Introduction to Bayesian Statistics - 11

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Multiprocessing with **emcee**

Useful python package `multiprocessing` (https://docs.python.org/3/library/multiprocessing.html)

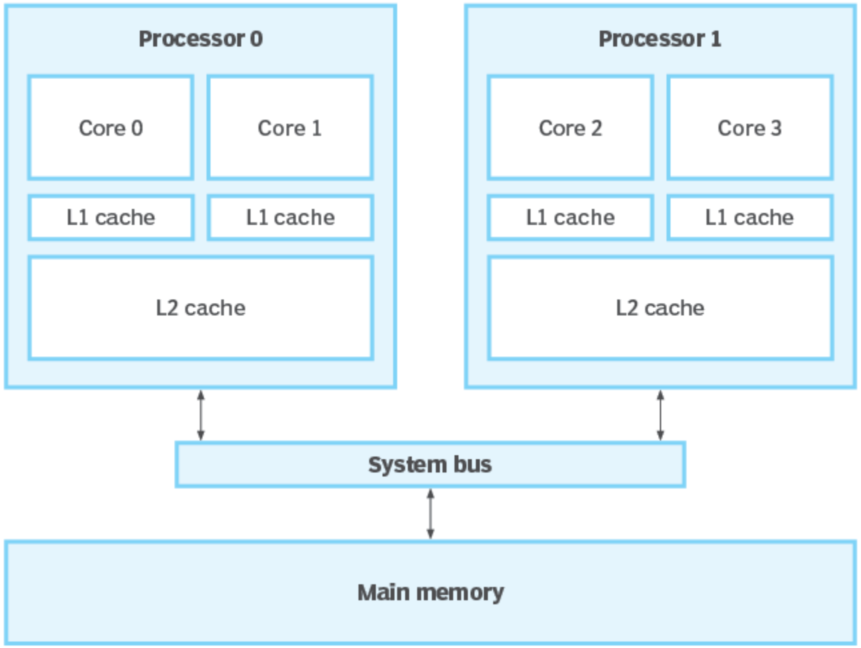
Here we use the package ONLY in the context of **emcee**.

```
import multiprocessing
import time

def cube(x):
    return x**3

if __name__ == "__main__":
    pool = multiprocessing.Pool()
    start_time = time.perf_counter()
    processes = [pool.apply_async(cube, args=(x,)) for x in range(1,1000)]
    result = [p.get() for p in processes]
    finish_time = time.perf_counter()
    print(f"Program finished in {finish_time-start_time} seconds")
    print(result)
```

Multiprocessing means using two or more central processing units (CPUs) in a single computer system.

Its definition can vary, but generally it refers to the ability to support multiple CPUs and to the capacity to distribute work among them.

Present–day multicore processors can easily have 12, 24 or more cores on the same motherboard, enabling concurrent processing of numerous tasks.



What are processes?

A process, or a running process, is a collection of instructions carried out by the computer processor.
A process's execution proceeds in a sequential manner, just as a computer program which is written in a text file, but when executed, it becomes a process and performs sequentially all the tasks crafted in the program.

A normal computer runs several processes continuously to manage operating system, and all hardware and applications installed on the machine. This can range from simple background tasks like a spell-checker or system event handlers to a complex application like Microsoft Word.

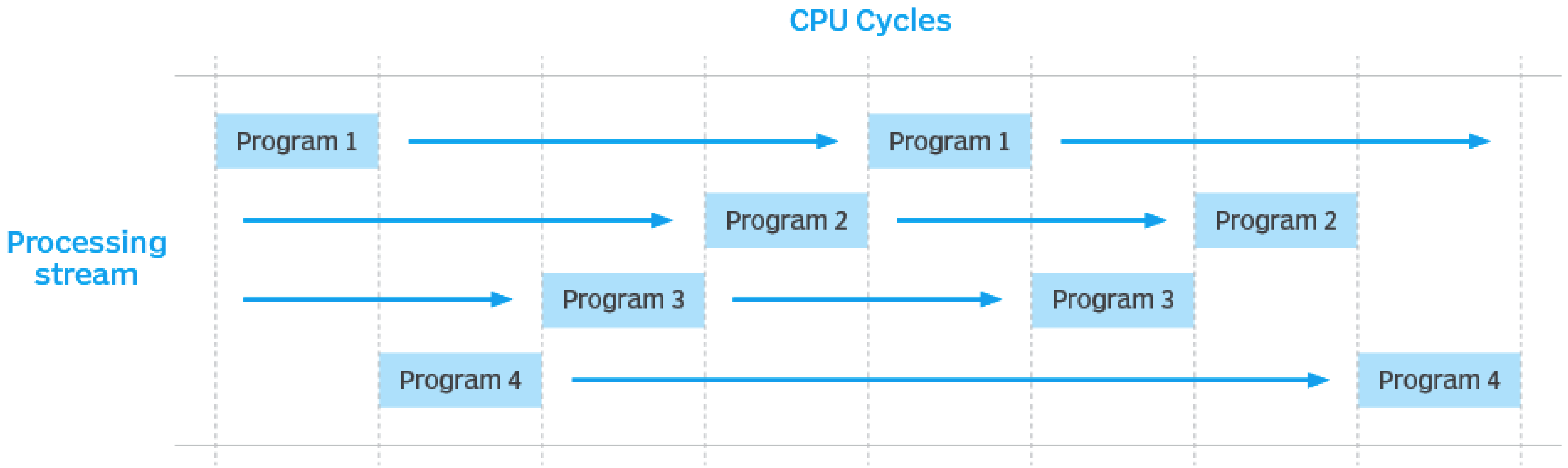
Processes vs. threads

According to a common misconception processes and threads are the same while, in fact, they are different execution sequences. Here is a list of differences between a processes and a threads:

- A process is part of a running program, while a thread belongs to a given process.
- Threads are small when compared to processes.
- A process takes much longer to terminate.
- The creation of a process takes longer than the creation of a thread.
- Processes require more time for context switching.
- Unlike threads, which share memory, processes have their own memory space.
- Data is not shared across processes, but it is between threads.

