Introduction to Bayesian Statistics - 4

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A few simple applications of Bayesian techniques

1. analytical straight-line fit
2. weighted mean
3. Miscalibrated Gaussian measurement errors
4. search for weak signals in spectra
5. expert elicitation
6. The lost flight AF 477

1. Analytical straight-line fit

$$y_i = ax_i + b + \varepsilon_i$$

y_i measured value

x_i independent variable (“exactly” known)

a, b fit parametes: eventually we expect to find pdf’s for these parameters

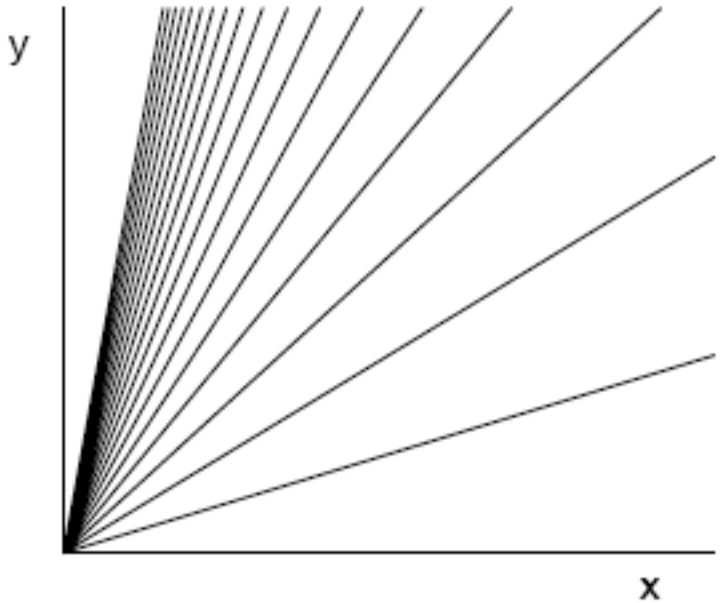
ε_i statistical error

$\langle \varepsilon_i \rangle = 0; \quad \langle \varepsilon_i^2 \rangle = \sigma^2 \quad \Rightarrow$ the statistical measurement error has a Gaussian distribution

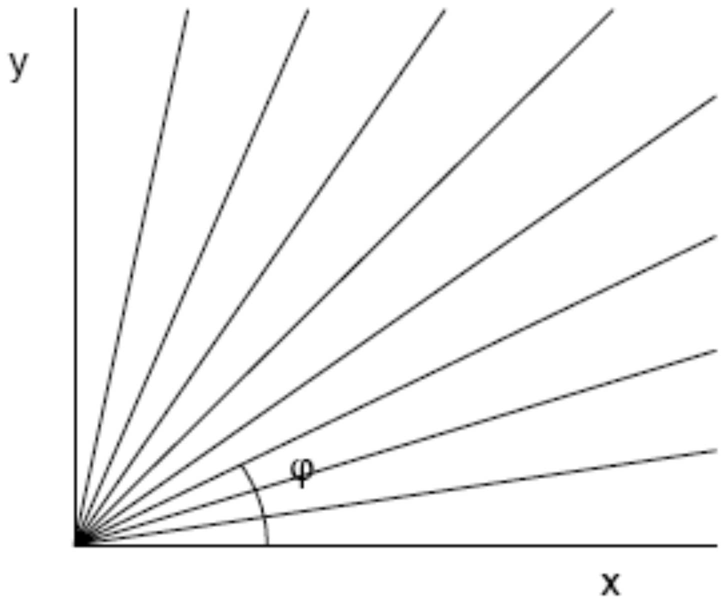
likelihood

$$p\left(\mathbf{y} \mid a,b,\mathbf{x},\sigma\right) = \left(2\pi\sigma^2\right)^{-N/2} \exp\left[-\frac{1}{2\sigma^2}\sum_{i=1}^{N}\left(y_i - a x_i - b\right)^2\right]$$

prior angular distribution



uniform *a*



uniform angle

The uniform distribution of a introduces an angular bias. The least informative choice corresponds to a uniform angular distribution

$$p_\varphi(\varphi) = \frac{1}{\pi}; \quad -\frac{\pi}{2} \le \varphi < \frac{\pi}{2}$$

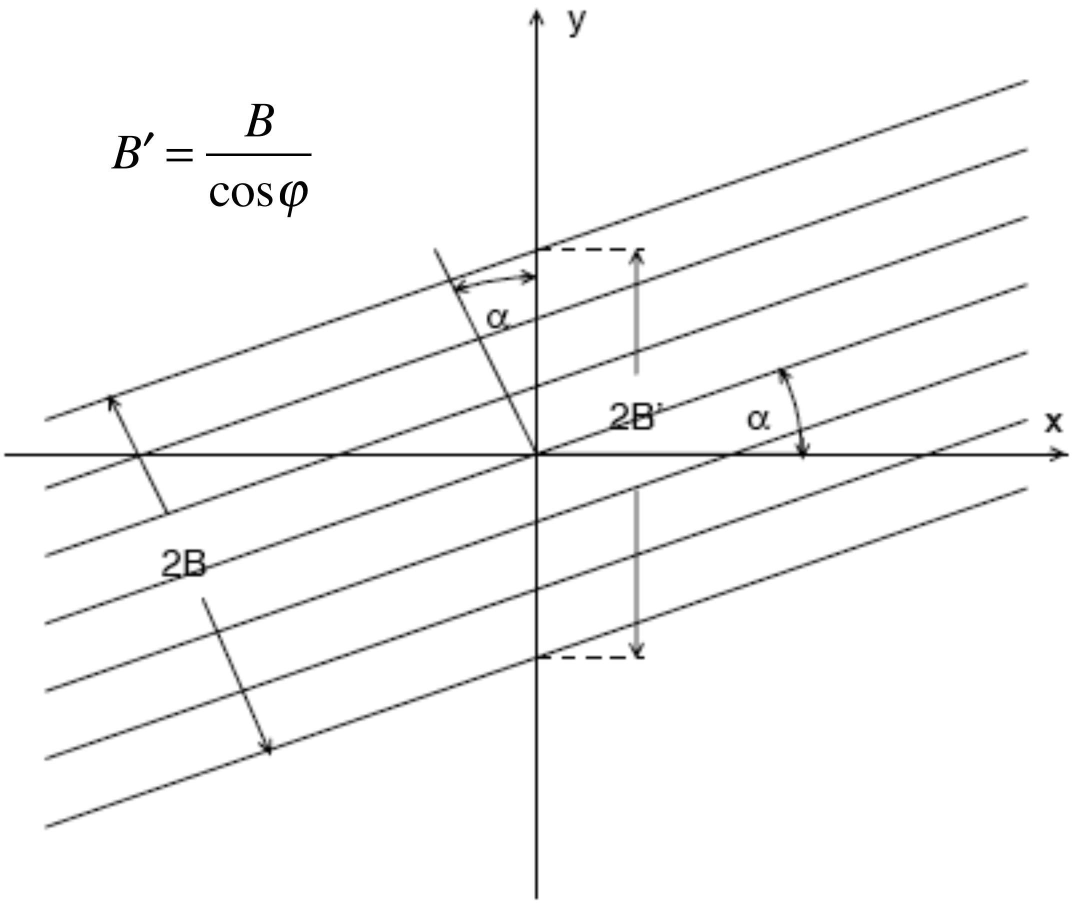
and we obtain the distribution of *a* with the transformation method:

$$a = \tan\varphi$$

$$\Rightarrow \quad p_\varphi(\varphi)d\varphi = p_a(a)da = p_a(a)d(\tan\varphi) = p_a(a)\sec^2\varphi d\varphi$$

$$\Rightarrow \quad p_a(a) = \frac{1}{\pi\sec^2\varphi} = \frac{1}{\pi(1+\tan^2\varphi)} = \frac{1}{\pi(1+a^2)}$$

prior distribution of *b*: improper uniform distribution, related to the distribution of *a*



$$B' = \frac{B}{\cos\varphi}$$

$$p(b \mid a=0) = \frac{1}{2B}; \quad p(b \mid a) = \frac{1}{2B'} = \frac{\cos\varphi}{2B} = \frac{1}{2B} \cdot \frac{1}{\sqrt{1+a^2}}$$

we obtain the posterior from Bayes' theorem

$$p(a,b \mid \mathbf{y},\mathbf{x},\sigma) = \frac{p(\mathbf{y} \mid a,b,\mathbf{x},\sigma)}{\int_{-\infty}^{+\infty} da \int_{-B/\cos\varphi}^{B/\cos\varphi} db\; p(\mathbf{y} \mid a,b,\mathbf{x},\sigma)\cdot p(a,b)} \cdot p(a,b)$$

where the prior is

$$p(a,b) = p(b \mid a)\cdot p(a) = \left(\frac{1}{2B}\cdot\frac{1}{\sqrt{1+a^2}}\right)\left(\frac{1}{\pi\left(1+a^2\right)}\right)$$

$$\propto \frac{1}{\left(1+a^2\right)^{3/2}}$$

finally we find

$$p\left(a,b\,|\,\mathbf{y},\mathbf{x},\sigma\right)=\frac{\exp\left[-\frac{1}{2\sigma^2}\sum_{i=1}^{N}\left(y_i-ax_i-b\right)^2\right]}{\left\{\int_{-\infty}^{+\infty}da\int_{-B/\cos\varphi}^{B/\cos\varphi}db\,\exp\left[-\frac{1}{2\sigma^2}\sum_{i=1}^{N}\left(y_i-ax_i-b\right)^2\right]\cdot\frac{1}{\left(1+a^2\right)^{3/2}}\right\}}\cdot\frac{1}{\left(1+a^2\right)^{3/2}}$$

$$\approx\frac{\frac{1}{\left(1+a^2\right)^{3/2}}\exp\left[-\frac{1}{2\sigma^2}\sum_{i=1}^{N}\left(y_i-ax_i-b\right)^2\right]}{\left\{\int_{-\infty}^{+\infty}\frac{da}{\left(1+a^2\right)^{3/2}}\int_{-\infty}^{+\infty}db\,\exp\left[-\frac{1}{2\sigma^2}\sum_{i=1}^{N}\left(y_i-ax_i-b\right)^2\right]\right\}}$$

This expression has a partly Gaussian structure, and now we rearrange the quadratic expression in the exponential

$$\sum_{i=1}^{N}(y_i - ax_i - b)^2 = \sum_{i=1}^{N}\left[(y_i - ax_i)^2 - 2b(y_i - ax_i) + b^2\right]$$

$$= \sum_{i=1}^{N}(y_i - ax_i)^2 - 2b\sum_{i=1}^{N}(y_i - ax_i) + Nb^2$$

$$= N\left\{\left[b^2 - 2b\frac{1}{N}\sum_{i=1}^{N}(y_i - ax_i) + \left(\frac{1}{N}\sum_{i=1}^{N}(y_i - ax_i)\right)^2\right] + \frac{1}{N}\sum_{i=1}^{N}(y_i - ax_i)^2 - \left(\frac{1}{N}\sum_{i=1}^{N}(y_i - ax_i)\right)^2\right\}$$

$$= N\left\{\left(b - \frac{1}{N}\sum_{i=1}^{N}(y_i - ax_i)\right)^2 + \frac{1}{N}\sum_{i=1}^{N}(y_i - ax_i)^2 - \left(\frac{1}{N}\sum_{i=1}^{N}(y_i - ax_i)\right)^2\right\}$$

$$= N\left(b - \frac{1}{N}\sum_{i=1}^{N}(y_i - ax_i)\right)^2 + N\left(\frac{1}{N}\sum_{i=1}^{N}y_i^2 - 2a\frac{1}{N}\sum_{i=1}^{N}x_i y_i + a^2\frac{1}{N}\sum_{i=1}^{N}x_i^2\right) - N\left(\frac{1}{N}\sum_{i=1}^{N}y_i - a\frac{1}{N}\sum_{i=1}^{N}x_i\right)^2$$

$$= N\left(b - \frac{1}{N}\sum_{i=1}^{N}(y_i - ax_i)\right)^2 + N\left(\operatorname{var} y - 2a\operatorname{cov}(x,y) + a^2 \operatorname{var} x\right)$$

therefore the normalization integral becomes

$$\int_{-\infty}^{+\infty}\frac{da}{(1+a^2)^{3/2}}\exp\left[-\frac{N}{2\sigma^2}\left(\operatorname{var} y - 2a\operatorname{cov}(x,y) + a^2\operatorname{var} x\right)\right]\int_{-\infty}^{+\infty} db\,\exp\left[-\frac{N}{2\sigma^2}\left(b - \frac{1}{N}\sum_{i=1}^{N}(y_i - ax_i)\right)^2\right]$$

$$= \sqrt{\frac{2\pi\sigma^2}{N}}\int_{-\infty}^{+\infty}\frac{da}{(1+a^2)^{3/2}}\exp\left[-\frac{N}{2\sigma^2}\left(\operatorname{var} y - 2a\operatorname{cov}(x,y) + a^2\operatorname{var} x\right)\right]$$

Approximate integration of the remaining integral

$$\int_{-\infty}^{+\infty} \frac{da}{\left(1+a^2\right)^{3/2}} \exp\left[-\frac{N}{2\sigma^2}\left(\operatorname{var} y - 2a\operatorname{cov}(x,y) + a^2 \operatorname{var} x\right)\right]$$

We evaluate this integral by integrating about the peak of the integrand, assuming that the peak is narrow.

We start with the logarithm of the integrand, we find its maximum and we Taylor expand about the maximum

$$\Phi(a) = -\frac{3}{2}\ln\left(1+a^2\right) - \frac{N}{2\sigma^2}\left(\operatorname{var} y - 2a\operatorname{cov}(x,y) + a^2 \operatorname{var} x\right)$$

$$\Phi(a) = -\frac{3}{2}\ln\left(1+a^2\right) - \frac{N}{2\sigma^2}\left(\operatorname{var} y - 2a\operatorname{cov}(x,y) + a^2 \operatorname{var} x\right)$$

$$\frac{d\Phi}{da} = -\frac{3a}{1+a^2} + \frac{N}{\sigma^2}\left(\operatorname{cov}(x,y) - a\operatorname{var} x\right) = 0$$

we find *a* from this cubic equation

note that when N>>1 the peak is at position $a_0 \approx \frac{\operatorname{cov}(x,y)}{\operatorname{var} x}$

We use the Newton-Raphson method for the solution of the cubic equation:

$$f(a_0) = -\frac{3a_0}{1+a_0^2}$$

$$f'(a_0) = -3\frac{1-a_0^2}{(1+a_0^2)^2} - \frac{N}{\sigma^2}\operatorname{var} x \approx -\frac{N}{\sigma^2}\operatorname{var} x$$

then

$$\delta a_1 = -\frac{3a_0}{1+a_0^2}\frac{\sigma^2}{N \operatorname{var} x} \qquad\qquad a_1 = a_0 - \frac{3a_0}{1+a_0^2}\frac{\sigma^2}{N \operatorname{var} x} \tag{1}$$

Now, to complete the expansion, we must evaluate the second derivative at a_1:

$$\frac{d^2\Phi}{da^2} = -3\frac{1-a_1^2}{(1+a_1^2)^2} - \frac{N}{\sigma^2}\operatorname{var} x = -\frac{1}{\sigma_1^2} \tag{2}$$

$$\Phi(a) \approx \Phi(a_1) + \frac{1}{2}\frac{d^2\Phi}{da^2}\bigg|_{a_1}(a-a_1)^2 = \Phi(a_1) - \frac{(a-a_1)^2}{2\sigma_1^2}$$

we find this by using equations (1) and (2)

Now we complete the evaluation of the integral

$$\int_{-\infty}^{+\infty} \frac{da}{\left(1+a^2\right)^{3/2}} \exp\left[-\frac{N}{2\sigma^2}\left(\operatorname{var} y - 2a\operatorname{cov}(x,y) + a^2 \operatorname{var} x\right)\right]$$

$$= \int_{-\infty}^{+\infty} \exp\left[\Phi(a)\right] da$$

$$\approx \int_{-\infty}^{+\infty} \exp\left[\Phi(a_1) - \frac{\left(a-a_1\right)^2}{2\sigma_1^2}\right] da = \sqrt{2\pi\sigma_1^2}\, \exp\left[\Phi(a_1)\right]$$

and finally we find the posterior distribution.

