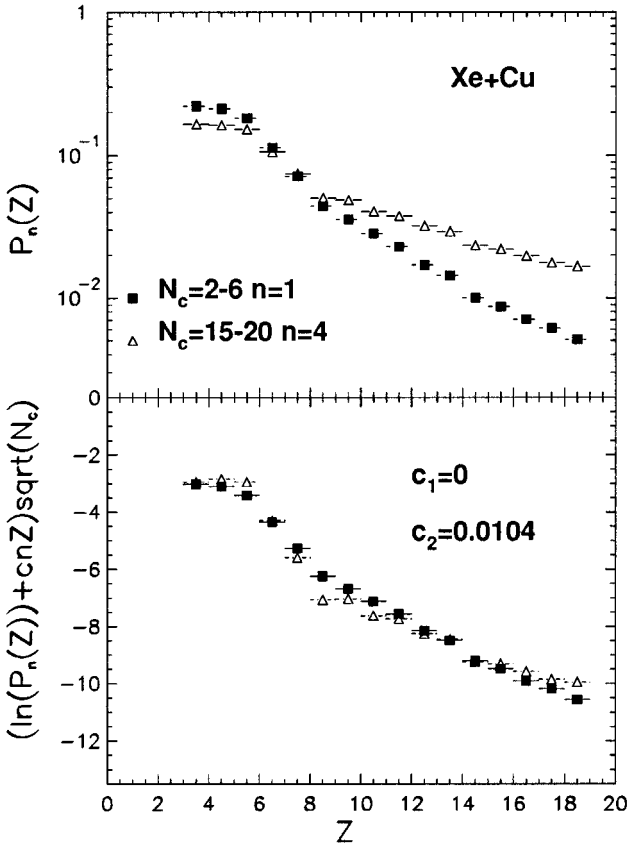

FIG. 3. Examples of charge distributions for two different values of fragment multiplicity n and total charge multiplicity N_c bins (upper part) and their corresponding scaled forms (lower part) for the reaction Xe+Cu at 30 MeV/nucleon.

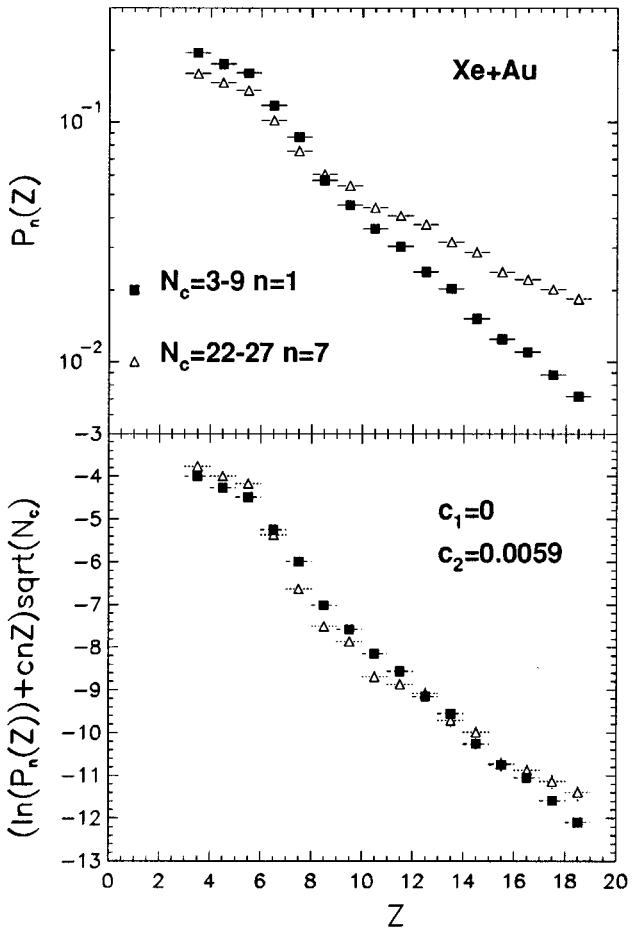

FIG. 4. Same as Fig. 3, for the reaction Xe+Au at 30 MeV/nucleon.