Multiprocessing can also be confused with multitasking or time sharing, the management of programs and the system services they request as tasks that can be interleaved, and with multithreading, the management of multiple execution paths through the computer or of multiple users sharing the same copy of a program.

Illustration of multithreading with 4 threads



PRO's

Multiprocessing environments are widely adopted and offer a many advantages such as increased speed, throughput and reliability. Common benefits of multiprocessing include the following:

- **Reliability**. If one processor fails in a multiprocessor system, the other processors can pick up the slack and continue to function. While the shutting down of one processor might cause a gradual slowdown, the system can still function smoothly. This makes multiprocessing systems highly reliable.

- **Increased throughput**. Throughput is the number of processes executed at a given time. Given that multiprocessor systems use many CPUs to handle data, increased performance is expected when the system uses parallel processing. This means more tasks can be accomplished in a shorter amount of time, as they're divided among different processors.

- **Cost saving**. Multiprocessing systems are more economical compared to multiple single processor systems. This is because multiple processors within a single system share the same memory, disk space, buses and peripherals.

CON's

While multiprocessing improves system performance and reliability, it does come with the following challenges:

- **Expensive**. Systems with multiple processors may be expensive. Having just one processor is less expensive than having two or more.
- **Deadlocks**. In systems with multiple processors, a deadlock can occur if one processor attempts to access an I/O device while another processor is trying to use it.
- **Extra memory requirements**. Due to their improved computing capability, multiprocessor computers are widely used. However, they require more memory. In multiprocessing architectures, memory is shared across all processes and each processor needs memory space. All processors work together and have direct access to the main memory, which causes an increase in memory usage.
- **Complex operating system**. In multiprocessing OSs, each CPU has its own operating system, which assigns each processor with several minor tasks and the load is distributed among the processors. However, the use of multiple processors makes it more complex for the OSs to function.

Radiocarbon From Cosmic Radiation[1]

E. C. Anderson and W. F. Libby

Institute for Nuclear Studies and *Department of Chemistry, University of Chicago*

S. Weinhouse, A. F. Reid, A. D. Kirshenbaum, and A. V. Grosse

Houdry Process Corporation, Marcus Hook, Pennsylvania

It has recently been suggested (*2*) that neutrons produced by cosmic radiation should form radiocarbon by the reaction $N^{14}(n,p)C^{14}$ in such amounts that all carbon in living matter (and in other chemical forms in exchange equilibrium with atmospheric CO_2 on a 5,000-year time scale) should be radioactive to the extent of 1–10 disintegrations/minute/gram. In view of the 5,000-year half-life of radiocarbon (*3, 4*), it was further expected that it should be absent from such geologically "old" carbon sources as petroleum, coal, or limestone. The existence of such "cosmic radioelements" was anticipated by one of us (*1*) shortly after the discovery of artificial radioactivity.

These predictions were investigated by examining the radioactivity of two series of isotopically enriched samples of methane. The first series was derived from petroleum methane (referred to as petromethane) and the other from methane from the Patapsco Sewage Plant of the city of Baltimore (referred to as biomethane).

Measurements on the enriched biomethane samples established the activity of "living" carbon to be 10.5 disintegrations/minute/gram, in good agreement with the predicted value. On the other hand, enrichment of the petromethane by a factor of 25 failed to show activity beyond the limits of experimental error, in line with the theory that cosmic rays produce our activity.

The two methane thermal diffusion plants located at the Houdry Laboratories were used. Six hundred l. of biomethane were necessary to obtain about 20 l. of 12 per cent $C^{13}H_4$; these were purified in a high-vacuum system and reconcentrated to 65–70 per cent $C^{13}H_4$. Experimental values of $C^{13}H_4$ separation were used to determine the operating constants of the thermal diffusion columns. Upon extension of the separation theory to $C^{14}H_4$ concentration, the C^{14} enrichment was calculated, jointly with D. Tanguy, of Houdry's Special Projects Division, using these constants. The over-all error in the computed factor was probably about 20 per cent.

Possibilities of contamination were essentially excluded by running the plants and purification apparatus alternatively with bio- and petromethane, as indicated by the take-off dates of Table 1.

The samples were measured in a brass-cylinder Geiger counter, with glass heads, of 1,900-cc. volume, surrounded by 1.5-inch-thick lead shield to reduce the background. Alternate readings were taken at 6.5 cm. Hg pressure with the two series of samples. A standard external sample of U_3O_8 was used to check that the counter sensitivity was the same for all fillings.

