Transmission and reflection coefficients in optics

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October 25, 2024

Suppose that there is a wave propagating towards the interface that divides two media as shown in the figure and suppose that it has a maximum amplitude equal to 1, so that the transmitted wave has amplitude $\mathcal{T}$ and the reflected wave has amplitude $\mathcal{R}$.

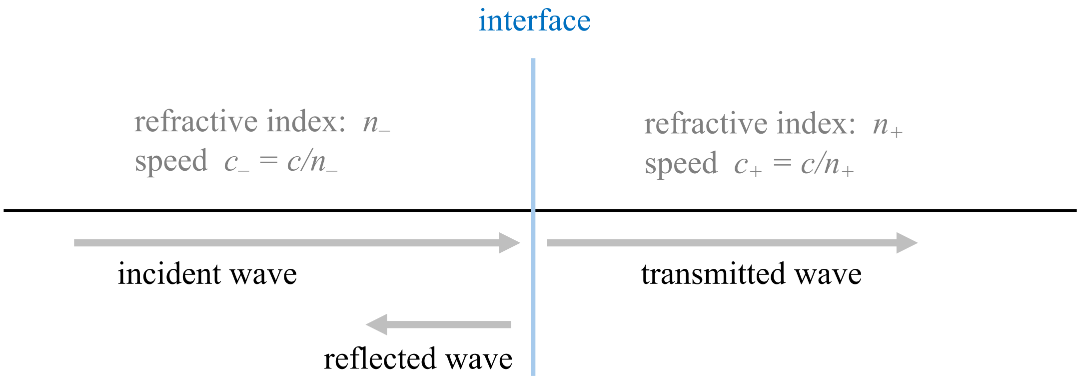

Figure 1: Schematization of the amplitude and direction of the waves near an interface between two media.

Letting $\Phi(t - x/c_-)$ be the incoming field, then the reflected field is $\mathcal{R}\Phi(t + x/c_-)$, and the transmitted field is $\mathcal{T}\Phi(t - x/c_+)$, and we obtain the total field on the left of the interface

$$\Phi(t - x/c_-) + \mathcal{R}\Phi(t + x/c_-), \tag{1}$$

while the field on the opposite side is

$$\mathcal{T}\Phi(t - x/c_+). \tag{2}$$

In general, both the field and its time derivative must be continuous at the interface — which we set at $x = 0$ — therefore at the interface

$$\Phi(t) + \mathcal{R}\Phi(t) = \mathcal{T}\Phi(t) \tag{3}$$

$$-\frac{1}{c_-}\Phi'(t) + \frac{1}{c_-}\mathcal{R}\Phi'(t) = -\frac{1}{c_+}\mathcal{T}\Phi'(t), \tag{4}$$

and assuming that neither Φ nor its derivative be identically zero, we obtain

$$1+\mathcal{R}=\mathcal{T} \tag{5}$$

$$-\frac{1}{c_-}+\frac{1}{c_-}\mathcal{R}=-\frac{1}{c_+}\mathcal{T}, \tag{6}$$

i.e.,

$$\mathcal{R}=\frac{c_+-c_-}{c_++c_-} \tag{7}$$

$$\mathcal{T}=\frac{2c_+}{c_++c_-}. \tag{8}$$

In the case of light, $c_\pm = c/n_\pm$, and we obtain the well–known formulas for the the reflection and transmission coefficients at an interface

$$\mathcal{R}=\frac{n_--n_+}{n_-+n_+} \tag{9}$$

$$\mathcal{T}=\frac{2n_-}{n_-+n_+}. \tag{10}$$

In particular, we see that the sign of the reflection coefficient depends on the relative magnitude of the refractive index on the side of the incoming field.