$\langle N_c\rangle \propto T$. In our case the former limit may be applicable. Thus we tested the thermal scaling of Eq. (6) with the substitution of E_t with N_c.

The N_c values considered in the present analysis range from 3 to 20 for the Cu target and from 3 to 27 for Au. The transverse energy corresponding to these N_c values reaches 250 and 350 MeV for Cu and Au, respectively. As the excitation energy cannot be reliably extracted from the experimental data, it has been evaluated in the framework of dynamical model calculations: for central collisions it is ≈750 MeV for Cu [6] and ≈1500 MeV for the Au target. These values agree with the previsions of the Viola systematics.

In many of these reactions, invariant velocity plots indicate the presence of two sources. Thus one may be led to believe that binomial reducibility and thermal scaling ought to be tested on each of the two sources individually. However, experimental evidence shows that both reducibility and thermal scaling hold for both sources combined. Of course, if the emission probability p is the same for both sources, this is a mathematical truth. For sufficiently different values of p, simulations show that the source with large p dominates and controls all distributions.

Figures 3 and 4 show the application of this thermal scaling to distributions with the widest difference in n and N_c. The scaling seems to work reasonably well for both targets.

Two tentative conclusions can be drawn from the present analysis of fragment multiplicities and charge distributions: (a) reducibility of the multiplicity distributions $P_c \rightarrow p$ is accompanied by reducibility of the charge distributions $P_n(Z)$ according to Eq. (6) with c values increasing with increasing N_c and (b) thermal scaling of $P_n(Z)$ according to Eq. (6) seems to occur.

The most intriguing result, however, is the possible absence of a temperature independent entropy term ($c=0$) for the lowest N_C bins, and the distinct presence of it ($c>0$) for the highest N_c bins. If the transition is real, one might be led to speculate about a possible difference in the reaction mechanism.

The way in which charge conservation is implemented in a sequential emission or in chemical equilibrium could be quite different. In the former case, the first fragment cannot know about the number and size of the fragments that may follow it. Thus one expects $c=0$. In the latter case, as in a multifragment transition state, each fragment may be acutely aware of all the others and the burden of charge conservation should be evenly spread among the fragments. In this case $c>0$.

Regarding the aspect of sequentiality versus simultaneity, we note that an analysis of different observables for the Xe+Cu reaction indicates [6] a mean fragment emission time of $\approx$200 fm/c, consistent with a fast, but still sequential, time scale. Thus the relevance of the previous speculation to the reactions in question is unclear.

Another possibility could be the following. In the liquid-vapor co-existence region, the properties of the vapor are not affected by the requirement of overall mass conservation, since the liquid in equilibrium adjusts its masses with changing volume just to ensure such global conservation. This translates into the parameter c being equal to zero. On the other hand, in the overheated vapor region there is only one phase which must ensure mass conservation, at least grand canonically. Thus, in this region $c>0$.

Simulations on finite percolating systems [7] show that the resulting data can be analyzed just as described here. One obtains $c=0$ when a large (percolating) cluster is present, and $c>0$ when the percolating cluster is absent. Similarly an evaporation simulation following the binomial scheme, modified in such a way to prevent the emission of fragments larger than the emitting sources, yields $c=0$ if an evaporation residue is present, and $c>0$ otherwise.

Finally one could consider the actual values of c for the cases in which it is different than zero. In the case of 110 MeV/nucleon Ar+Au, $c\approx0.016$ leading to a value of $Z_0 \approx 60$ [see Eq. (8)], while in the present case the largest values of c lead to $Z_0\approx95$ for the Cu target and to $Z_0\approx170$ for the Au target. The correlation of Z_0 with the total size of the system is interesting. However, dimensionality may be relevant to this aspect of the problem. Percolation calculations show that $c=k/Z_0$ with k different than 1. Similar results are shown by the binomial evaporation simulation reported above. Therefore the correlation between c and the size of the system remains still open. Furthermore it is likely that there might be a substantial averaging over the excitation energy that could be very different for N_c and E_t. Thus a comparison between the c values for the Ar+Au case and those obtained in the present cases may be premature.

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