TABLE 1

Source	Sample No.	Calculated C^{14} enrichment	C^{13} concentration from mass spectrometer (%)	% CH_4 in gas before final purification	Date taken	Total count rate, including background (disintegrations/minute)
Petro-methane	I	1	1.04	99.6	10/16/46	340.6 ±1.0
	II	1	1.04	99.6	"	342.6 ±1.0
	III	25	6.55	97.2	1/ 6/47	345.8 ±1.3
	III	1	1.04	99.4	12/ 5/46	342.9 ±2.0
Bio-methane	I	10	7.36	93.6	10/17/46	348.7 ±1.3
	VII	32	11.02	99.9	12/ 2/46	364.0 ±1.5
	VIII	260	63.5	97.2	2/10/47	562.0 ±2.9

Our data are given in Table 1. There is no significant activity in the petromethane, whereas the biomethane has a definite, easily measurable activity increasing linearly with the calculated C^{14} enrichment.

To eliminate the possibility that the activity was due to tritium, the most concentrated methane, sample Bio VIII was burned and converted to $CaCO_3$. The activity of the $CaCO_3$, as measured in a screen wall counter, established the radioactivity as carbon rather than hydrogen. Further identification of this radioactivity was obtained by a measurement of its absorption in aluminum, which agreed with that of synthetic C^{14}. The data thus establish the activity as being carried in carbon in a molecule of mass 18, present in the original biomethane in very low concentration. The agreement of the absorption data with those for C^{14} further confirms the identification of the activity with C^{14}.

The possibility that the C^{14} found in Baltimore sewage is due to contamination is not entirely excluded, although it is remote. Our sample was taken on September 2, 1946, at which time, according to P. C. Aebersold, no C^{14} from the Atomic Energy Commission had been received by anyone in Baltimore. The possibility of its origin from the atomic piles or bombs is excluded when one realizes that our activity corresponds to the existence in nature of some 10^8 curies, or 20 metric tons—an amount far larger than any synthetic source could have produced to date.

The discovery of cosmic-ray carbon has a number of interesting implications in the biological, geological, and meteorological fields; a number of these are being explored, particularly

[1] The authors are grateful to George E. Finck and R. J. Trautman, Department of Public Works of the city of Baltimore, to H. G. Swope, secretary of the A.C.S. Committee on Water, Sewage, and Sanitation, and to Abel Wolman, Johns Hopkins University, for their cooperation; to the Sun Oil Company for the use of its thermal diffusion columns; to W. H. Johnston for assistance in the counting measurements; and to J. T. Dooley for technical assistance.

Radiocarbon dating

Willard F. Libby
Facts

Photo from the Nobel Foundation archive.

Willard Frank Libby
Nobel Prize in Chemistry 1960

Born: 17 December 1908, Grand Valley, CO, USA

Died: 8 September 1980, Los Angeles, CA, USA

Affiliation at the time of the award: University of California, Los Angeles, CA, USA

Prize motivation: “for his method to use carbon-14 for age determination in archaeology, geology, geophysics, and other branches of science”

Prize share: 1/1

Work

Carbon is a fundamental component in all living material. In nature there are two variants, or isotopes: carbon-12, which is stable, and carbon-14, which is radioactive. Carbon-14 forms in the atmosphere when acted upon by cosmic radiation and then deteriorates. When an organism dies and the supply of carbon from the atmosphere ceases, the content of carbon-14 declines through radioactive decay at a fixed rate. In 1949 Willard Libby developed a method for applying this to determine the age of fossils and archeological relics.

Radiocarbon dating

WILLARD F. LIBBY

Radiocarbon dating

Nobel Lecture, December 12, 1960

Introduction

Radiocarbon dating had its origin in a study of the possible effects that cosmic rays might have on the earth and on the earth's atmosphere. We were interested in testing whether any of the various effects which might be predicted could actually be found and used. Initially the problem seemed rather difficult, for ignorance of billion-electron-volt nuclear physics (cosmic ray energies are in this range) was so abysmal at the time and incidentally fourteen years later still is so abysmal, that it is nearly impossible to predict with any certainty the effects of the collisions of the multibillion-volt primary cosmic radiation with air.

Formation of radiocarbon

However, in 1939, just before the war, Professor Serge Korff of New York University and others discovered that the cosmic rays produce secondary neutrons in their initial collisions with the top of the atmosphere.

