Supplementary text for the handout "Special Relativity and notation"

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This supplementary text contains the solution to the exercise in the handout "Special Relativity and notation". The basic information that we recall here are the following

- The CMB temperature is 2.72548 ± 0.00057 K.
- The modal CMB photon energy is $E_\gamma \approx 6.626 \times 10^{-4}$ eV.
- The mean free path ℓ is related to the total cross–section and the particle number density n by the equation $\ell = 1/n\sigma$.

Exercise:

1. Using the well–know formulas for the density of black body photons at a given temperature T, find the CMB photon density (Hint: recall that the energy density is proportional to the irradiance: $I = cu/4$, and that the Stefan–Boltzmann constant is $\sigma_B \approx 5.67 \times 10^{-8}$ W m^{-2} K^{-4}).
2. At a proton energy $E_p \approx 10^{20}$ eV, the photoproduction cross section for the process

$$p + \gamma \to p + \pi^0,$$

   is about 2×10^{-28} cm^2, and nearly energy–independent ; using this information and the density of CMB photons, find the corresponding mean free path of protons.

Solution:

1. For a perfect blackbody, the irradiance at temperature T is $I = \sigma_B T^4$, therefore for CMB photons we find $I \approx 3.13 \times 10^{-6}$ W m^{-2}. The corresponding energy density is $u = 4I/c \approx 4.17 \times 10^{-14}$ J m^{-3}. Converting to eV we find $u \approx 2.6 \times 10^5$eV m^{-3}. Finally, dividing by the energy of the individual photons (approximately equal to the modal energy) we find $n \approx 4 \times 10^8$ photons m^{-3} = 400 photons cm^{-3}.
2. From the formula for the mean free path, we find $\ell \approx 1.3 \times 10^{25}$ cm $= 1.3 \times 10^{23}$ m $\approx 1.3 \times 10^7$ light–years.

The mean free path of ultrahigh energy cosmic rays can be compared with the distance to the nearest galaxy cluster, the Virgo cluster, whose centroid is approximately 4 times as far (see Fig. 1). Now, the question is *where do the ultrahigh cosmic rays come from?* The experiments finds excesses in some directions in the sky, e.g., in the direction of Ursa Major, but no correspondence to any possible source.

Figure 1: NGC 4654, in the constellation Virgo. NGC 4654 is just north of the celestial equator, making it visible from the northern hemisphere and most of the southern hemisphere. The galaxy is around 55 million light–years from Earth. From `https://science.nasa.gov/missions/hubble/hubble-views-a-vibrant-virgo-cluster-galaxy/`.