Moreover

$$p\left(a,b\,|\,\mathbf{y},\mathbf{x},\sigma\right) \propto \frac{1}{\left(1+a^2\right)^{3/2}} \exp\left[-\frac{1}{2\sigma^2}\sum_{i=1}^{N}\left(y_i - a x_i - b\right)^2\right]$$

$$\approx \exp\left[-\Phi\left(a_1\right) - \frac{\left(a-a_1\right)^2}{2\sigma_1^2}\right] \exp\left[-\frac{N}{2\sigma^2}\left(b - \frac{1}{N}\sum_{i=1}^{N}\left(y_i - a_1 x_i\right)\right)^2\right]$$

and thus we see that:

$$\langle a \rangle = a_1; \quad \operatorname{var} a = \sigma_1^2;$$

$$\langle b \rangle = \frac{1}{N}\sum_{i=1}^{N}\left(y_i - a_1 x_i\right); \quad \operatorname{var} b = \frac{\sigma^2}{N}$$

2. Weighted mean

We consider known Gaussian errors

The likelihood function is

$$P(\mathbf{d} \mid \mu, \boldsymbol{\sigma}) = \prod_{k=1}^{N} \frac{1}{\sqrt{2\pi\sigma_k^2}} \exp\left[-\frac{(d_k - \mu)^2}{2\sigma_k^2} \right]$$

$$= \frac{1}{(2\pi)^{N/2}} \left(\prod_{k=1}^{N} \frac{1}{\sigma_k} \right) \exp\left[-\frac{1}{2} \sum_{k=1}^{N} \frac{(d_k - \mu)^2}{\sigma_k^2} \right]$$

Using an improper uniform prior we find

$$P(\mu \mid \mathbf{d},\boldsymbol{\sigma}) = \frac{P(\mathbf{d} \mid \boldsymbol{\sigma},\mu)}{\int_\mu P(\mathbf{d} \mid \boldsymbol{\sigma},\mu) \cdot P(\mu)\, d\mu} \cdot P(\mu) \rightarrow \frac{P(\mathbf{d} \mid \boldsymbol{\sigma},\mu)}{\int_\mu P(\mathbf{d} \mid \boldsymbol{\sigma},\mu)\, d\mu}$$

and then

$$P(\mu \mid \mathbf{d},\boldsymbol{\sigma}) = \frac{\exp\left[-\frac{1}{2}\sum_{k=1}^{N}\frac{(d_k-\mu)^2}{\sigma_k^2}\right]}{\int_{-\infty}^{+\infty}\exp\left[-\frac{1}{2}\sum_{k=1}^{N}\frac{(d_k-\mu)^2}{\sigma_k^2}\right] d\mu}$$

the exponent can be rearranged as usual

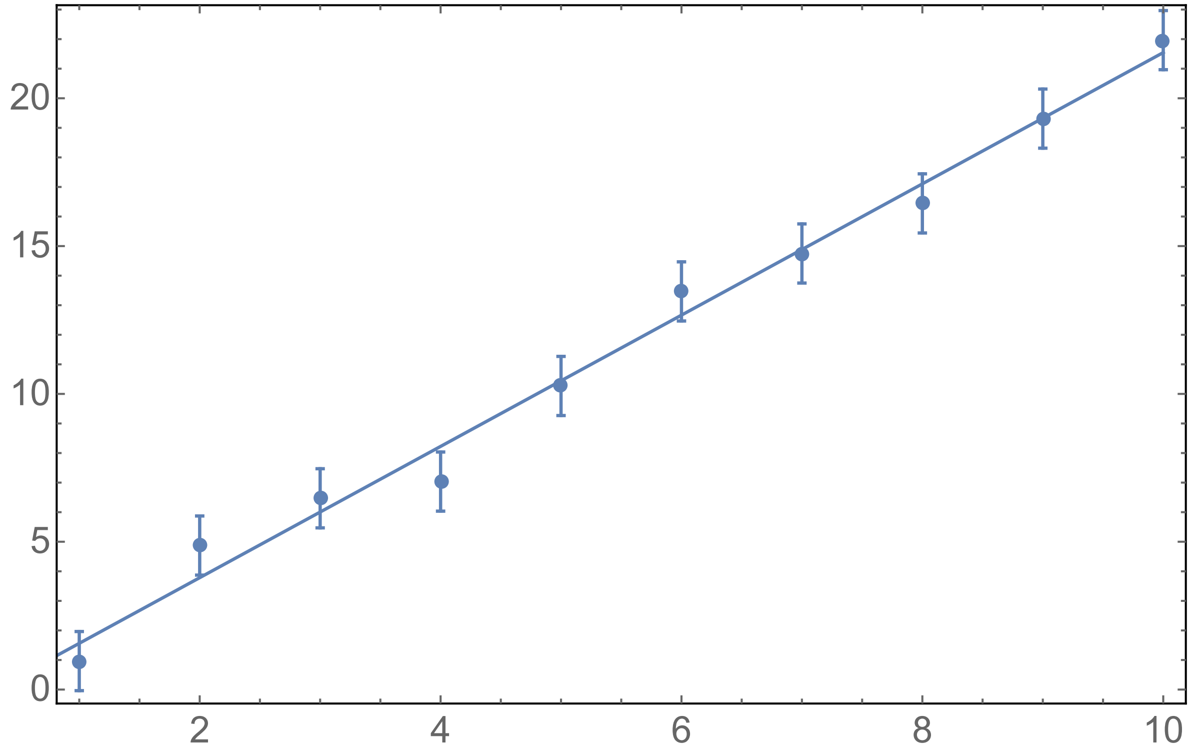
$$
\begin{aligned}
\sum_{k=1}^{N} \frac{\left(d_k-\mu\right)^2}{\sigma_k^2} &= \sum_{k=1}^{N} \frac{d_k^2-2\mu d_k+\mu^2}{\sigma_k^2} = \mu^2 \sum_{k=1}^{N} \frac{1}{\sigma_k^2} - 2\mu \sum_{k=1}^{N} \frac{d_k}{\sigma_k^2} + \sum_{k=1}^{N} \frac{d_k^2}{\sigma_k^2} \\
&= \left(\sum_{k=1}^{N} \frac{1}{\sigma_k^2}\right)\left[\mu^2 - 2\mu\left(\sum_{k=1}^{N} \frac{d_k}{\sigma_k^2} \Big/ \sum_{k=1}^{N} \frac{1}{\sigma_k^2}\right) + \left(\sum_{k=1}^{N} \frac{d_k}{\sigma_k^2} \Big/ \sum_{k=1}^{N} \frac{1}{\sigma_k^2}\right)^2\right] + \left[\left(\sum_{k=1}^{N} \frac{d_k}{\sigma_k^2}\right)^2 \Big/ \sum_{k=1}^{N} \frac{1}{\sigma_k^2} - \left(\sum_{k=1}^{N} \frac{d_k}{\sigma_k^2}\right)^2\right] \\
&= \left(\sum_{k=1}^{N} \frac{1}{\sigma_k^2}\right)\left[\mu - \left(\sum_{k=1}^{N} \frac{d_k}{\sigma_k^2} \Big/ \sum_{k=1}^{N} \frac{1}{\sigma_k^2}\right)\right]^2 + \left[\left(\sum_{k=1}^{N} \frac{d_k}{\sigma_k^2}\right)^2 \Big/ \sum_{k=1}^{N} \frac{1}{\sigma_k^2} - \left(\sum_{k=1}^{N} \frac{d_k}{\sigma_k^2}\right)^2\right]
\end{aligned}
$$

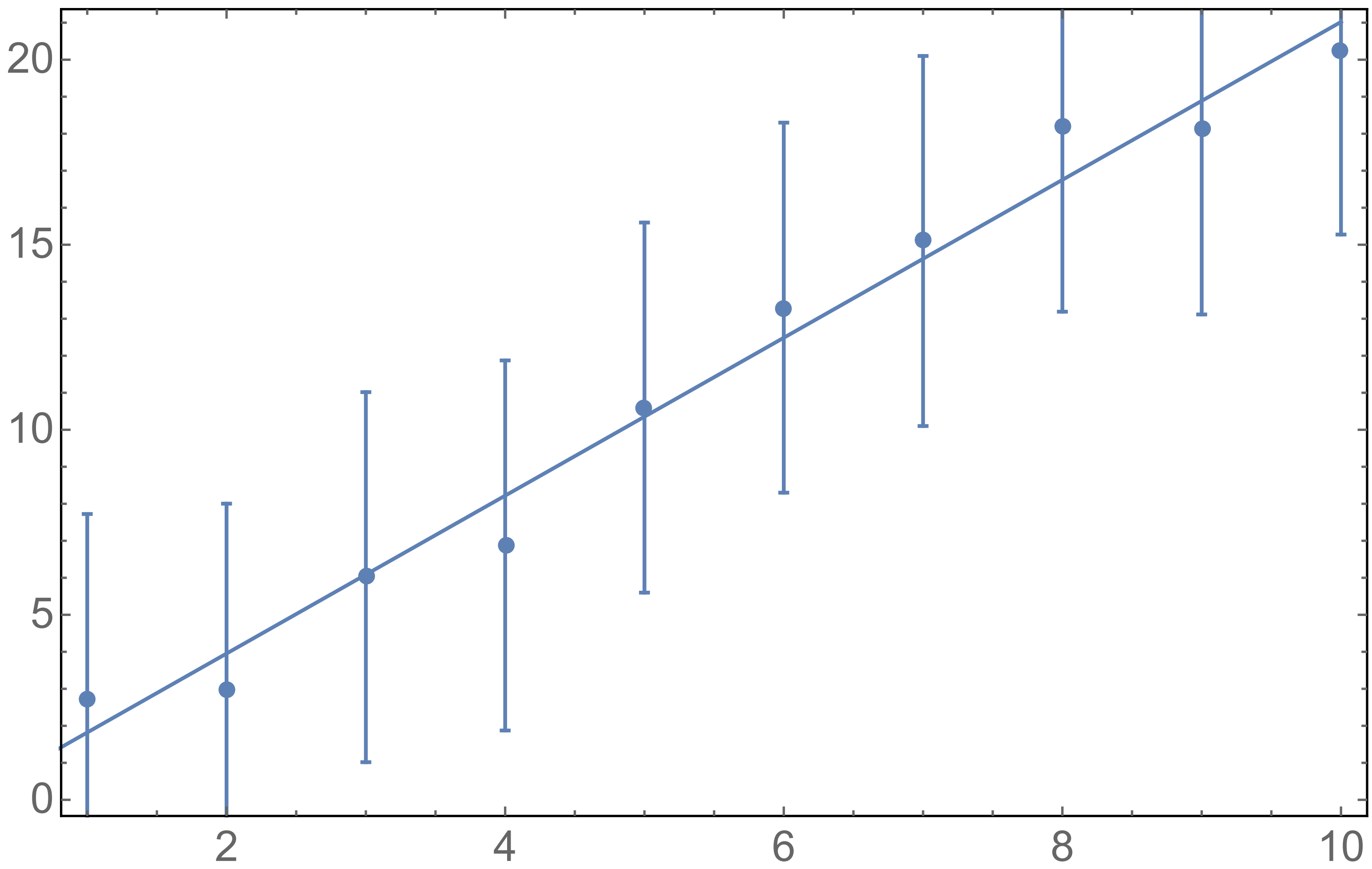
thus we see that the mean has a Gaussian posterior distribution and that

$$
\bar{\mu} = \frac{\displaystyle\sum_{k=1}^{N} \frac{d_k}{\sigma_k^2}}{\displaystyle\sum_{k=1}^{N} \frac{1}{\sigma_k^2}}; \qquad \sigma_\mu^2 = \left(\sum_{k=1}^{N} \frac{1}{\sigma_k^2}\right)^{-1}
$$

3. Miscalibrated Gaussian measurement errors

Now we consider a case where we must find the mean value with given measurement errors, and where the errors are Gaussian and they are systematically multiplied by an unknown scale factor.





20
15
10
5
0
2
4
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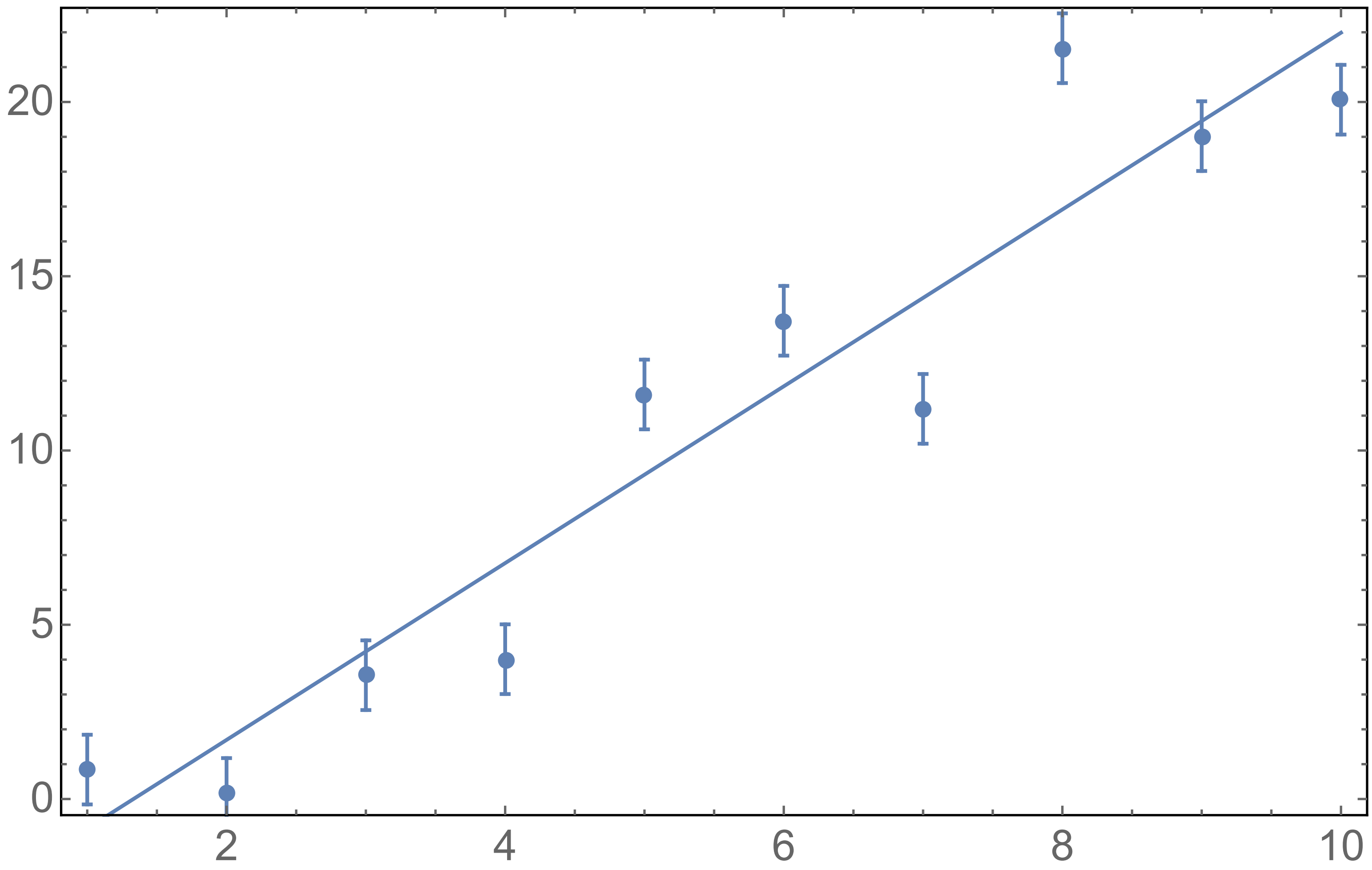
8
10



20
15
10
5
0
2
4
6
8
10

The likelihood has a Gaussian structure

$$P(\mathbf{d} \mid \mu, \boldsymbol{\sigma}, \alpha) = \prod_{k=1}^{N} \frac{1}{\sqrt{2\pi\alpha^2\sigma_k^2}} \exp\left[-\frac{(d_k - \mu)^2}{2\alpha^2\sigma_k^2} \right]$$

$$= \frac{1}{(2\pi)^{N/2}\alpha^N} \left(\prod_{k=1}^{N} \frac{1}{\sigma_k} \right) \exp\left[-\frac{1}{2\alpha^2} \sum_{k=1}^{N} \frac{(d_k - \mu)^2}{\sigma_k^2} \right]$$

we must rearrange the exponent as usual ...

$$\sum_{k=1}^{N} \frac{(d_k - \mu)^2}{\sigma_k^2} = \sum_{k=1}^{N} \frac{d_k^2}{\sigma_k^2} - 2\mu \sum_{k=1}^{N} \frac{d_k}{\sigma_k^2} + \mu^2 \sum_{k=1}^{N} \frac{1}{\sigma_k^2} = \frac{ND}{\sigma_M^2} - 2\mu \frac{NM}{\sigma_M^2} + \mu^2 \frac{N}{\sigma_M^2}$$

$$= \frac{N}{\sigma_M^2}\left(D - 2\mu M + \mu^2\right)$$

$$\text{dove} \quad \frac{1}{\sigma_M^2} = \frac{1}{N}\sum_{k=1}^{N} \frac{1}{\sigma_k^2}; \quad M = \sum_{k=1}^{N} \frac{d_k}{\sigma_k^2} \Bigg/ \sum_{k=1}^{N} \frac{1}{\sigma_k^2}; \quad D = \sum_{k=1}^{N} \frac{d_k^2}{\sigma_k^2} \Bigg/ \sum_{k=1}^{N} \frac{1}{\sigma_k^2}$$

therefore the likelihood is

$$P(\mathbf{d} \mid \mu, \boldsymbol{\sigma}, \alpha) = \frac{1}{(2\pi)^{N/2} \alpha^N} \left(\prod_{k=1}^{N} \frac{1}{\sigma_k} \right) \exp\left[-\frac{N}{2\alpha^2 \sigma_M^2} \left(D - 2\mu M + \mu^2 \right) \right]$$