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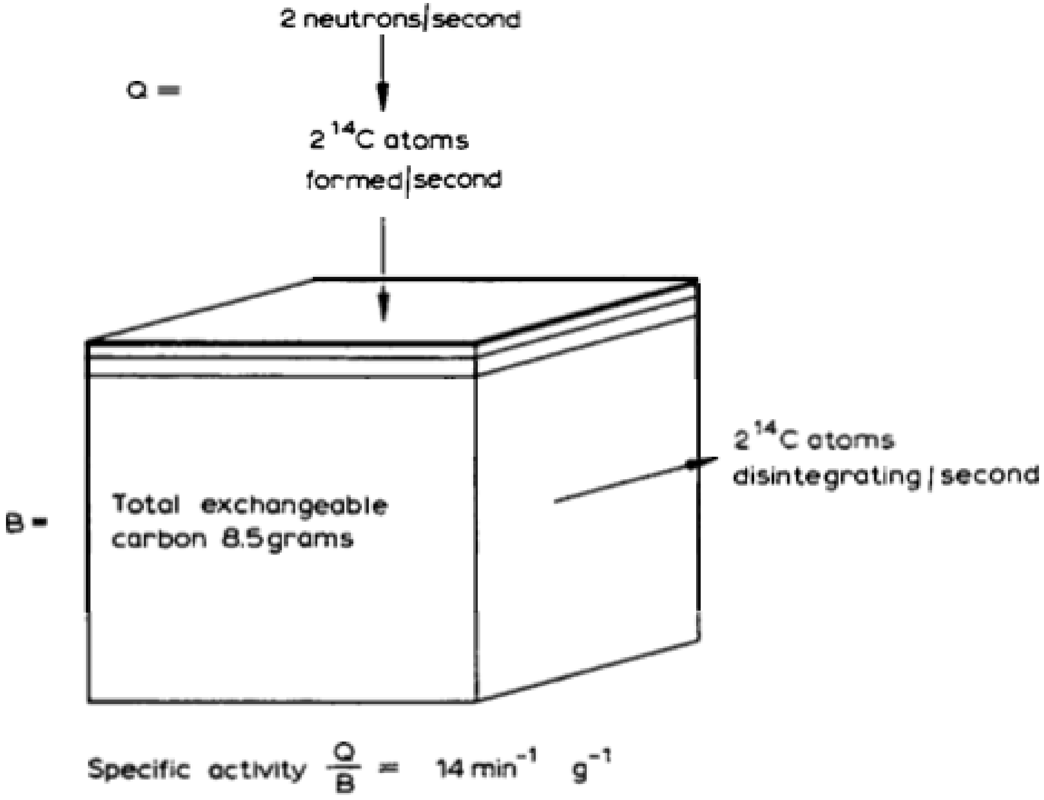
To return to radiocarbon dating - knowing that there are about 2 neutrons formed per square centimeter per second, each of which forms a carbon-14 atom, and assuming that the cosmic rays have been bombarding the atmosphere for a very long time in terms of the lifetime of carbon-14 (carbon-14 has a half-life of about 5,600 years) - we can see that a steady-state condition should have been established, in which the rate of formation of carbon-14 would be equal to the rate at which it disappears to reform nitrogen-14. This allows us to calculate quantitatively how much carbon-14 should exist on earth (see Fig. 1); and since the 2 atoms per second per cm^2 go into a mixing reservoir with about 8.5 grams of carbon per cm^2, this gives an expected specific activity of living matter of 2.0/8.5 disintegrations per second per gram of carbon.

Table I. Carbon reservoir make-up (g C/cm^2).

	Anderson and Libby	*W. W. Rubey*
Ocean, «carbonate»	7.25	6.95
Ocean, dissolved organic	0.59 }	
Biosphere	0.33 }	0.78
Humus	0.20	
Atmosphere	0.12	0.125
Total	8.5	7.9



(from the Nobel Lecture of W. F. Libby, link)

Age Determinations by Radiocarbon Content: Checks with Samples of Known Age

J. R. Arnold and W. F. Libby

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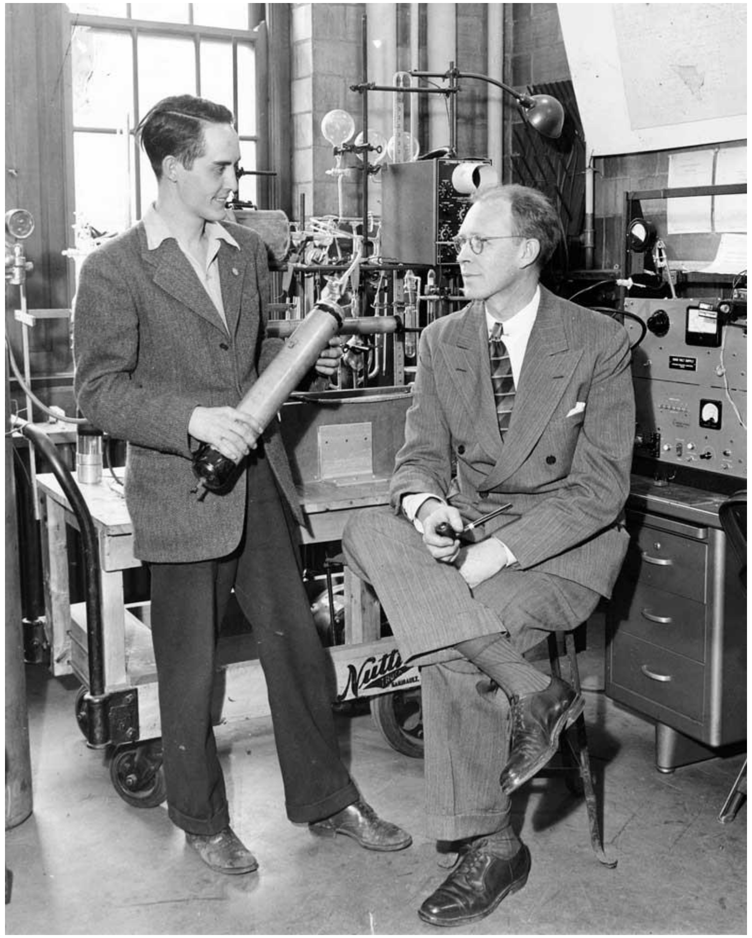


