Sign conventions

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October 31, 2023

The authoritative book by Misner, Thorne and Wheeler (MTW) [1] is often used as a reference in GR. MTW analyzed the different sign conventions used by different authors and came up with a "sign convention scheme", which is defined by the following equations

$$\eta_{\mu\nu} = [S1] \times \text{diag}(-1, +1, +1, +1) \tag{1}$$

$$R^{\mu}_{\alpha\beta\gamma} = [S2] \times \left(\partial_{\beta}\Gamma^{\mu}_{\alpha\gamma} - \partial_{\gamma}\Gamma^{\mu}_{\alpha\beta} + \Gamma^{\mu}_{\beta\sigma}\Gamma^{\sigma}_{\alpha\gamma} - \Gamma^{\mu}_{\gamma\sigma}\Gamma^{\sigma}_{\alpha\beta}\right)$$
(2)

$$G_{\mu\nu} = [S3] \times \kappa T_{\mu\nu} \tag{3}$$

where the Einstein gravitational constant is defined as

$$\kappa = \frac{8\pi G}{c^4} \approx 2.0766 \times 10^{-43} \mathrm{N}^{-1} \tag{4}$$

Sign [S3] sets the condition on the kind of contraction of the Riemann tensor that we adopt for the definition of the Ricci tensor.

In this course I adopt the following convention, [S1] = -1, [S2] = +1, and [S3] = +1. The last choice implies that

$$R_{\mu\nu} = R^{\alpha}_{\mu\nu\alpha} \tag{5}$$

this is the same as in the book by W. Rindler [2].

References

- Charles W Misner, Kip S Thorne, and John Archibald Wheeler. Gravitation. Macmillan, 1973.
- [2] Wolfgang Rindler. Essential relativity: special, general, and cosmological. Springer Science & Business Media, 1977.