Now we estimate the scale factor from Bayes' theorem

$$P(\alpha \mid \mathbf{d},\boldsymbol{\sigma}) = \frac{P(\mathbf{d} \mid \boldsymbol{\sigma},\alpha)}{\int_{\alpha} P(\mathbf{d} \mid \boldsymbol{\sigma},\alpha) \cdot P(\alpha)\, d\alpha} \cdot P(\alpha)$$

however we need first to marginalize the likelihood with respect to the mean, which in this case is a ***nuisance parameter***

we take a uniform prior for the mean

$$P(\mathbf{d} \mid \boldsymbol{\sigma},\alpha) = \int_{\mu} P(\mathbf{d} \mid \mu,\boldsymbol{\sigma},\alpha) P(\mu \vert \boldsymbol{\sigma},\alpha)\, d\mu$$

$$= \frac{1}{W} \int_{\mu_{\min}}^{\mu_{\max}} P(\mathbf{d} \mid \mu,\boldsymbol{\sigma},\alpha)\, d\mu$$

$$\approx \frac{1}{W} \frac{1}{(2\pi)^{N/2} \alpha^N} \left(\prod_{k=1}^{N} \frac{1}{\sigma_k} \right) \int_{-\infty}^{+\infty} \exp\left[-\frac{N}{2\alpha^2 \sigma_M^2} \left(D - 2\mu M + \mu^2 \right) \right] d\mu$$

$$\left(W = \mu_{\max} - \mu_{\min} \right)$$

as usual ...

$$D - 2\mu M + \mu^2 = \mu^2 - 2\mu M + M^2 + D - M^2$$
$$= (\mu - M)^2 + D - M^2$$

... therefore the likelihood is:

$$P(\mathbf{d} \mid \boldsymbol{\sigma}, \alpha) \approx \frac{1}{W} \frac{1}{(2\pi)^{N/2} \alpha^N} \left(\prod_{k=1}^{N} \frac{1}{\sigma_k} \right) \int_{-\infty}^{+\infty} \exp\left\{ -\frac{N}{2\alpha^2 \sigma_M^2} \left[(\mu - M)^2 + D - M^2 \right] \right\} d\mu$$
$$= \frac{1}{W} \frac{1}{(2\pi)^{N/2} \alpha^N} \left(\prod_{k=1}^{N} \frac{1}{\sigma_k} \right) \exp\left(-\frac{N(D - M^2)}{2\alpha^2 \sigma_M^2} \right) \sqrt{\frac{2\pi \alpha^2 \sigma_M^2}{N}}$$

$$P(\alpha \mid \mathbf{d},\boldsymbol{\sigma}) = \frac{P(\mathbf{d} \mid \boldsymbol{\sigma},\alpha)}{\int_\alpha P(\mathbf{d} \mid \boldsymbol{\sigma},\alpha)\cdot P(\alpha)} \cdot P(\alpha)$$

$$= \frac{\frac{1}{\alpha^{N-1}} \exp\left(-\frac{N(D-M^2)}{2\alpha^2 \boldsymbol{\sigma}_M^2}\right)}{\int_\alpha \frac{1}{\alpha^{N-1}} \exp\left(-\frac{N(D-M^2)}{2\alpha^2 \boldsymbol{\sigma}_M^2}\right) \cdot P(\alpha)\, d\alpha} \cdot P(\alpha)$$

$$P(\alpha) \propto \frac{1}{\alpha}$$

the prior should be scale-independent and therefore we take Jeffrey's prior

$$P(\alpha \mid \mathbf{d},\boldsymbol{\sigma}) = \frac{\frac{1}{\alpha^{N-1}}\exp\left(-\frac{A^2}{\alpha^2}\right)\cdot\frac{1}{\alpha}}{\int_\alpha \frac{1}{\alpha^{N-1}}\exp\left(-\frac{A^2}{\alpha^2}\right)\cdot\frac{1}{\alpha}\,d\alpha}; \qquad A^2 = \frac{N\left(D-M^2\right)}{2\sigma_M^2}$$

$$P(\alpha \mid \mathbf{d},\boldsymbol{\sigma}) \to \frac{\frac{1}{\alpha^{N-1}}\exp\left(-\frac{A^2}{\alpha^2}\right)\cdot\frac{1}{\alpha}}{\int_0^\infty \frac{1}{\alpha^{N-1}}\exp\left(-\frac{A^2}{\alpha^2}\right)\cdot\frac{1}{\alpha}\,d\alpha}$$

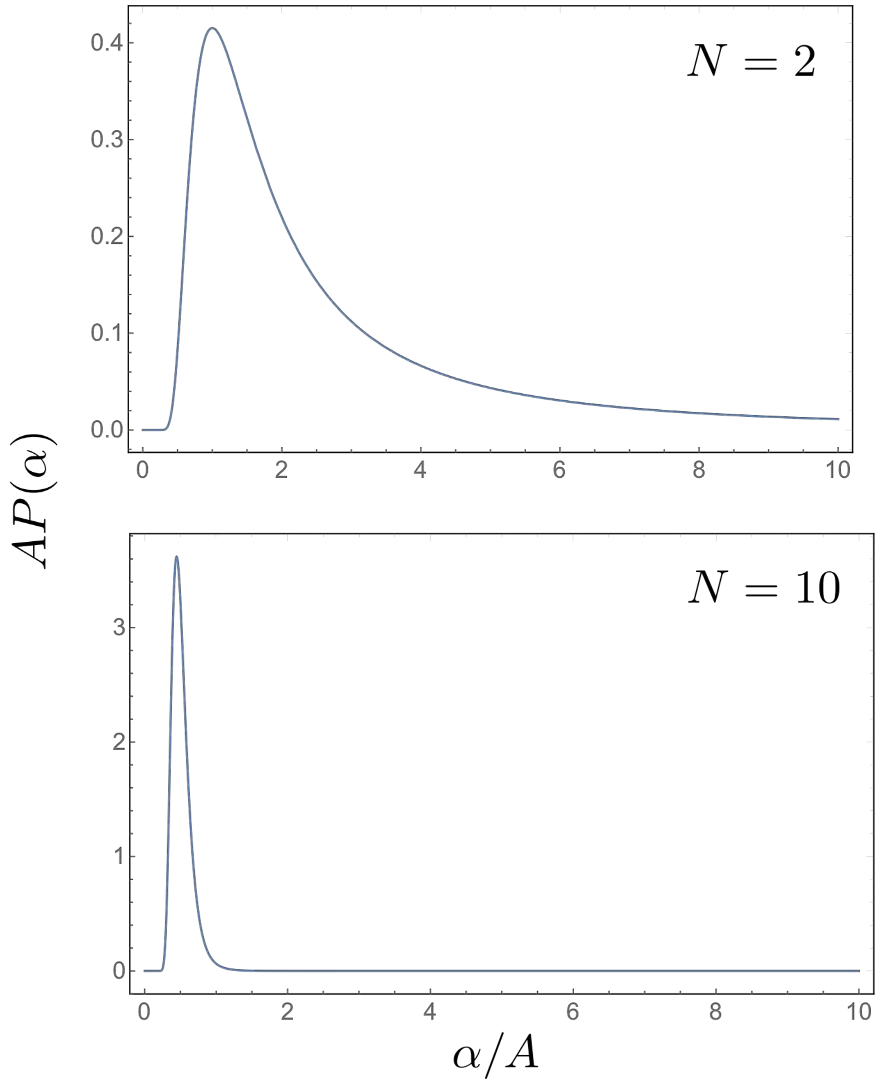
$$\int_0^\infty \frac{1}{\alpha^{N-1}} \exp\left(-\frac{A^2}{\alpha^2}\right) \cdot \frac{1}{\alpha} d\alpha = \int_0^\infty \frac{1}{\alpha^N} \exp\left(-\frac{A^2}{\alpha^2}\right) d\alpha$$

$$\frac{A^2}{\alpha^2} = x; \quad \alpha = \frac{A}{\sqrt{x}}; \quad d\alpha = -\frac{A}{2x^{3/2}} dx$$

$$\int_0^\infty \frac{x^{N/2}}{A^N} \exp(-x) \frac{A}{2x^{3/2}} dx = \frac{1}{2A^{N-1}} \int_0^\infty x^{\frac{N-1}{2}-1} \exp(-x) dx = \frac{1}{2A^{N-1}} \cdot \Gamma\left(\frac{N-1}{2}\right)$$

$$P(\alpha \mid \mathbf{d}, \boldsymbol{\sigma}) \to \frac{\dfrac{2A^{N-1}}{\alpha^N} \exp\left(-\dfrac{A^2}{\alpha^2}\right)}{\Gamma\left(\dfrac{N-1}{2}\right)}$$

$$P(\alpha|\mathbf{d},\boldsymbol{\sigma}) = \frac{(2A^{N-1}/\alpha^N)\exp(-A^2/\alpha^2)}{\Gamma\left[(N-1)/2)\right]}$$



we take the MAP estimate the scale parameter from the pdf

$$P(\alpha \mid \mathbf{d}, \boldsymbol{\sigma}) \to \frac{\dfrac{2A^{N-1}}{\alpha^N} \exp\left(-\dfrac{A^2}{\alpha^2}\right)}{\Gamma\left(\dfrac{N-1}{2}\right)}$$

$$\frac{d}{d\alpha} P(\alpha \mid \mathbf{d}, \boldsymbol{\sigma}) \propto -\frac{N}{\alpha^{N+1}} \exp\left(-\frac{A^2}{\alpha^2}\right) + \frac{2A^2}{\alpha^{N+3}} \exp\left(-\frac{A^2}{\alpha^2}\right) = 0$$

$$\Rightarrow \quad N\alpha^2 = 2A^2 \quad \Rightarrow \quad \alpha_{MAP} = \sqrt{\frac{2}{N}} A = \sqrt{\frac{\left(D - M^2\right)}{\sigma_M^2}}$$

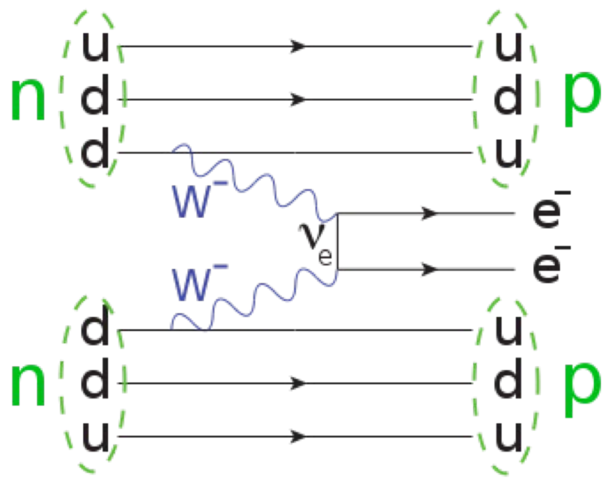
4. Search of signals in binned spectra: Bayesian analysis in the GERDA experiment

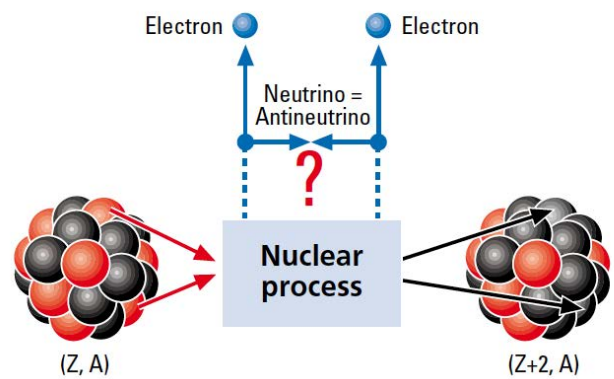
(Caldwell and Kröninger, PRD 74 (2006) 092003)

Consider the search for sparse signals in a spectrum where

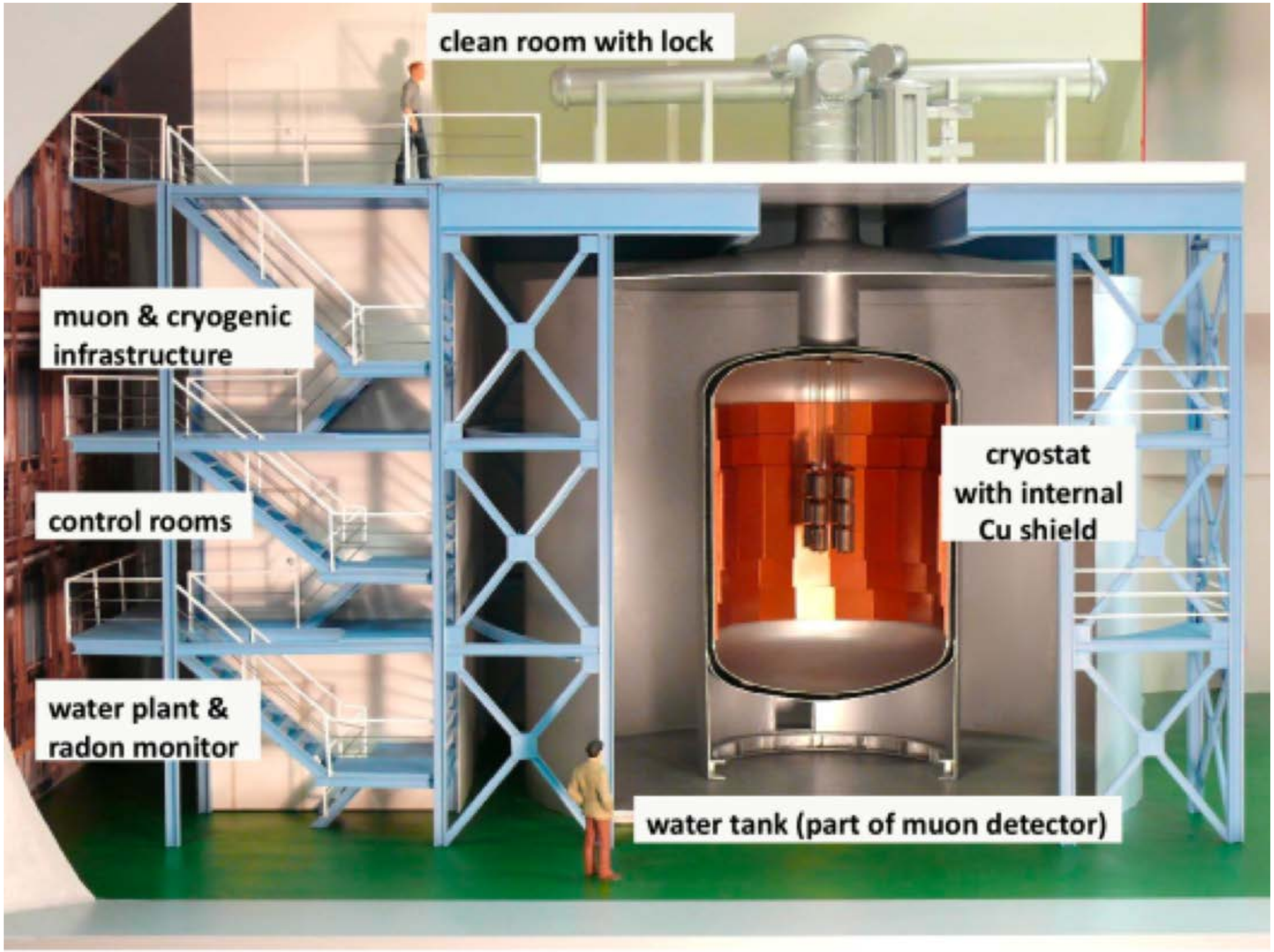
- The spectrum is confined to a certain region of interest.
- The spectral shape of a possible signal is known.
- The spectral shape of the background is known.
- The spectrum is divided into bins and the event numbers in the bins follow Poisson distributions.

The work of Caldwell and Kröninger has been carried out in the context of GERDA (GERmanium Detector Array), an experiment that aims to detect weak signals from neutrinoless beta decay in germanium detectors kept in a very low background environment.