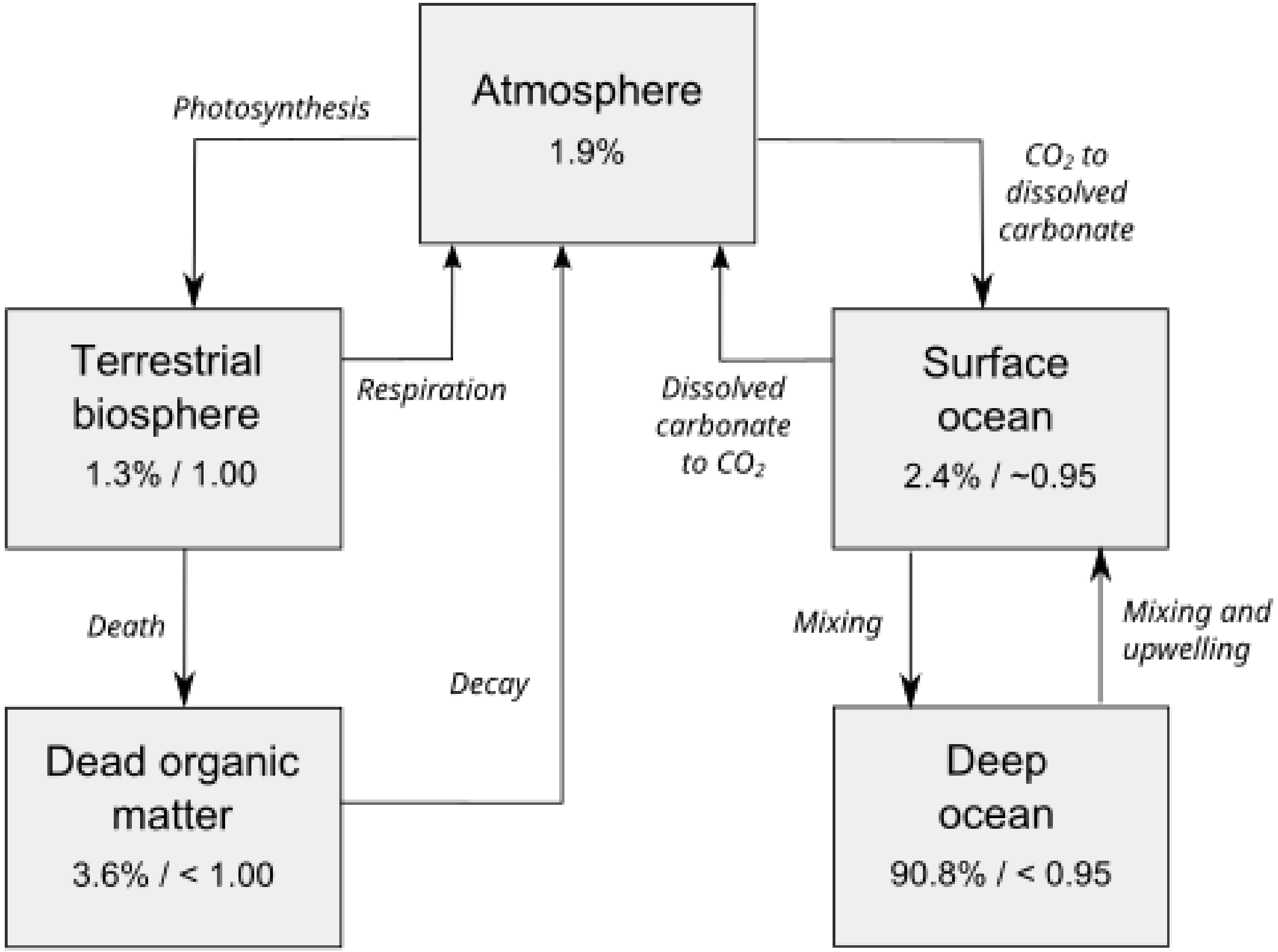
Willard F. Libby (right), the physical chemist who conceived of radiocarbon dating, with graduate student Ernest Anderson.
(from this webpage of the ACS)

TABLE 1

AGE DETERMINATIONS ON SAMPLES OF KNOWN AGE

Sample	Specific activity (cpm/g of carbon) Found	Specific activity (cpm/g of carbon) Expected	Age (years) Found	Age (years) Expected
Tree Ring	11.10 ± 0.31 11.52 ± 0.35 11.34 ± 0.25 10.15 ± 0.44 11.08 ± 0.31 Average: 10.99 ± 0.15	10.65	1100 ± 150	1372 ± 50 (577 ± 50 A.D.)
Ptolemy	9.5 ± 0.45	9.67	2300 ± 450	2149 ± 150 (200 ± 150 B.C.)
Tayinat	8.97 ± 0.31 9.03 ± 0.30 9.53 ± 0.32 Average: 9.18 ± 0.18	9.10	2600 ± 150	2624 ± 50 (675 ± 50 B.C.)
Redwood	8.81 ± 0.26 8.56 ± 0.22 Average: 8.68 ± 0.17	8.78	3005 ± 165	2928 ± 52 (979 ± 52 B.C.)
Sesostris	7.73 ± 0.36 8.21 ± 0.50 Average: 7.97 ± 0.30	7.90	3700 ± 400	3792 ± 50 (1843 ± 50 B.C.)
Zoser: Sneferu		7.15	4750 ± 250	
Zoser	7.88 ± 0.74 7.36 ± 0.53			4650 ± 75 (2700 ± 75 B.C.)
Sneferu	6.95 ± 0.40 7.42 ± 0.38 6.26 ± 0.41 Average: 7.04 ± 0.20			4575 ± 75 (2625 ± 75 B.C.) Average: 4600 ± 75 (2650 ± 75 B.C.)

https://en.wikipedia.org/wiki/Radiocarbon_dating#/media/File:Carbon_exchange_reservoir_2.svg



Percentages show the fraction of the total carbon reservoir of each type.
Numbers after slash show ratio of ^{14}C to ^{12}C as fraction of atmospheric ratio.

