The False Alarm Rate

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Here we consider the False Alarm Rate (FAR) and related concepts. The FAR is used to define the statistical significance of detected gravitational–wave events.

- The False Alarm Rate (FAR) is the rate of false detections. It is a rate of coincidences similar to a detected one that would appear as statistical fluctuations. The FAR is measured as the number of such detections over time, so its units are number/year or number/second (or any other practical time unit)
- The inverse FAR is defined as $\mathrm{iFAR} = 1/\mathrm{FAR}$. The iFAR has units of time.
- The False Alarm Probability (FAP) is the probability of false detections. With an observation time T, the mean value of false detections is $\mathrm{FAR} \times T$, therefore the (Poisson) probability of no false detection during this time is $e^{-\mathrm{FAR}\times T}$, and finally the probability of at least one false detection is $\mathrm{FAP} = 1 - e^{-\mathrm{FAR}\times T}$.
- If the number of false detections *corresponding to a given FAR value* is $N_0 = \mathrm{FAR}_0 \times T$, then the iFAR is $\mathrm{iFAR}_0 = T/N_0$, and therefore

$$\mathrm{iFAR} = \frac{T}{N} = \mathrm{iFAR}_0 \frac{N_0}{N}. \tag{1}$$

  This means that a log–log plot of iFAR vs. N is a straight line with slope -1. The fluctuation of N can be modeled with a Poisson distribution. An example plot is shown in figure 1. Note that this is not a cumulative plot.

Equation (1) can be put into a cumulative form as follows:

$$N(\mathrm{FAR}) = T\times\mathrm{FAR} \quad \Rightarrow \quad N(\mathrm{FAR} \le \mathrm{FAR}_0) \approx \int_0^{N(\mathrm{FAR}_0)} N' dN' = \frac{1}{2} N^2(\mathrm{FAR}_0) = \frac{T^2/2}{\mathrm{iFAR}_0^2} \tag{2}$$

and therefore

$$N(\mathrm{iFAR} > \mathrm{iFAR}_0) \approx \frac{T^2/2}{\mathrm{iFAR}_0^2}. \tag{3}$$

which, in a log-log plot is again a straight line, with slope -2.

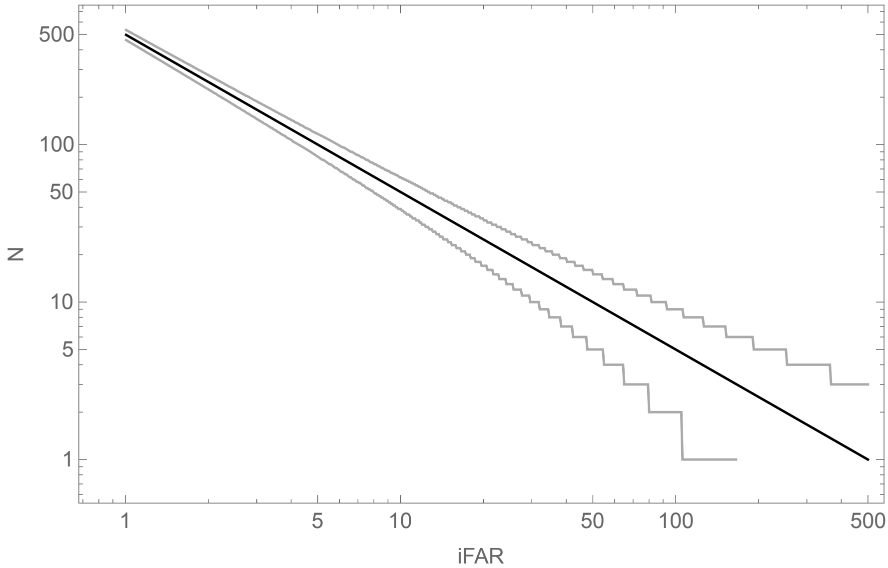

Figure 1: Differential (i.e., not cumulative) plot of the number of events vs. iFAR, with $\mathrm{iFAR}_0 \times N_0 = 500$. The gray curves delimit the 90% probability band.

The cumulative plot is the sum of Poisson processes with different rates. Is this sum still a Poisson process? Yes, it is, we see this by recalling that the characteristic function of the Poisson distribution with average a is $F(z) = e^{(z-1)a}$, thefore the sum of two Poisson processes with averages a and b has the characteristic function

$$F(z) = e^{(z-1)a} e^{(z-1)b} = e^{(z-1)(a+b)} \tag{4}$$

which is again a Poisson process with average $a + b$.