Feynman diagram of neutrinoless double-beta decay, with two neutrons decaying to two protons. ***The only emitted products in this process are two electrons, which can occur if the neutrino and antineutrino are the same particle*** *(i.e. Majorana neutrinos) so the same neutrino can be emitted and absorbed within the nucleus.*
In conventional double-beta decay, two antineutrinos - one arising from each W vertex - are emitted from the nucleus, in addition to the two electrons. The detection of neutrinoless double-beta decay is thus a sensitive test of whether neutrinos are Majorana particles. (from Wikipedia)



Artist’s view of GERDA
http://www.mpi-hd.mpg.de/gerda/

We introduce the normalized spectral shapes of background and signal

$$f_B(E); \quad f_S(E);$$

(flat spectrum, known signal shape). Then we find the average number of events in each bin

$$\nu_i(B,S) = \nu_i(E_i, \Delta E_i, B, S) = B \int_{E_i}^{E_i+\Delta E_i} f_B(E)\,dE + S \int_{E_i}^{E_i+\Delta E_i} f_S(E)\,dE$$

An observed spectrum is defined by the numbers of counts in each bin: $\{n_i\}_{i=1,n}$ and since we assume a Poisson statistics in each bin, we find the following likelihoods for a given spectral observation

$$p(spectrum|B,I) = \prod_{i=1}^{N} \frac{\left[\nu_i(B,0)\right]^{n_i}}{n_i!} \exp\left[-\nu_i(B,0)\right]$$

$$p(spectrum|B,S,I) = \prod_{i=1}^{N} \frac{\left[\nu_i(B,S)\right]^{n_i}}{n_i!} \exp\left[-\nu_i(B,S)\right]$$

A specific spectral shape depends on the **average number of background (B) and signal (S) events**, and we can write

distribution for the average B

$$p\,(\text{spectrum}|H_{\text{bkg}}, I) = \int_B p\,(\text{spectrum}|B, I)\, p_B(B) dB$$

$$p\,(\text{spectrum}|H_{\text{bs}}, I) = \int_{B,S} p\,(\text{spectrum}|B, S, I)\, p_B(B) p_S(S) dB dS$$

distribution for the average S

the possible spectra are the results of many possibile choices of the background and of the signal rates, and therefore of the average number of background and signal events; here we marginalize over these dependencies

The competing hypotheses (observation of binned energy spectra) are

- H_{bkg} = background only
- H_{bs} = background + signal

then

$$p\left(H_{bkg}\middle|spectrum,I\right)=\frac{p\left(spectrum\middle|H_{bkg},I\right)}{p\left(spectrum\middle|I\right)}p\left(H_{bkg}\middle|I\right)$$

$$p\left(H_{bs}\middle|spectrum,I\right)=\frac{p\left(spectrum\middle|H_{bs},I\right)}{p\left(spectrum\middle|I\right)}p\left(H_{bs}\middle|I\right)$$

Bayes

$$p\left(spectrum\middle|I\right)=p\left(spectrum\middle|H_{bkg},I\right)p\left(H_{bkg}\middle|I\right)+p\left(spectrum\middle|H_{bs},I\right)p\left(H_{bs}\middle|I\right)$$

Evidence

Then we find the complete likelihood functions:

$$p\left(spectrum \middle| H_{bkg}, I\right) = \int_B p\left(spectrum \middle| B, I\right) p_B(B)\, dB$$

$$= \int_B \prod_{i=1}^{N} \frac{\left[\nu_i(B,0)\right]^{n_i}}{n_i!} \exp\left[-\nu_i(B,0)\right] p_B(B)\, dB$$

$$p\left(spectrum \middle| H_{bs}, I\right) = \int_B p\left(spectrum \middle| B, S, I\right) p_B(B)\, p_S(S)\, dB$$

$$= \int_B \prod_{i=1}^{N} \frac{\left[\nu_i(B,S)\right]^{n_i}}{n_i!} \exp\left[-\nu_i(B,S)\right] p_B(B)\, p_S(S)\, dB$$

The final, complete expressions are:

$$p\left(H_{bkg}\middle|spectrum,I\right)=\frac{p\left(spectrum\middle|H_{bkg},I\right)}{p\left(spectrum\middle|I\right)}p\left(H_{bkg}\middle|I\right)$$

$$=\frac{\int_B\prod_{i=1}^{N}\frac{\left[\nu_i(B,0)\right]^{n_i}}{n_i!}\exp\left[-\nu_i(B,0)\right]p_B(B)dB}{\int_B\prod_{i=1}^{N}\frac{\left[\nu_i(B,0)\right]^{n_i}}{n_i!}\exp\left[-\nu_i(B,0)\right]p_B(B)dB\ p\left(H_{bkg}\middle|I\right)+\int_B\prod_{i=1}^{N}\frac{\left[\nu_i(B,S)\right]^{n_i}}{n_i!}\exp\left[-\nu_i(B,S)\right]p_B(B)p_S(S)dB\ p\left(H_{bs}\middle|I\right)}p\left(H_{bkg}\middle|I\right)$$

$$p\left(H_{bs}\middle|spectrum,I\right)=\frac{p\left(spectrum\middle|H_{bs},I\right)}{p\left(spectrum\middle|I\right)}p\left(H_{bs}\middle|I\right)$$

$$=\frac{\int_B\prod_{i=1}^{N}\frac{\left[\nu_i(B,S)\right]^{n_i}}{n_i!}\exp\left[-\nu_i(B,S)\right]p_B(B)p_S(S)dB}{\int_B\prod_{i=1}^{N}\frac{\left[\nu_i(B,0)\right]^{n_i}}{n_i!}\exp\left[-\nu_i(B,0)\right]p_B(B)dB\ p\left(H_{bkg}\middle|I\right)+\int_B\prod_{i=1}^{N}\frac{\left[\nu_i(B,S)\right]^{n_i}}{n_i!}\exp\left[-\nu_i(B,S)\right]p_B(B)p_S(S)dB\ p\left(H_{bs}\middle|I\right)}p\left(H_{bs}\middle|I\right)$$

One can use these expressions to test hypotheses (by means of Bayes factors), and find values for B and S.

5. Expert elicitation

(Morgan, PNAS 111 (2014) 7176)

Consider a problem where no experimental data exist, and where you wish to make an informed guess.

You can rely on expert opinion and ask the expert to provide his/her own estimate of a probability distribution.

You can also rely on a population of experts and construct averaged probability distributions from their guesses.

The Interpretation of Probability

A subjectivist or Bayesian interpretation of probability (5, 26–28) is used when one makes subjective probabilistic assessments of the present or future value of uncertain quantities, the state of the world, or the nature of the processes that govern the world. In such situations, probability is viewed as a statement of an individual's belief, informed by all formal and informal evidence that he or she has available. Although subjective, such judgments cannot be arbitrary. They must conform to the laws of probability. Further, when large quantities of evidence are available on identical repeated events, one's subjective probability should converge to the classical frequentist interpretation of probability.

Expert elicitation is a complex procedure and one should resort to it only when absolutely necessary.

Morgan concludes that

"*... it may be tempting to view expert elicitation as a low-cost, low-effort alternative to conducting serious research and analysis, it is neither. Rather, expert elicitation should build on and use the best available research and analysis and be undertaken only when, given those, the state of knowledge will remain insufficient to support timely informed assessment and decision making.*"

Partial list of problems in expert elicitation

1. Words convey different meanings

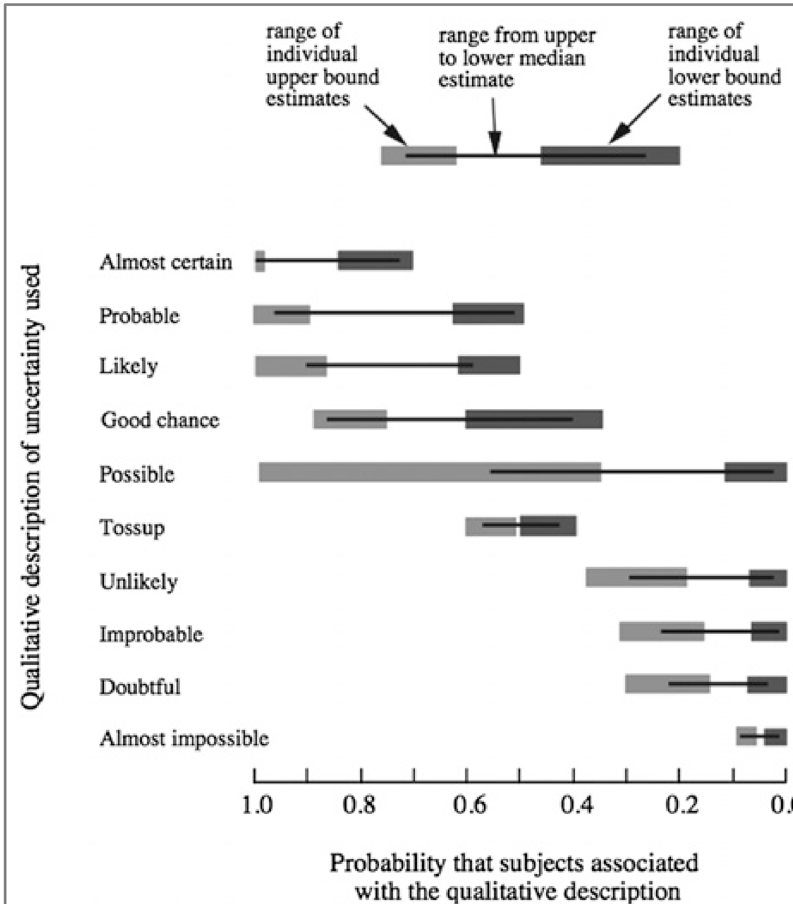


Fig. 1. The range of numerical probabilities that respondents attached to qualitative probability words in the absence of any specific context are shown. Note the very wide ranges of probability that were associated with some of these words. Figure redrawn from Wallsten et al. (30).

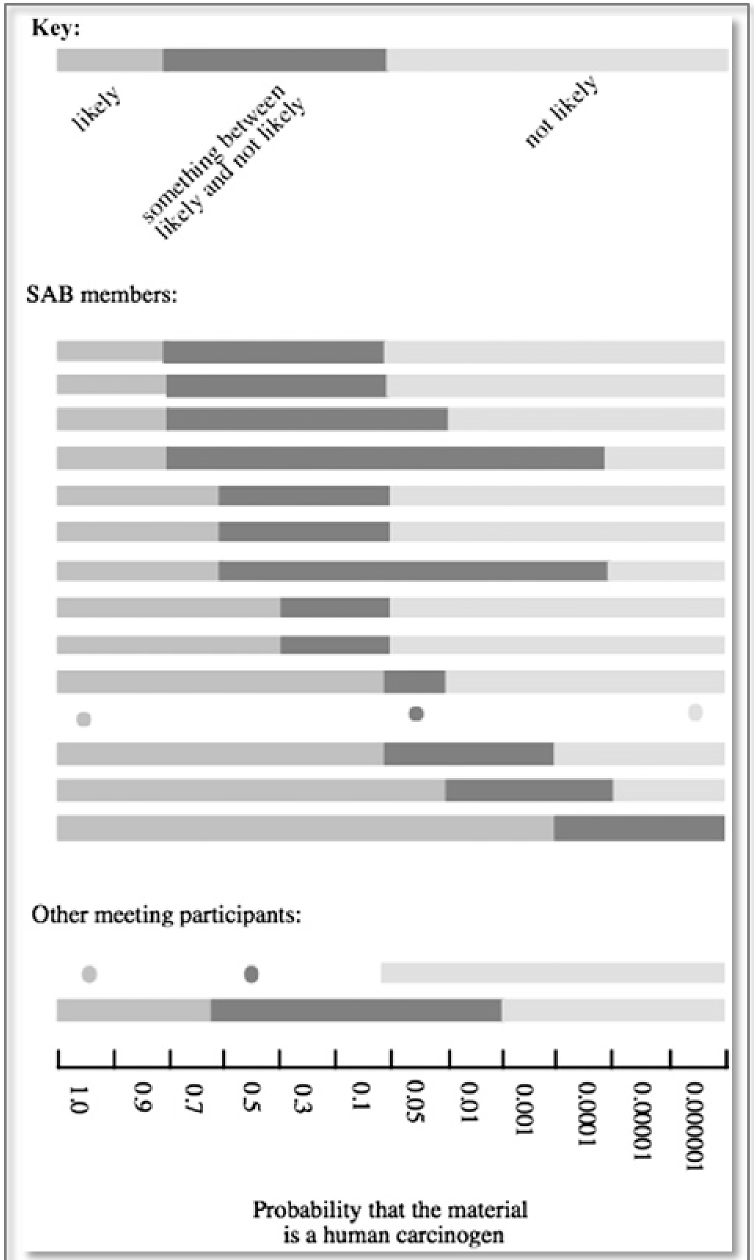


Fig. 2. Results obtained by Morgan (32) when members of the Executive Committee of the EPA Science Advisory Board were asked to assign numerical probabilities to uncertainty words that had been proposed for use with EPA cancer guidelines (33). Note that even in this relatively small and expert group, the minimum probability associated with the word "likely" spans 4 orders of magnitude, the maximum probability associated with the word "not likely" spans more than 5 orders of magnitude, and there is an overlap of the probabilities the different experts associated with the two words.

2. Cognitive heuristics and bias

... When presented with an estimation task, if people start with a first value (i.e., an anchor) and then adjust up and down from that value, they typically do not adjust sufficiently. ...

To minimize the influence of this heuristic when eliciting probability distributions, it is standard procedure not to begin with questions that ask about "best" or most probable values but rather to first ask about extremes: "What is the highest (lowest) value you can imagine for coefficient X?" or "Please give me a value for coefficient X for which you think there is only one chance in 100 that actual value could be larger (smaller)."

Having obtained an estimate of an upper (lower) bound, it is then standard practice to ask the expert to imagine that the uncertainty about the coefficient's value has been resolved and the actual value has turned out to be 10% or 15% larger (smaller) than the bound they offered. We then ask the expert, "Can you offer an explanation of how that might be possible?"

Sometimes experts can offer a perfectly plausible physical explanation, at which point we ask them to revise their bound. After obtaining estimates of upper and lower bounds on the value of a coefficient of interest, we then go on to elicit intermediate values across the probability distribution ["What is the probability that the value of X is greater (less) than Y?"].

3. Ubiquitous overconfidence

... One reason for adopting this rather elaborate procedure is that there is strong evidence that most such judgments are overconfident.

A standard measure of overconfidence is the surprise index: the fraction of true values that lie outside an assessor's 98% confidence interval when answering questions for which the true answer is known (e.g., the length of the Panama Canal). Fig. 3 reports a summary of results from 21 different studies involving over 10,000 such assessment questions. Note that none yield the target value for the surprise index of 2% and over half yielded values of 30% or more!

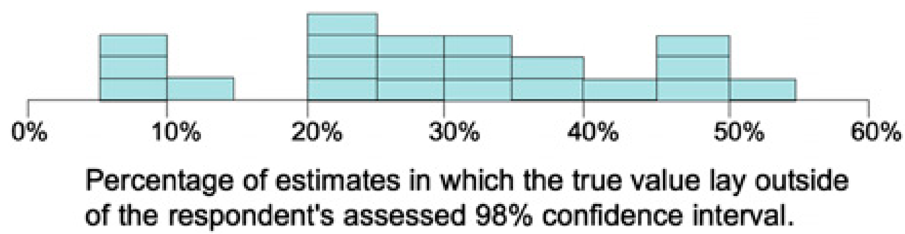


Fig. 3. Summary of the value of the surprise index (ideal value = 2%) observed in 21 different studies involving over 10,000 assessment questions. These results indicate clearly the ubiquitous tendency to overconfidence (i.e., assessed probability distributions that are too narrow). A more detailed summary is provided in Morgan and Henrion (39).