Important considerations

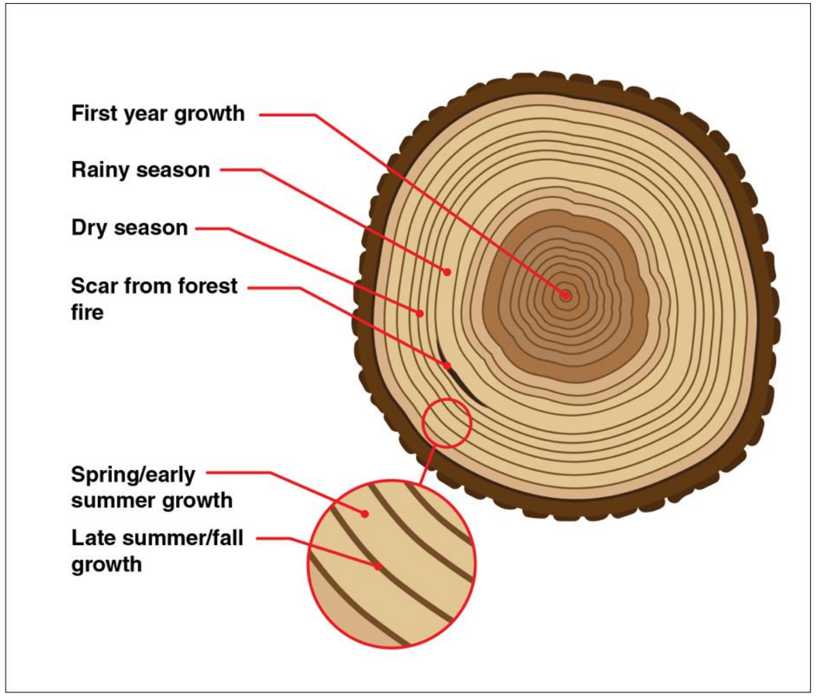
In its basic form, the method assumes

- constancy of cosmic radiation
- no contamination of samples

However, this is not always true, issues at stake are

- space-time variations of the $^{14}C/^{12}C$ ratio in the atmosphere
  - impact of the industrial age
  - impact of nuclear bomb tests
- isotopic fractionation (uneven intake/metabolism of ^{14}C and ^{12}C in different environments/organisms)
- variations of the $^{14}C/^{12}C$ ratio in different parts of the carbon reservoir
  - marine effect
  - hemisphere effect
- contamination

Calibration methods are needed, such as tree rings.



The likelihood function

In this plot, we observe data where both the collection time and the fraction of C-14 are subject to measurement errors.

Let T be the "true" collection time, and μ be the "true" C-14 fraction, then the measured values (times and fractions) have pdf's

$$t(T) \sim N(T, \sigma_t)$$
$$y(\mu) \sim N(\mu, \sigma_y)$$

while

$$\mu \sim \exp(-T \ln 2/\tau_{1/2})$$

Then, we can write

$$p(t, y|, T) = p(t|y, T)p(y|T) = p(t|T)p(y|T)$$

$$L \propto \prod_k \exp\left(-\frac{(t_k - T_k)^2}{2\sigma_{t,k}^2}\right) \exp\left(-\frac{(y_k - \mu_k)^2}{2\sigma_{\mu,k}^2}\right)$$

$$\text{with} \quad \mu_k = A \exp\left(-\frac{T_k \ln 2}{\tau_{1/2}}\right)$$

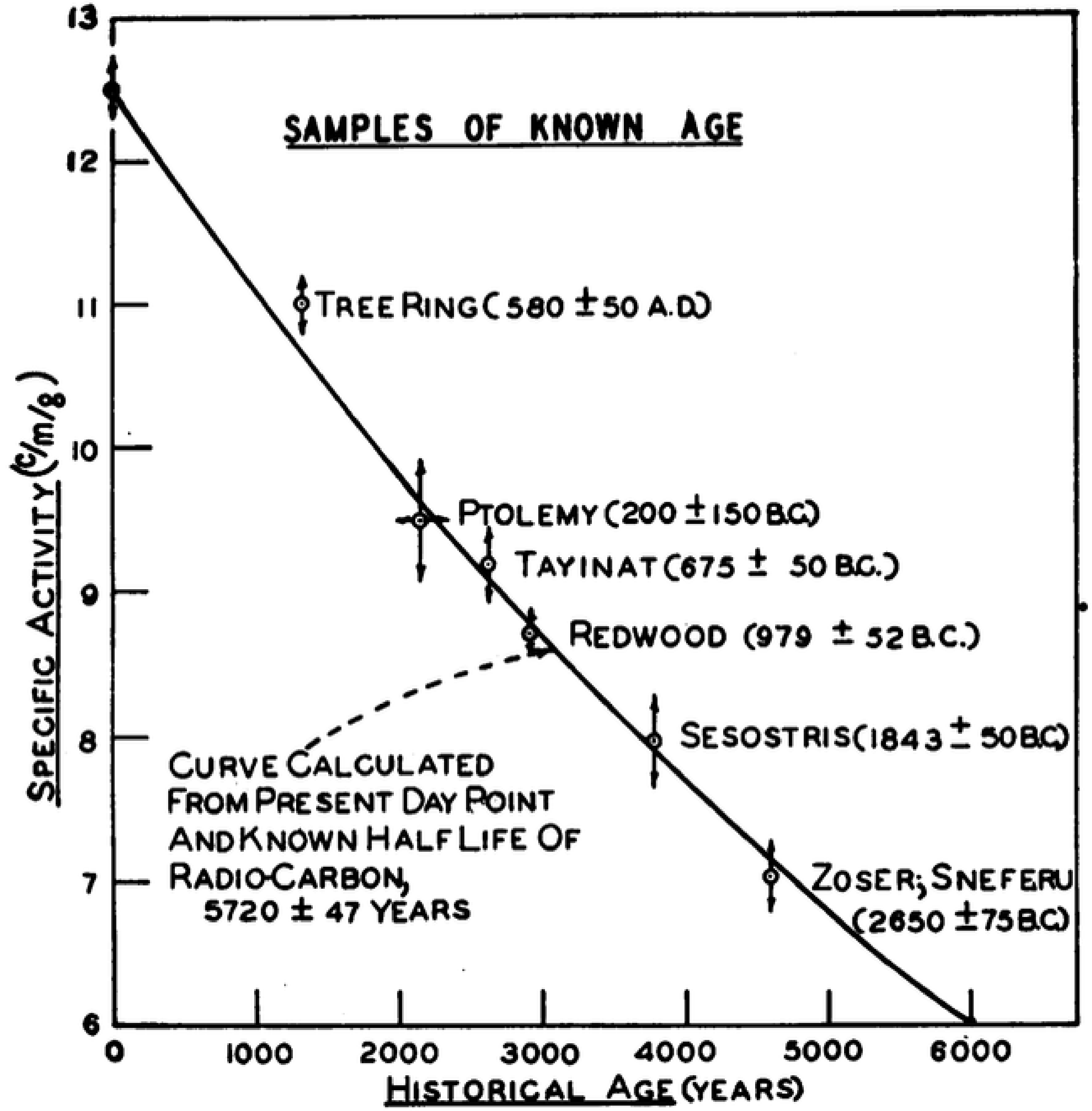


FIG. 1. Specific activities for samples of known age.

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Research

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Modelling cosmic radiation events in the tree-ring radiocarbon record

Qingyuan Zhang[1], Utkarsh Sharma[1], Jordan A. Dennis[1], Andrea Scifo[2], Margot Kuitems[2], Ulf Büntgen[3,4,5,6], Mathew J. Owens[7], Michael W. Dee[2] and Benjamin J. S. Pope[1,8]

Annually resolved measurements of the radiocarbon content in tree-rings have revealed rare sharp rises in carbon-14 production. These 'Miyake events' are likely produced by rare increases in cosmic radiation from the Sun or other energetic astrophysical sources. The radiocarbon produced is not only circulated through the Earth's atmosphere and oceans, but also absorbed by the biosphere and locked in the annual growth rings of trees. To interpret high-resolution tree-ring radiocarbon measurements therefore necessitates modelling the entire global carbon cycle. Here, we introduce '`ticktack`' (https://github.com/SharmaLlama/ticktack/), the first open-source Python package that connects box models of the carbon cycle with modern Bayesian inference tools. We use this to analyse all public annual 14C tree data, and infer posterior parameters for all six known Miyake events. They do not show a consistent relationship to the solar cycle, and several display extended durations that challenge either astrophysical or geophysical models.