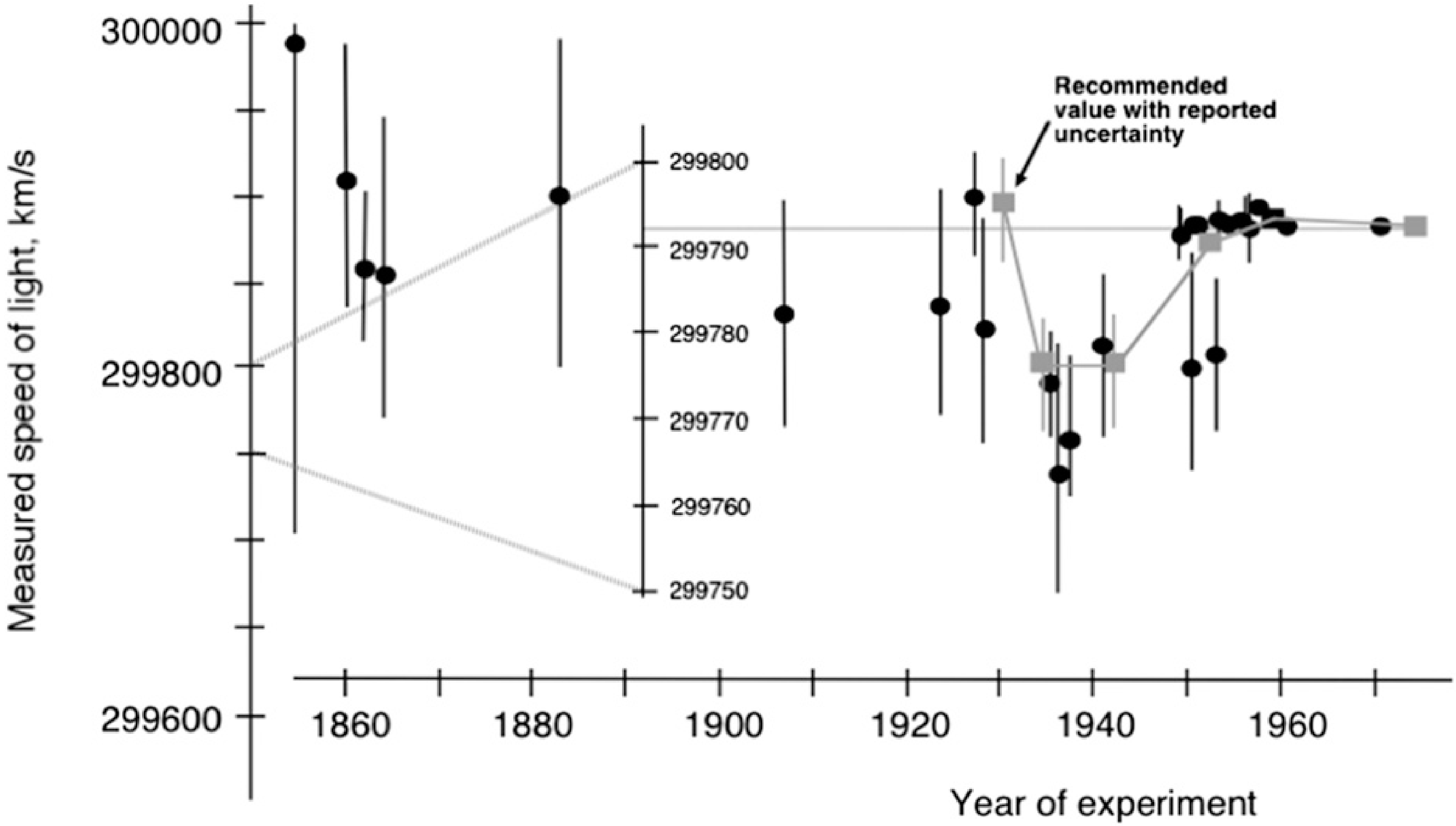


Fig. 4. Published estimates of the speed of light. The light gray boxes that start in 1930 are the recommended values from the particle physics group that presumably include an effort to consider uncertainty arising from systematic error (40). Note that for over two decades the reported confidence intervals on these recommended values did not include the present best-measured value. Henrion and Fischhoff (40), from which this figure is combined and redrawn, report that the same overconfidence is observed in the recommended values of a number of other physical constants.

4. Expert calibration

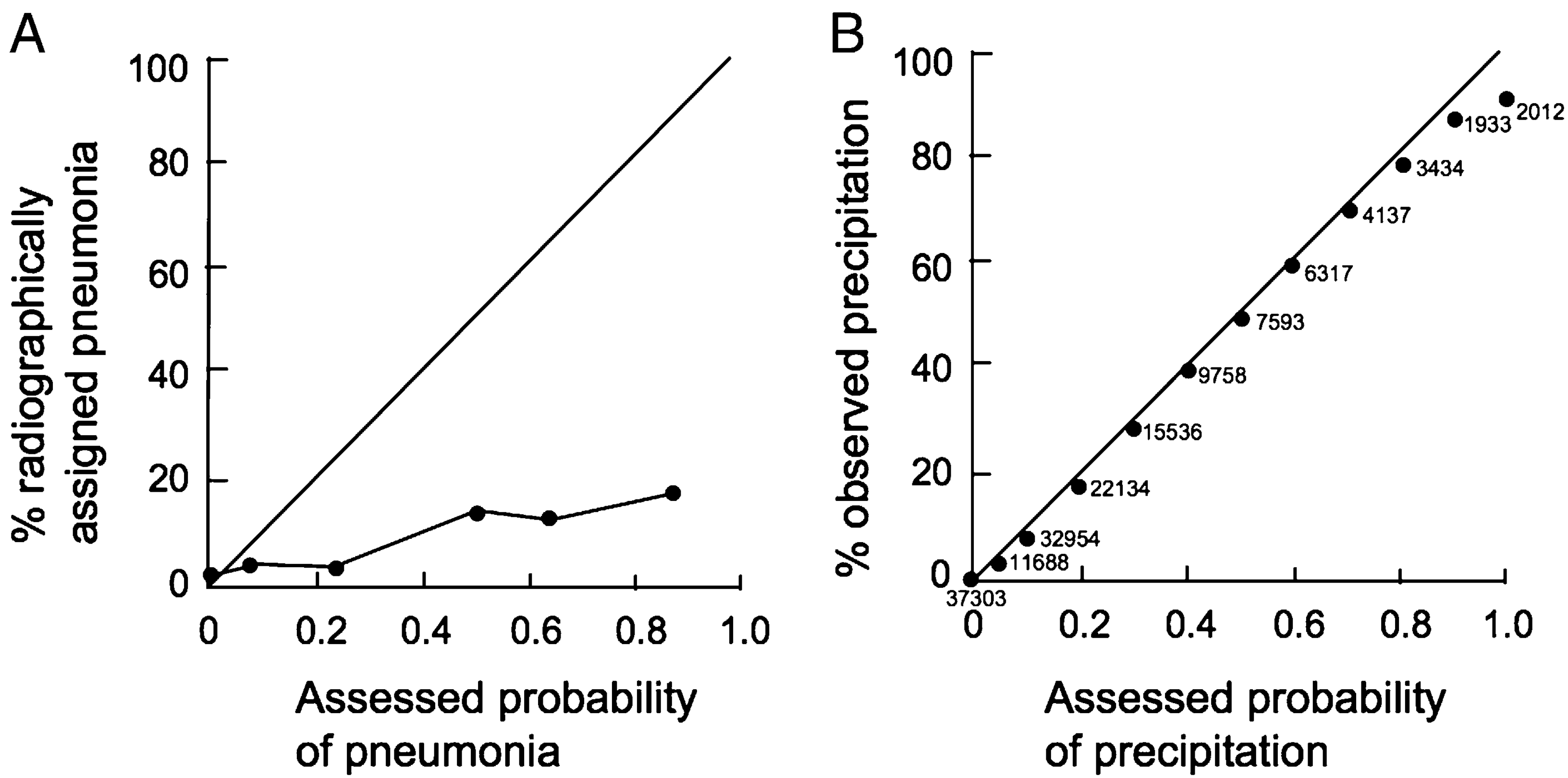


Fig. 5. Illustration of two extremes in expert calibration. (*A*) Assessment of probability of pneumonia (based on observed symptoms) in 1,531 first-time patients by nine physicians compared with radiographically assigned cases of pneumonia as reported by Christensen-Szalanski and Bushyhead (44). (*B*) Once-daily US Weather Service precipitation forecasts for 87 stations are compared with actual occurrence of precipitation (April 1977 to March 1979) as reported by Charba and Klein (43). The small numbers adjacent to each point report the number of forecasts.

5. Range of opinions in the scientific community

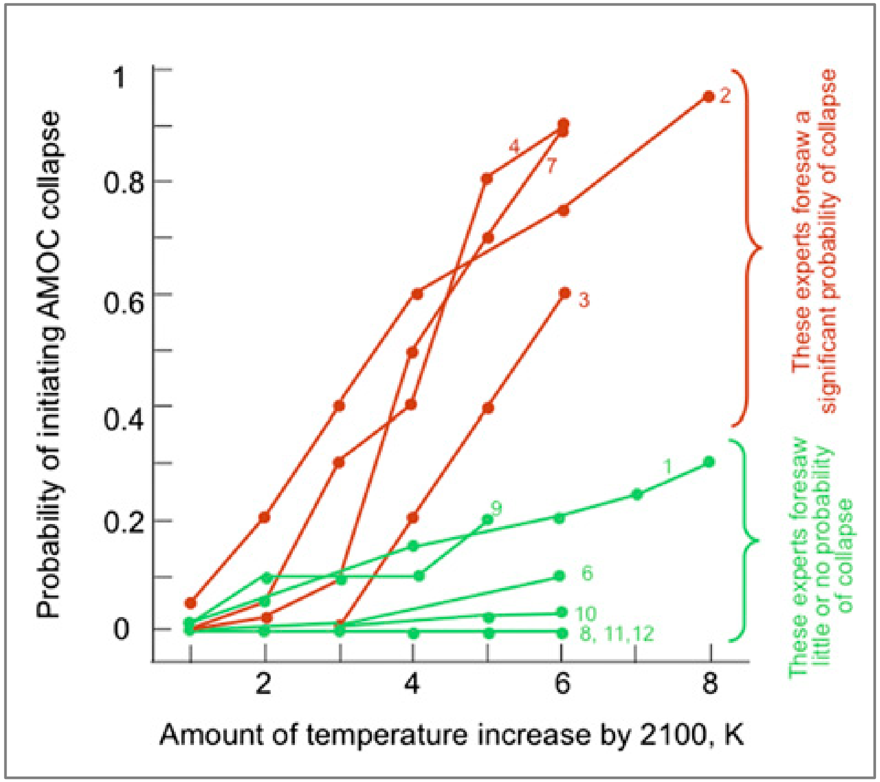


Fig. 8. Expert elicitation can be effective in displaying the range of opinions that exist within a scientific community. This plot displays clearly the two very different schools of thought that existed roughly a decade ago within the community of oceanographers about the probability "that a collapse of the AMOC will occur or will be irreversibly triggered as a function of the global mean temperature increase realized in the year 2100." Each curve shows the subjective judgments of one of 12 experts. Four experts (2, 3, 4, and 7 in red) foresaw a high probability of collapse, while seven experts (in red) foresaw little, if any, likelihood of collapse. Collapse was defined as a reduction in AMOC strength by more than 90% relative to present day. Figure redrawn from Zickfeld et al. (18).

(AMOC = Atlantic Meridional Overturning Circulation)

Expert elicitation and the Yucca Mountain nuclear waste depository

(Alley and Alley, “Too Hot to Touch – The Problem of High-Level Nuclear Waste, CUP, 2012)

Figure 19.2 Lathrop Wells cinder cone. Photograph by Greg Valentine.

FEDERAL LANDS IN SOUTHERN NEVADA

... In 1980, the first estimates of volcanic activity at the repository site put the annual probability at about 1 in 100 million. This was right at the Nuclear Regulatory Commission's cutoff point for inclusion in the TSPA, but not below it. By some accounts, 1 in 100 million is also roughly the same possibility as the ultimate low-probability, high-consequence event – global mass extinction from the impact of an asteroid or comet.

In the mid 1990s, DOE convened a panel of ten experts, mostly volcanologists, to conduct a formalized "ask-the-experts" approach to estimating the probability of volcanism and its uncertainty. The method, called expert elicitation, brings together a panel of experts and mathematically combines their individual estimates.

The goal is to obtain a probability distribution and range of uncertainty representative of the larger scientific community. Of course, the end result is affected by who serves on the panel, and the pool of qualified participants is not very large.

Using a formal nomination process, ten panel members were selected from a group of 70 scientists. Expertise mattered, but equally important were strong communication and interpersonal skills, as well as flexibility and impartiality.

The experts were asked to act as objective evaluators of the various theories. Their job was to listen to proponents of different positions and then weigh each of these theories in making their estimates.

After workshops and field trips to bring everyone up to speed, professional interviewers spent two days with each panel member extracting key information. Each of the ten experts independently arrived at an annual probability distribution for a volcanic event intersecting the repository. The average of these estimates was about 1 in 70 million, later revised to about 1 in 60 million. In the scheme of things, this was not far from the original 1 in 100 million estimate.

The expert elicitation did not end the debate. The results were challenged not only by the State of Nevada but also by scientists working for the NRC. Arguing that conservatism was needed, the NRC used an estimate of 1 event in 10 million in their assessments.

Still portending a rare event, the higher probability by NRC scientists came about, in part, from their assumption that faults and deep tectonic structures may provide pathways for the ascent of magma directly into Yucca Mountain. There was also disagreement about whether the time between eruptions was increasing or decreasing. Volcanism is known to be episodic. While most geoscientists consider the volcanism in the Yucca Mountain region to be waning, a few argued that we could be in the middle or end of a quiescent period. ...

6. The lost flight AF 477

Bayesian methods allow for incremental extension of our knowledge. They are ideally suited to search for lost objects by incorporating new knowledge in a natural way.

This has been strikingly demostrated by several headline-making cases, like the retrieval of the wreakage of the Air France Flight AF 447, which disappeared in the early morning of June 1st 2009.

(Data and info from Stone & al., *Stat. Science* **29** (2014) 69, and from the presentation of C. M. Keller)

O GLOBO

IRINEU MARINHO (1876-1925) RIO DE JANEIRO, TERÇA-FEIRA, 2 DE JUNHO DE 2009 • ANO LXXXIV • Nº 27.693 ROBERTO MARINHO (1904-2003)

CADERNO ESPECIAL

CHORO E DESESPERO nos aeroportos de Paris e do Rio: parentes dos passageiros do voo 447 da Air France buscam informações sobre o destino da aeronave que desapareceu quando sobrevoava o Atlântico

Tragédia e mistério na rota Rio-Paris

Sumiço de airbus da Air France no Atlântico com 228 pessoas choca, comove e intriga o mundo

• No maior desastre aéreo envolvendo brasileiros, o voo 447 da Air France que saiu do Rio com 228 pessoas, na noite de domingo, com destino à Cidade Luz, desapareceu do controle de radares enquanto sobrevoava o Oceano Atlântico, na fronteira entre o espaço aéreo brasileiro e o senegalês. O paradeiro da aeronave e as causas do acidente ainda eram desconhecidos até o fim da noite de ontem. A última informação que se tem é que uma mensagem automática de pane elétrica e despressurização foi enviada pela aeronave às 23h14m de domingo. A Air France diz que o avião pode ter sido atingido por um raio em meio a turbulência. Morreram 58 brasileiros, 61 franceses, 26 alemães, entre passageiros de mais 33 nacionalidades. A Aeronáutica informou, no fim da tarde de ontem, que o governo do Senegal teria localizado em seu mar territorial supostos pedaços de materiais, mas não confirmou que seriam do avião. O governador Sérgio Cabral e o prefeito Eduardo Paes decretaram luto por três dias. Entre as vítimas estão executivos de multinacionais como Luiz Roberto Anastácio (Michelin) e Erich Heine (Thyssenkrupp CSA), além do príncipe Pedro Luís de Orleans e Bragança, descendente de Dom Pedro II; o maestro Sílvio Barbato e Marcelo de Oliveira, chefe de gabinete do prefeito do Rio.

Abismo marinho pode tornar impossível recuperar destroços

Em tempo real, no site do GLOBO, a busca do avião desaparecido

A lista (ainda não oficial) dos passageiros do AF 447

Os maiores desastres da história da aviação comercial

Editoria de Arte

O FIM DO VOO 447

O voo 447 da Air France partiu do Rio às 19h30m de domingo e deveria ter pousado em Paris às 6h10m de ontem (horário de Brasília).