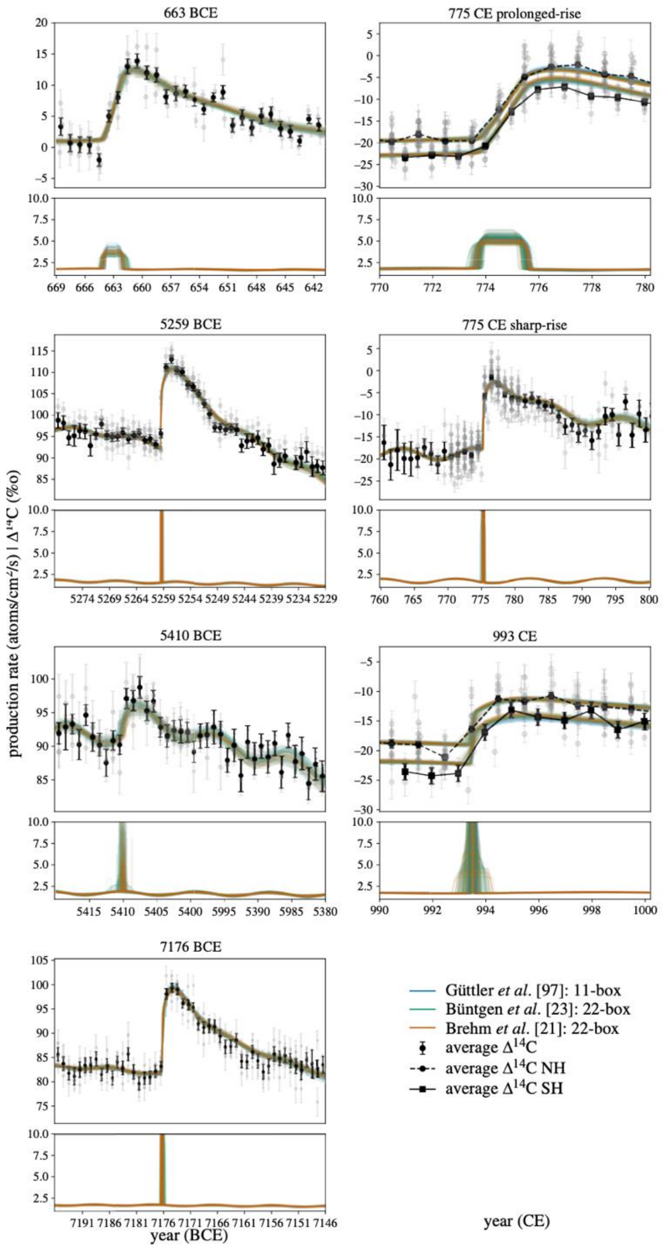

Figure 1. Results of MCMC fitting of a parametric Miyake event model to all six known events. Each is presented in a pair of panels. Top: the tree-ring data (individual trees in grey, mean in black) overlaid with colour-coded curves drawn at random from MCMC posterior samples for all three CBMs; they are in excellent agreement with one another and with the data. Bottom: radiocarbon production rate models drawn from the corresponding MCMC posterior samples, with the same colour bars. The 663 BCE event and a subset of the 774 CE event are consistent only with a production spike taking longer than a year. The 774 CE event is presented split into subsets of data showing a prolonged rise, and a sudden rise, which are incompatible in our models and analysed separately. (Online version in colour.)

MCMC code stored in https://github.com/edymil/Bayes-TS

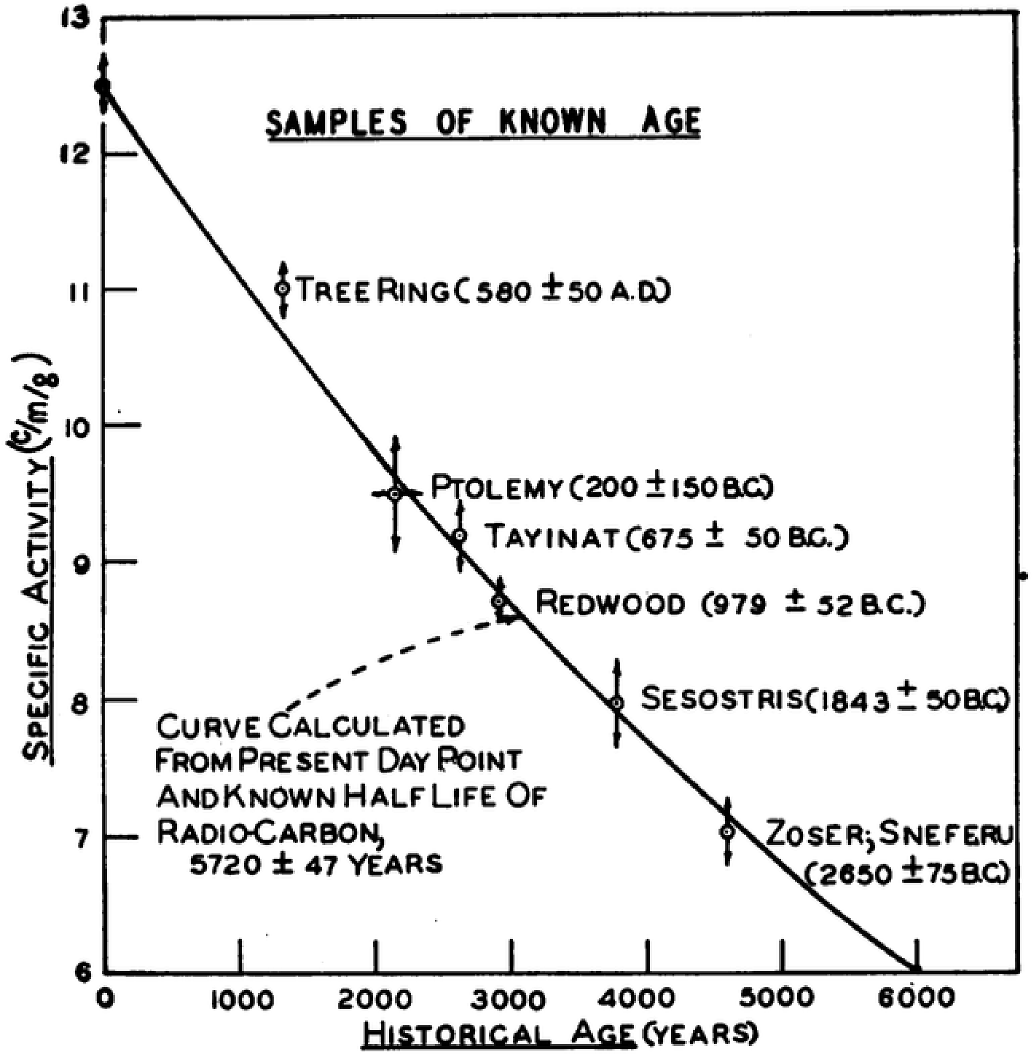

FIG. 1. Specific activities for samples of known age.