Paris
Dakar
ÁREA EM DESTAQUE
Rio

23h20m
Aeronave não fez contato com Dacar (Senegal)

23h14m
Uma mensagem automática é enviada para a companhia informando sobre falha elétrica e perda de pressurização

22h48m
É detectado pela última vez pelos radares brasileiros

22h33m
O piloto faz o último contato com o Centro de Controle de Recife (CINDACTA3)

Área do Centro de Controle de Dacar (Senegal)

ONDE ESTÃO SENDO FEITAS AS BUSCAS (a partir da última informação que se tem do avião)

Área do Centro de Controle do Atlântico (CINDACTA3)

FERNANDO DE NORONHA

Área do Centro de Controle de Recife (CINDACTA3)

RIO GRANDE DO NORTE

NATAL

AIR FRANCE

Destinos cruzados

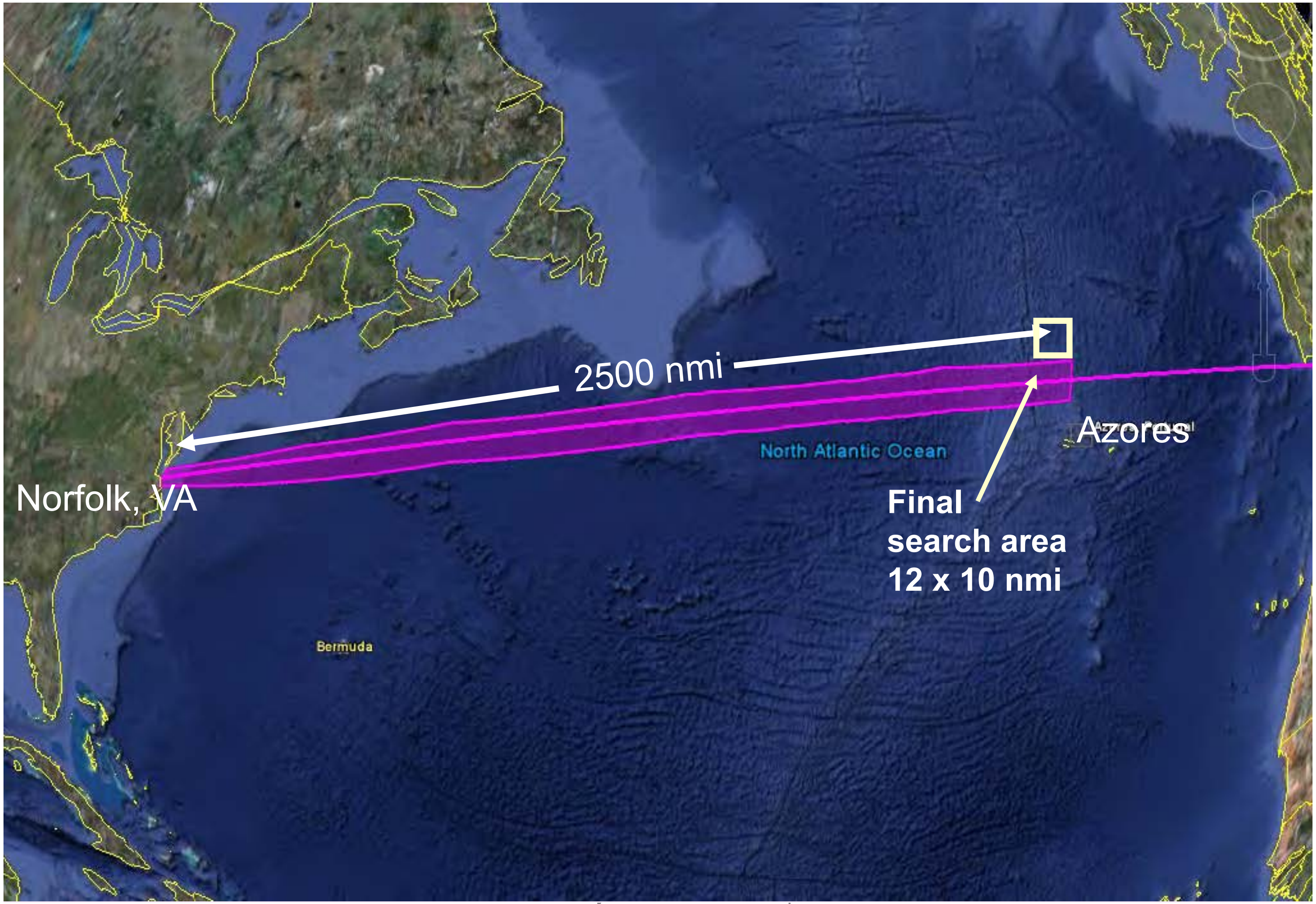
• Bianca Cotta e Carlos Eduardo de Melo casaram-se no sábado numa festa para 500 convidados e, no dia seguinte, viajaram para a sonhada lua de mel em Paris. Essa foi uma das centenas de histórias interrompidas na tragédia. Mas, em meio à tristeza, João Marcelo Calaça chorou: de alívio, por escapar da morte. Ele não pôde embarcar porque descobriu, só no aeroporto, que seu passaporte estava vencido.

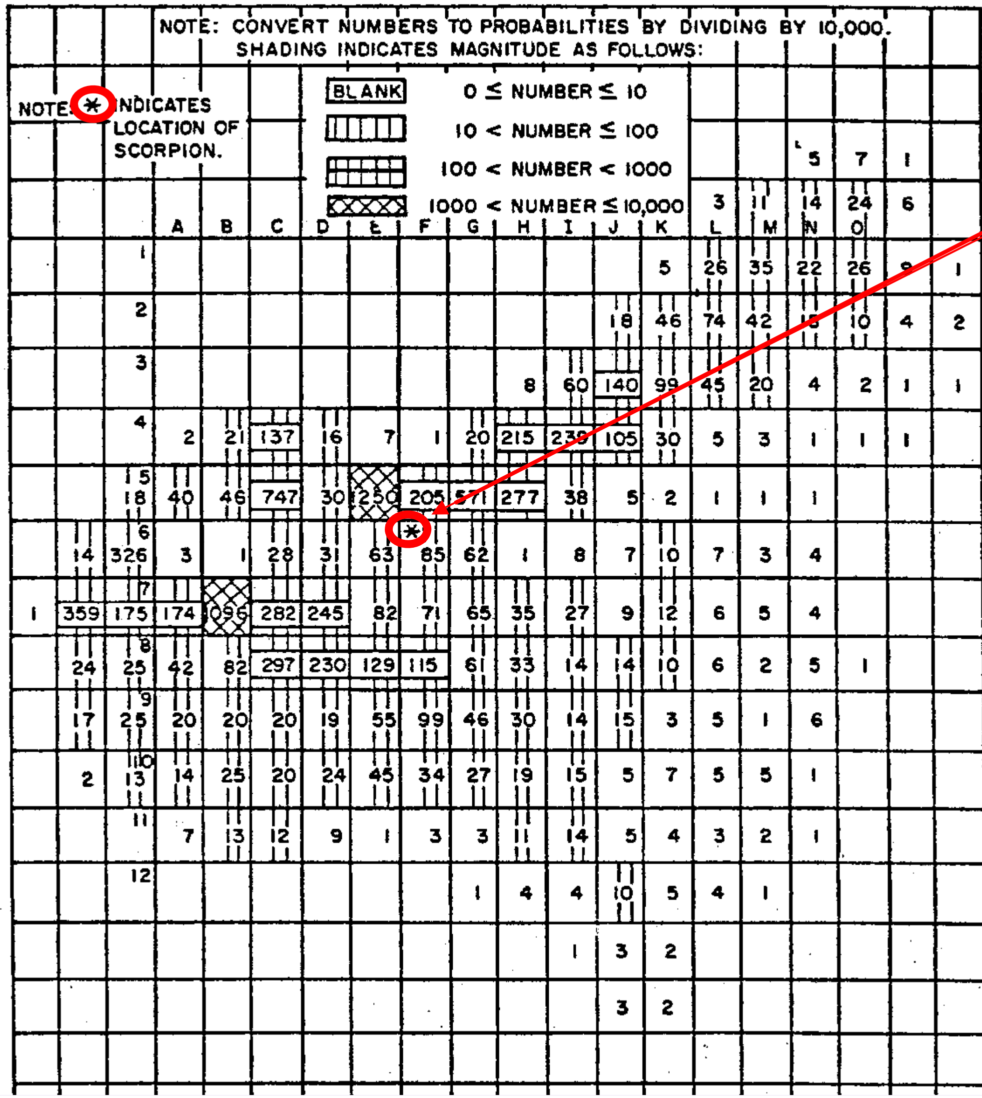
Sarkozy vai e Lula manda vice

• O presidente Lula, que estava em El Salvador, manteve sua participação na posse do presidente Maurício Funes, mas mandou o vice José Alencar ao Galeão. Em Paris, o presidente Sarkozy foi ao Charles de Gaulle conversar com parentes de vítimas.

What can be done in such a case? There are precedents that show that Bayes-inspired searches can be successful.

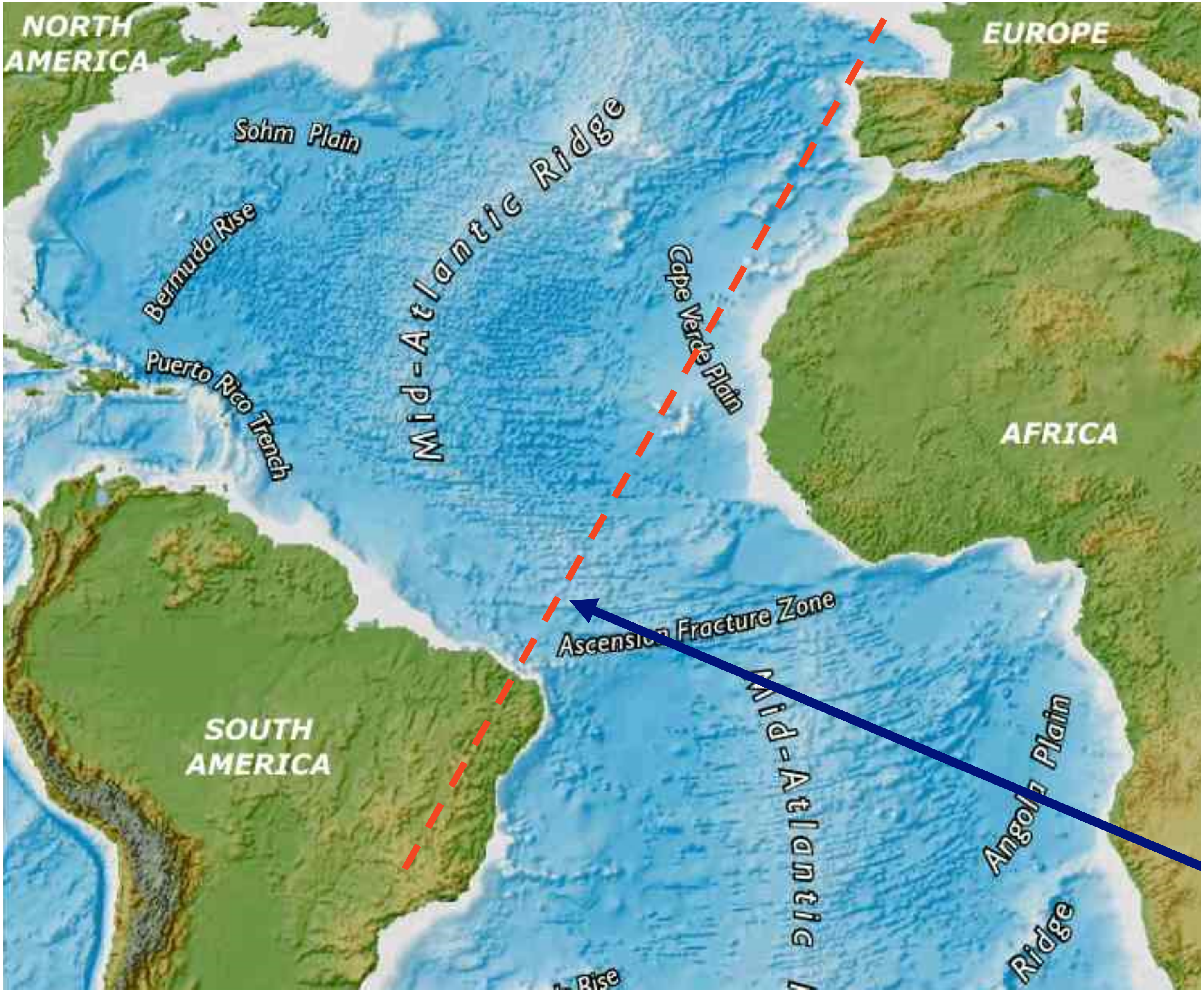
1968, the case of the missing USS Submarine Scorpion



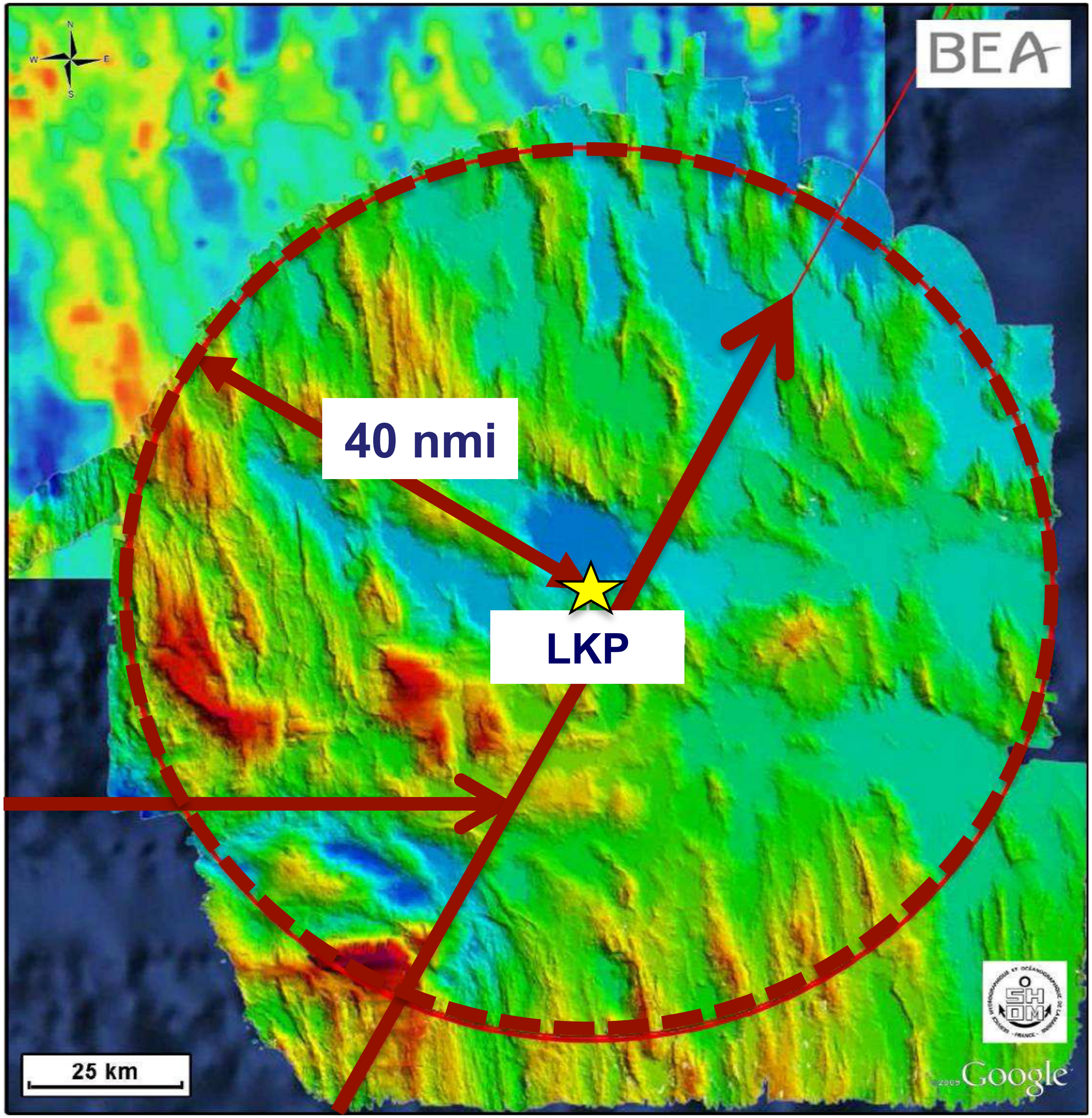


Scorpion located within 240 m of highest probability cell

AF 477 intended path



Last Known Position (LKP)
2.98° N,
30.59° W



Intended flight path

- Initial searches were unsuccessful.
- After one month (max. time for beacon batteries) nothing found
- Mid 2010, task assigned to Stone’s group
- Stone group delivers report at end of January 2011
- Search begins at end of March 2011
- Wreakage found after one week

A search procedure must specify

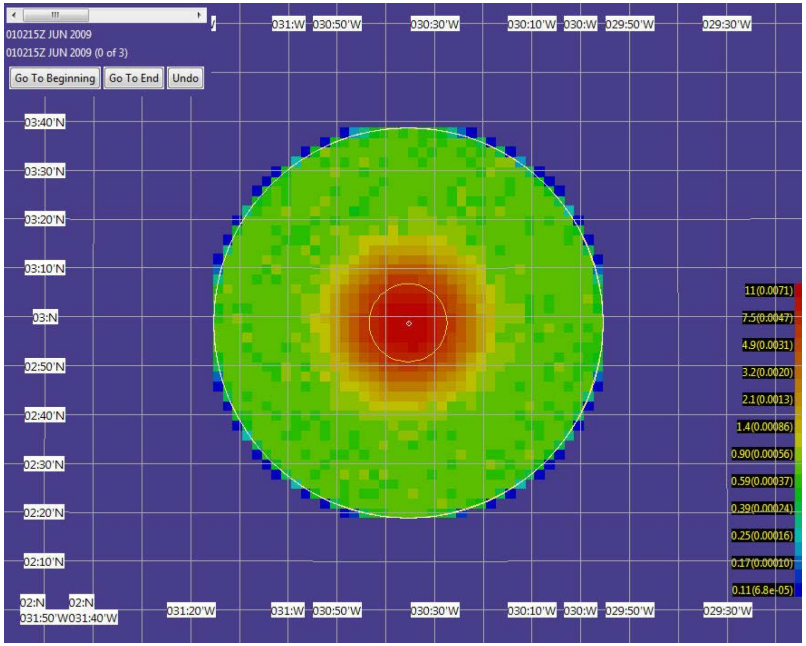
1. a prior distribution based on expert opinion and all available data

2. an updating method: this is based on an empirical representation (particle-based) of the prior PDF (by Monte Carlo simulation)

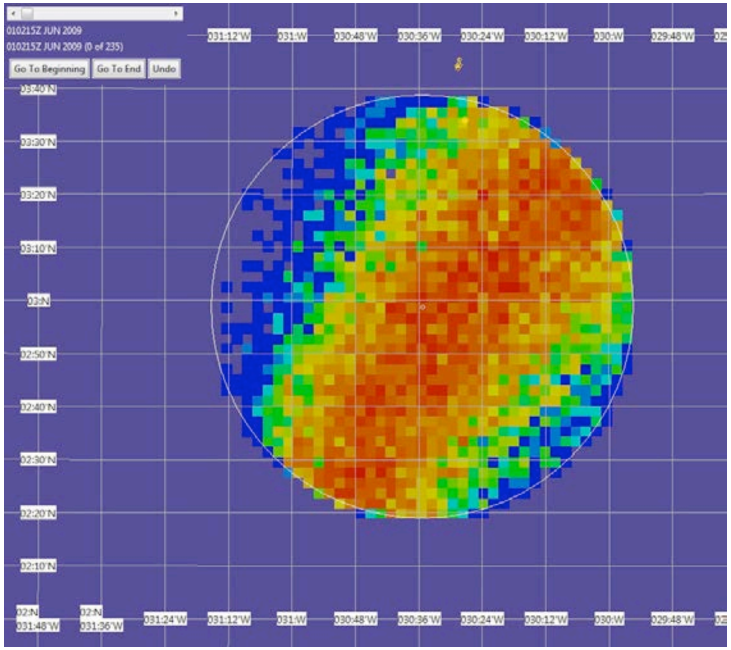
Each of the N particles is assigned an initial uniform prior $w_n = 1/N$ and this is updated with the probability $p_d(n)$ of finding the n-th particle (this requires a model for the search process)

$$\bar{w}_n = \frac{(1 - p_d(n))w_n}{\sum_{n'}(1 - p_d(n'))w_{n'}}$$

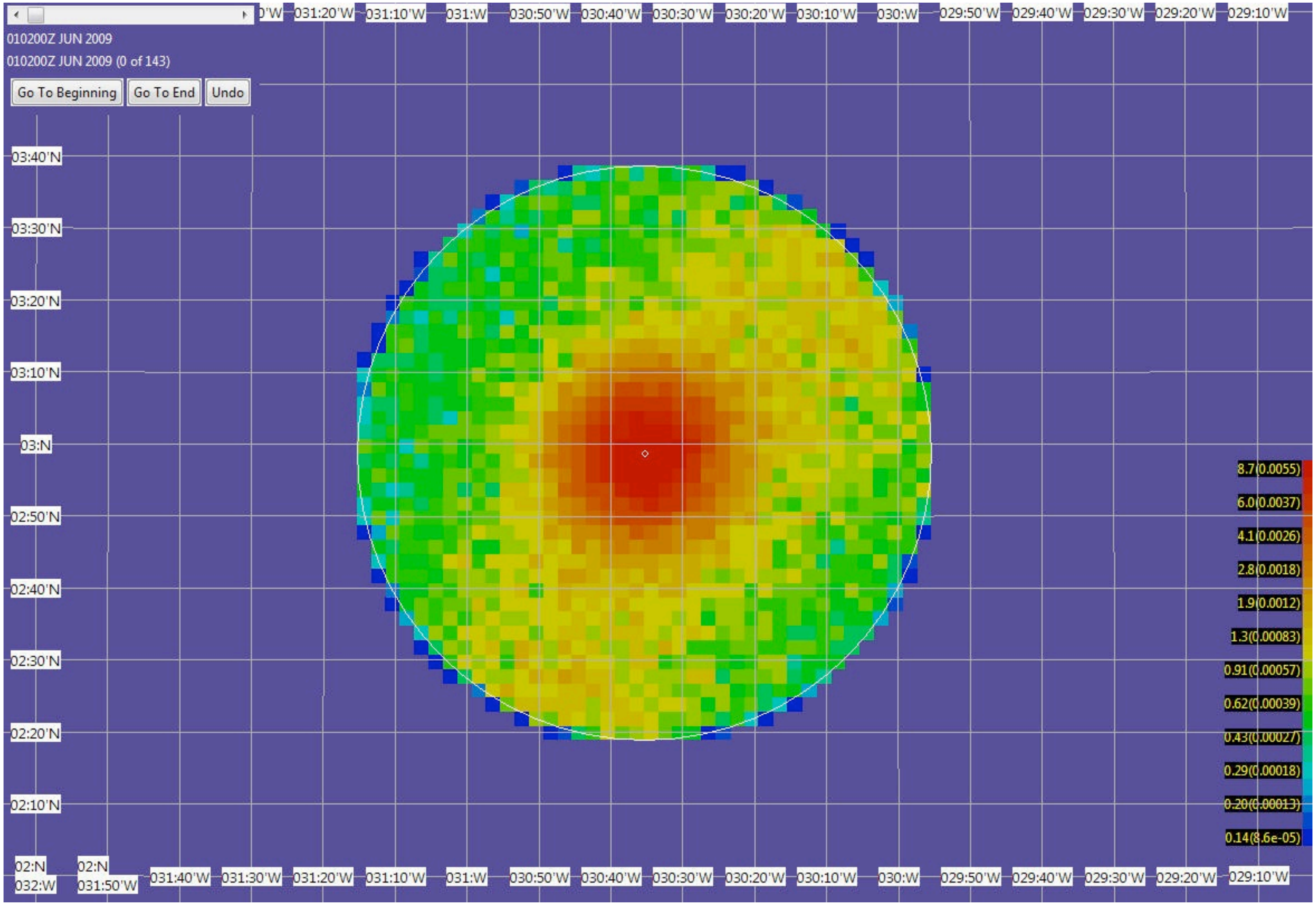
Adding the new weights in each cell of the search grid, we update the empirical PDF to its posterior value.



Flight path PDF



Reverse drift PDF



Prior PDF = 0.7 Flight path PDF + 0.3 Reverse drift PDF

Posterior after Phase 3 active sonar search

Posterior after Phase 2 active sonar search

Posterior after Phase 1 beacon search

Posterior after Surface Searches

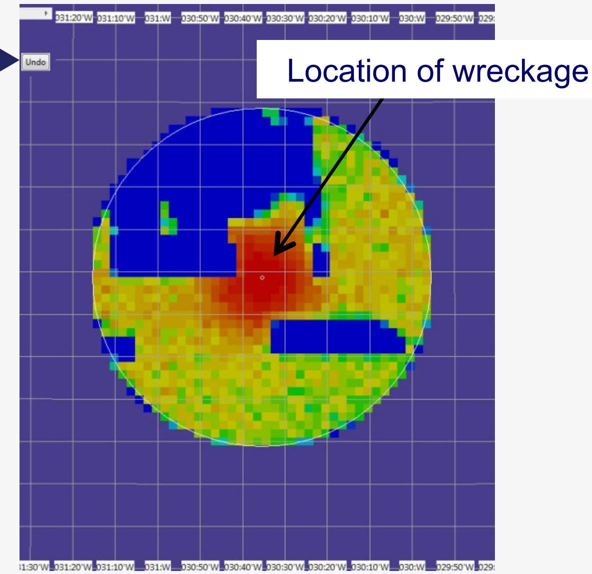
Prior Probability Distribution

35%
Uniform PDF over 40 nmi circle

35%
Circular normal PDF based on LOC accidents

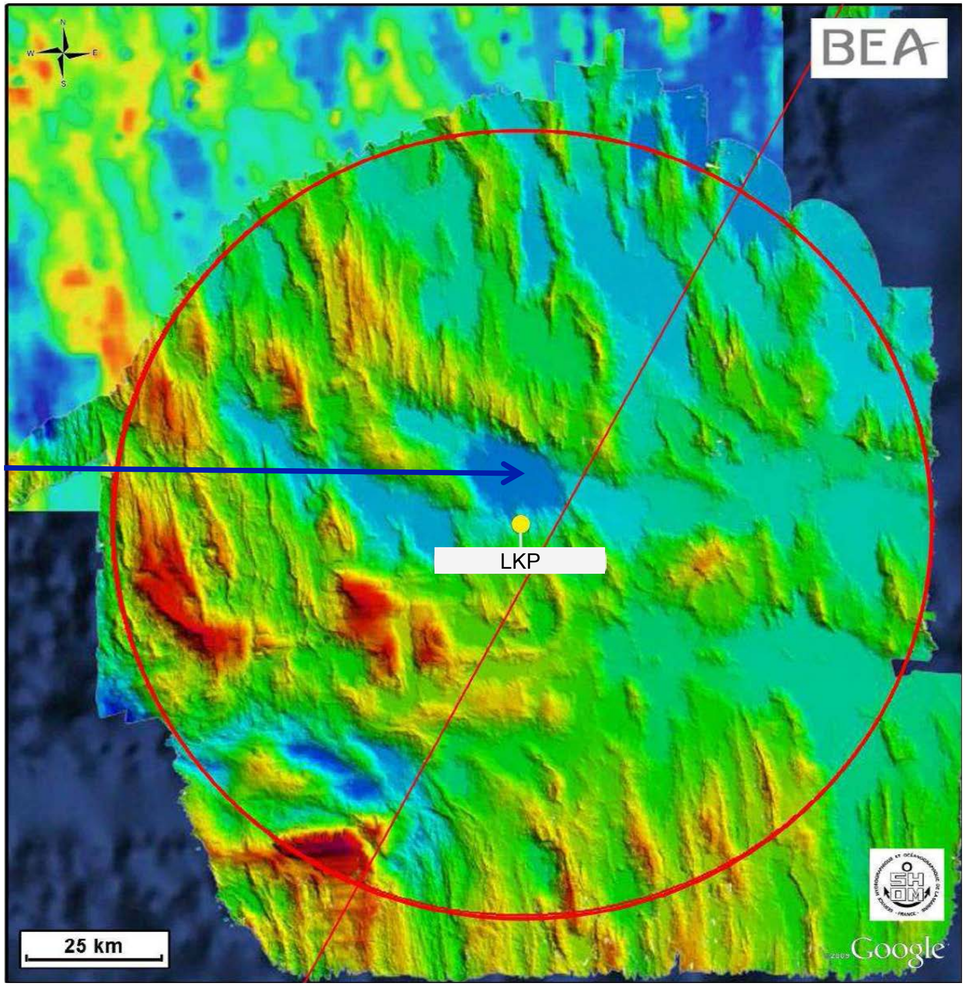
30%
PDF derived from reverse drift modeling

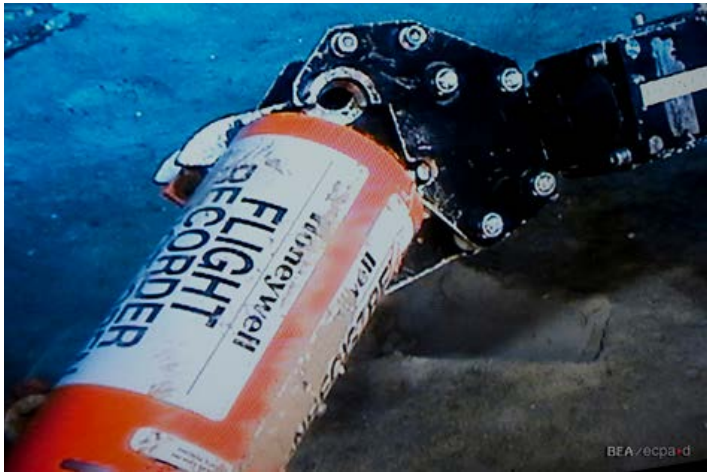
Flight dynamics Scenarios



Final PDF leading to the wreckage discovery

Approximate location of wreck, found April 3rd 2011





Flight Data Recorder (FDR) being recovered by a mechanical arm on the Remora 6000 Remotely Operated Vehicle

The Flight Data Recorder (FDR) and Cockpit Voice Recorder (CVR) provided valuable information on the accident.

Cockpit Voice Recorder (CVR) capsule with the Underwater Locator Beacon (ULB) still attached

CNN News Report (May 27th 2011)

The Air France flight from Rio de Janeiro to Paris that crashed in 2009 plummeted 38,000 ft in just three minutes and 30 seconds because pilots lost vital speed data, France's Bureau of Investigation and Analysis (BEA) said Friday.

Pilots on the aircraft got conflicting air speeds in the minutes leading up to the crash, the interim reports states. The aircraft climbed to 38,000 ft when "the stall warning was triggered and the airplane stalled," the report says.

Aviation experts are asking why the pilots responded to the stall by pulling the nose up instead of pushing it down to recover.

...

The speed displayed on the left primary flight display were "inconsistent" with those on the integrated standby instrument system (ISIS), the report says.

...

The aircraft experienced some “rolling” before stalling and then descending rapidly into the ocean. The descent lasted 3 minutes and 30 seconds and the engines remained operational, said the report. It plunged at 10,912 feet (3,300 meters) per minute.
...

Richard Quest, CNN’s aviation analyst, said: “For whatever reason the aircraft speed sensors failed and the A330 went into a high altitude stall. The pilot’s actions were unable to recover the aircraft and some might say, made the bad situation worse.

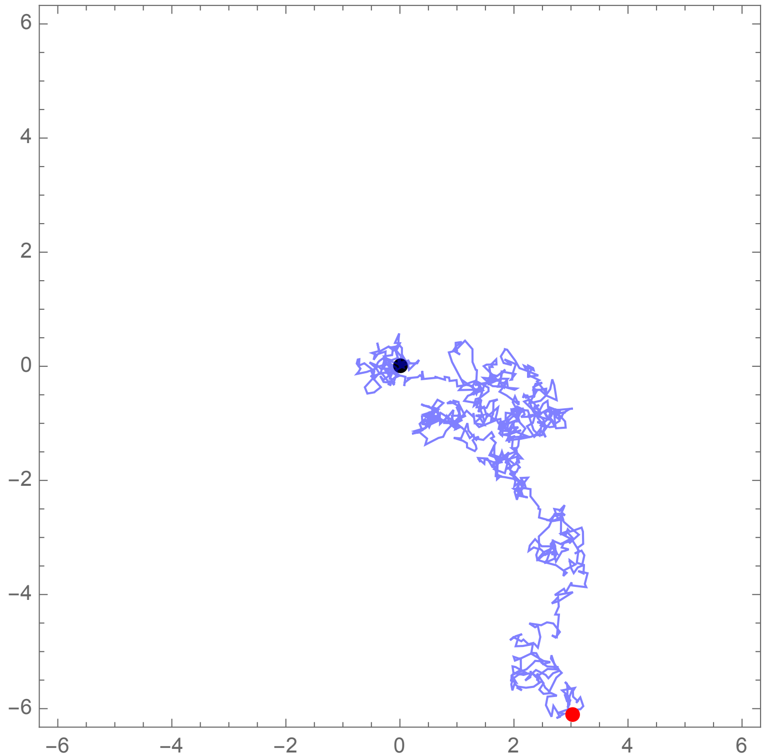
“The actual falling from the sky will have been horrific. This plane fell out of the sky.”

All 228 people aboard the Airbus A330 Flight 447 were killed on June 1, 2009.
...

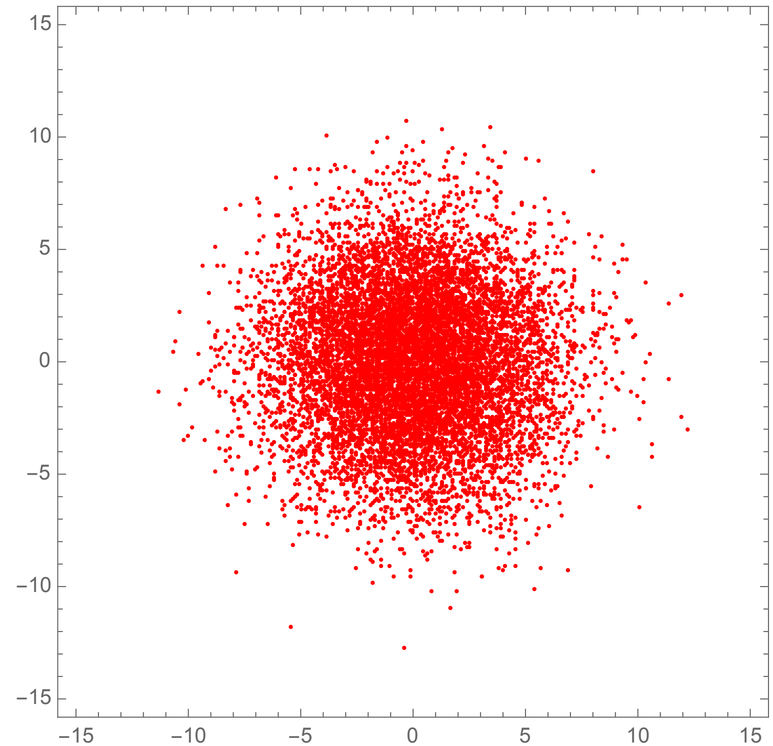
Air crash investigators at the Paris-based BEA have been working on the theory that the speed sensors, known as pitot tubes or probes, malfunctioned because of ice at high altitude.

Since the accident, Air France has replaced the pitots on its Airbus fleet with a newer model.
...

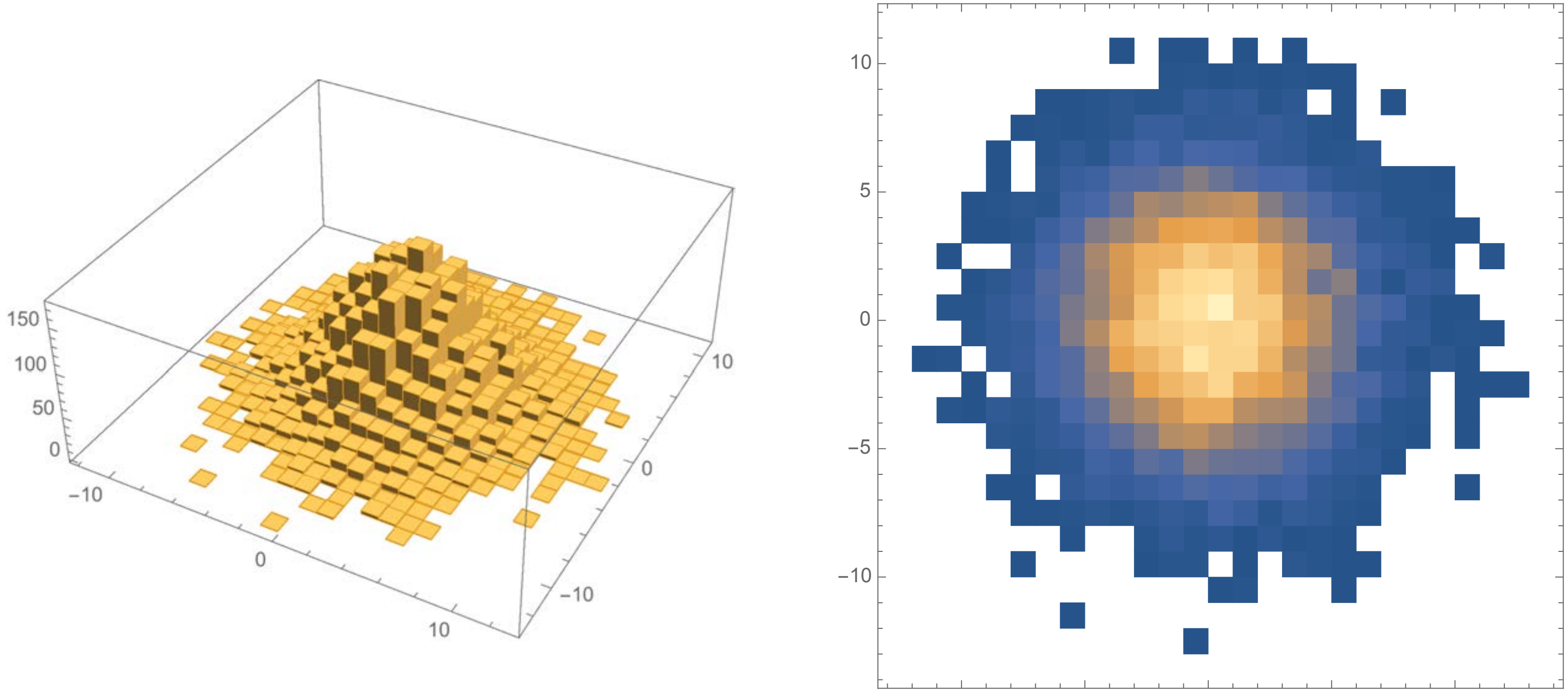
Elementary (artificial) examples of random motion and drift for the construction of simple a priori empirical distribution



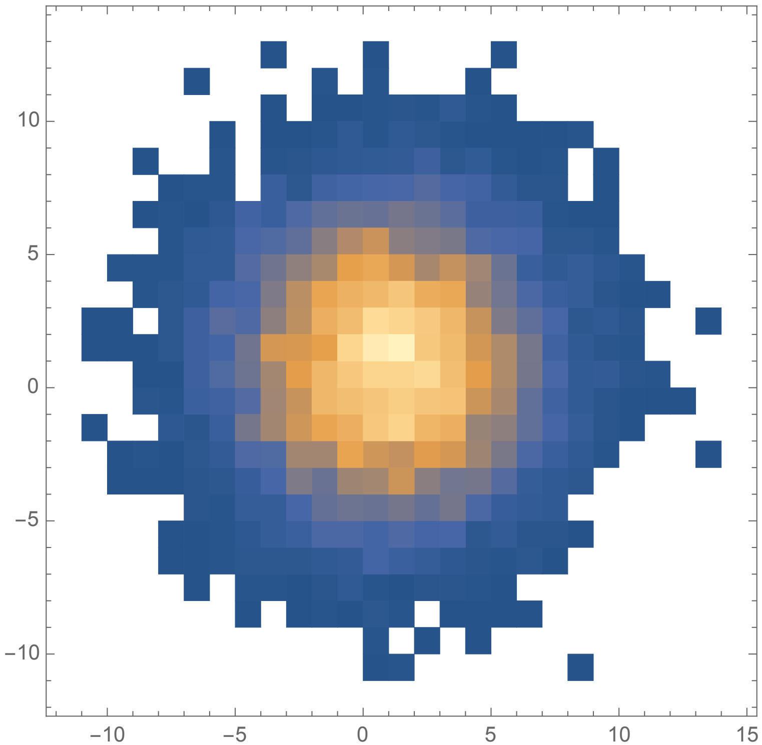
Random path from origin (black dot) to end (red dot)



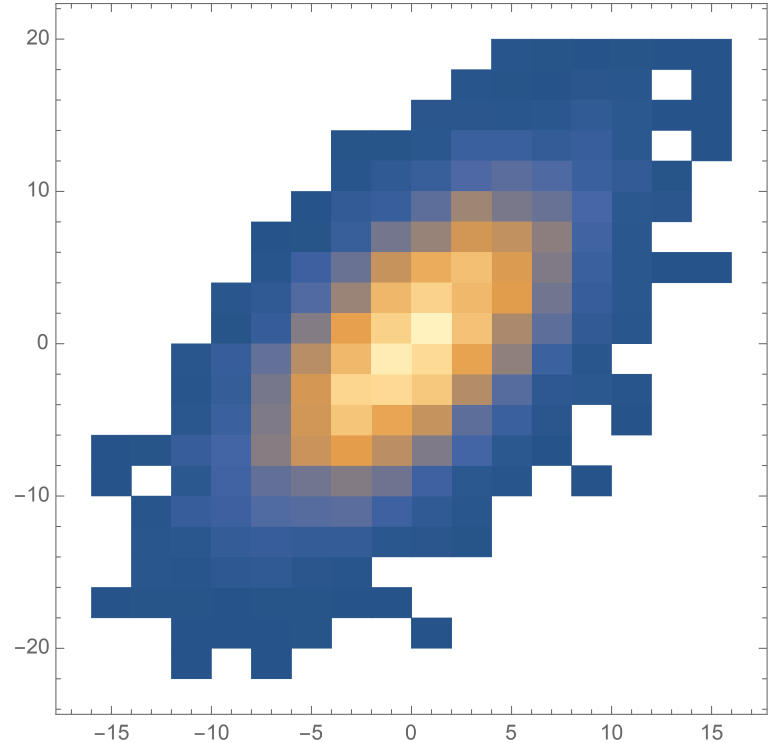
Endpoints of 10000 random paths with the same number of elementary steps



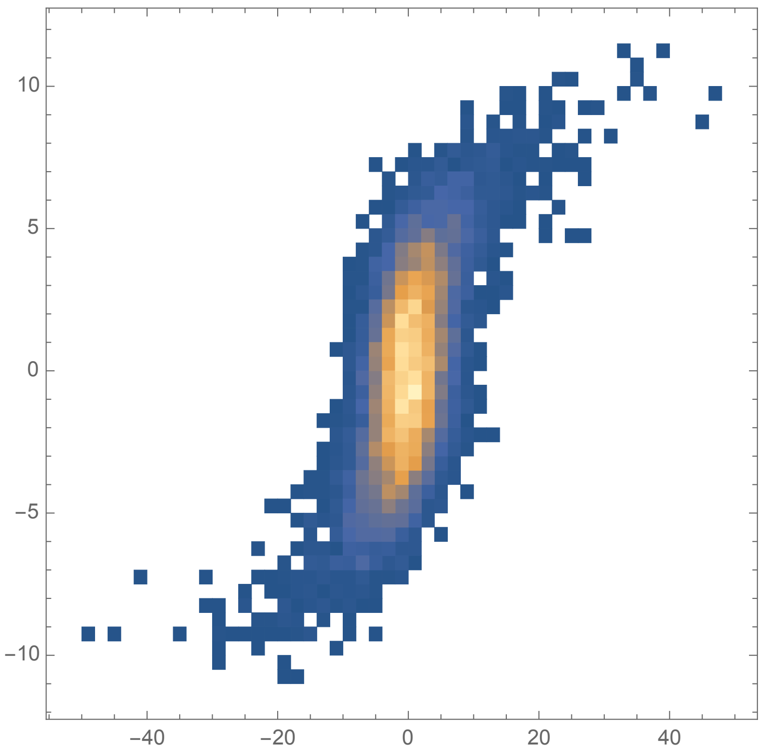
Reprentations of the empirical pdf



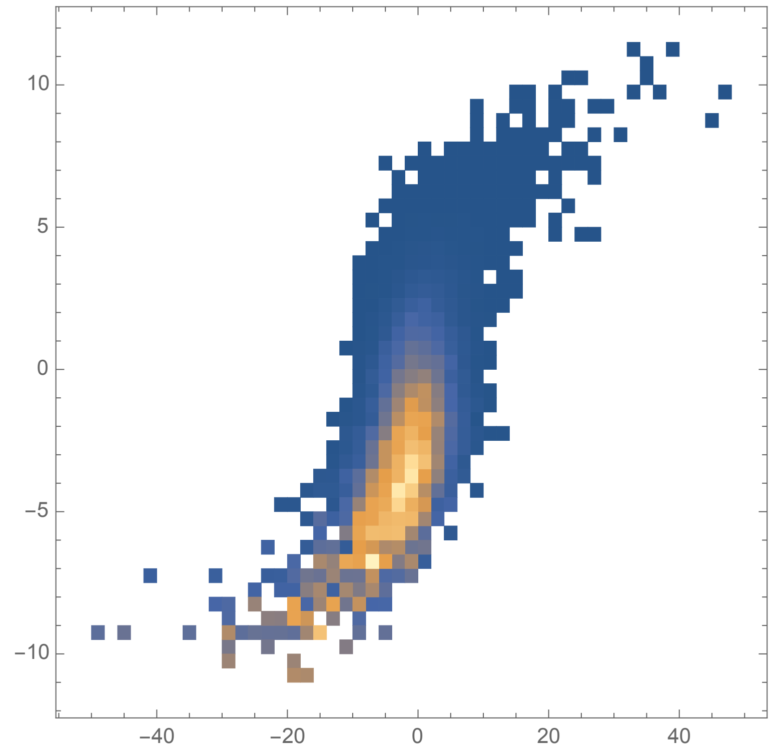
Small eastward drift



Position-dependent drift



Nonlinear position-dependent drift



Nonlinear position-dependent drift weighted with survival probability (higher in southern, warmer waters)

Search and Rescue Optimal Planning System

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Abstract – In 1974 the U.S. Coast Guard put into operation its first computerized search and rescue planning system CASP (Computer-Assisted Search Planning) which used a Bayesian approach implemented by a particle filter to produce probability distributions for the location of the search object. These distributions were used for planning search effort. In 2003, the Coast Guard started development of a new decision support system for managing search efforts called Search and Rescue Optimal Planning System (SAROPS). SAROPS has been operational since January, 2007 and is currently the only search planning tool that the Coast Guard uses for maritime searches. SAROPS represents a major advance in search planning technology. This paper reviews the technology behind the tool.

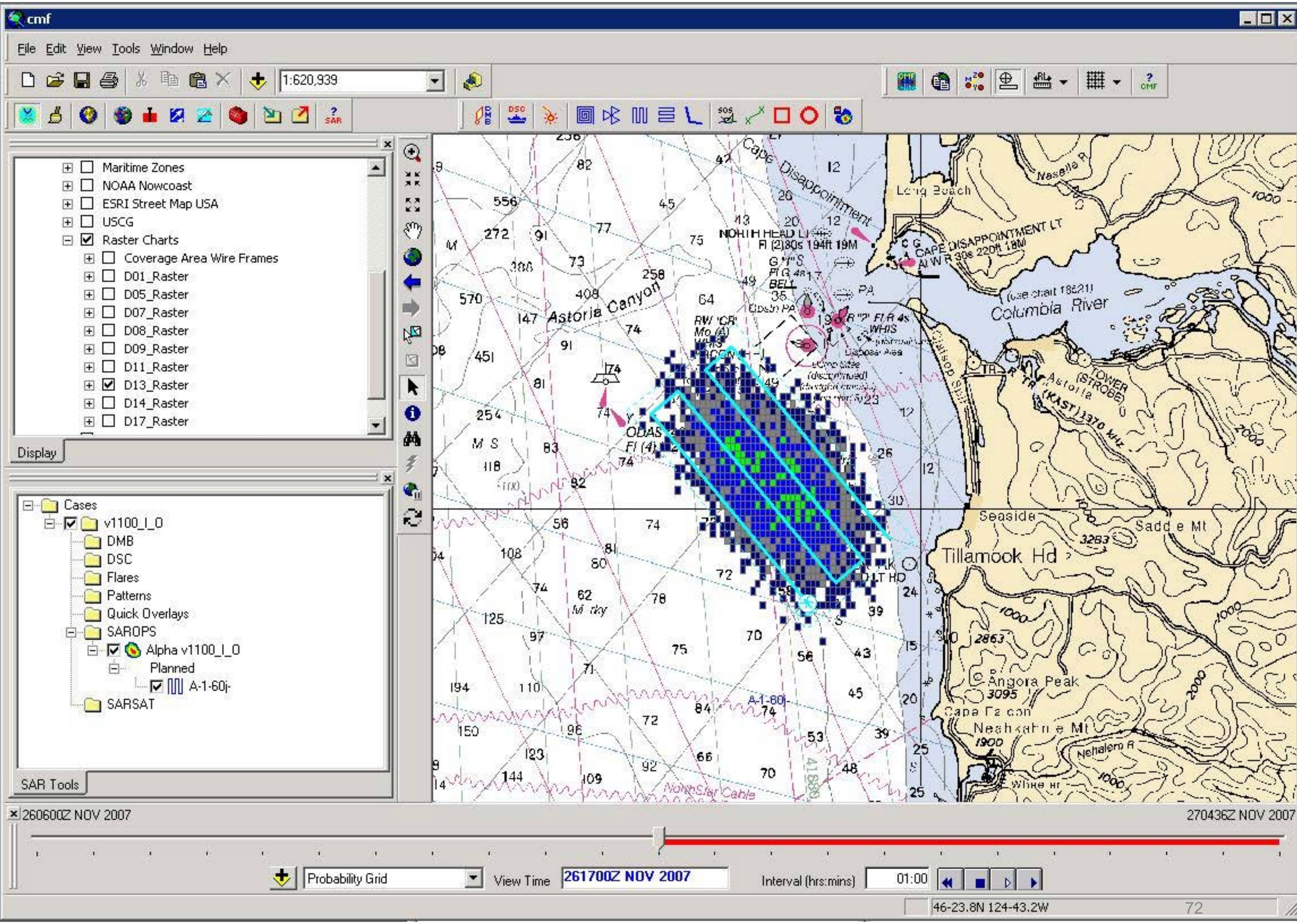
Published in: 13th Conference on Information Fusion (FUSION), 2010, DOI: 10.1109/ICIF.2010.5712114

The main elements of SAROPS
(from Kratze, Stone, and Frost)

EDS: SAROPS requires environmental estimates in order to account for possible drift of the search object and to estimate the detectability of the object by various search sensors, e.g., visual and radar. Estimates of ocean currents and winds are needed to account for drift of search objects. Wave height, cloud cover, sun, and rain all affect detectability of search objects. Water temperature is used to estimate survival times. The EDS provides the data on appropriate spatial and temporal grids to cover the area and time period of interest.

SIM: The simulator uses information about time and last known position of the search object and information about its intentions, to produce probability distributions (maps) for the object's location using a particle filter where each particle represents a possible path for the search object. The particle filter takes into account drift using estimates of winds and currents provided by the EDS. SIM includes information about the object's intended path, areas where trouble is likely to occur, areas where searches have already occurred, and more. It incorporates information from unsuccessful searches in a Bayesian fashion. All this is used to produce a probability distribution on paths. The paths and their weights define the object location distributions as a function of time. The distribution for a selected time is displayed as a set of rectangular cells with the probabilities associated with each cell indicated by a color scale.

Planner: SIM produces a probability distribution for the object's location at the time of the next search. The Planner uses this distribution along with a list of assigned search assets to produce operationally feasible search plans that maximize the increase in probability of detecting the object.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Display
Cases
v1100_I_0
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DSC
Flares
Patterns
Quick Overlays
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Probability Grid
View Time
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