The Role of Gravitation in Physics

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Chapter 14
Measurement of Classical Gravitation Fields
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Because of the principle of equivalence, one cannot ascribe a direct physical interpretation to the gravitational field insofar as it is characterized by Christoffel symbols $\Gamma^{\mu}_{\nu\rho}$. One can, however, give an invariant interpretation to the variations of the gravitational field. These variations are described by the Riemann tensor; therefore, measurements of the relative acceleration of neighboring free particles, which yield information about the variation of the field, will also yield information about the Riemann tensor.

Now the relative motion of free particles is given by the equation of geodesic deviation

$$\frac{\partial^2 \eta^\mu}{\partial \tau^2} + R^{\mu}_{\nu\rho\sigma} v^\nu \eta^\rho v^\sigma = 0 \quad (\mu, \nu, \rho, \sigma = 1, 2, 3, 4) \tag{14.1}$$

Here η^μ is the infinitesimal orthogonal displacement from the (geodesic) worldline ζ of a free particle to that of a neighboring similar particle. v^ν is the 4-velocity of the first particle, and τ the proper time along ζ. If now one introduces an orthonormal frame on ζ, v^μ being the timelike vector of the frame, and assumes that the frame is parallelly propagated along ζ (which insures that an observer using this frame will see things in as Newtonian a way as possible) then the equation of geodesic deviation (14.1) becomes

$$\frac{\partial^2 \eta^a}{\partial \tau^2} + R^{a}_{0b0} \eta^b = 0 \quad (a, b = 1, 2, 3,) \tag{14.2}$$

Here η^a are the physical components of the infinitesimal displacement and R^a_{0b0} some of the physical components of the Riemann tensor, referred to the orthonormal frame.

By measurements of the relative accelerations of several different pairs of particles, one may obtain full details about the Riemann tensor. One can thus very easily imagine an experiment for measuring the physical components of the Riemann tensor.

Now the Newtonian equation corresponding to (14.2) is

$$\frac{\partial^2 \eta^a}{\partial \tau^2} + \frac{\partial^2 v}{\partial x^a \partial x^b} \eta^b = 0 \qquad (14.3)$$

It is interesting that the empty-space field equations in the Newtonian and general relativity theories take the same form when one recognizes the correspondence $R^a_{0b0} \sim \frac{\partial^2 v}{\partial x^a \partial x^b}$ between equations (14.2) and (14.3), for the respective empty-space equations may be written $R^a_{0a0} = 0$ and $\frac{\partial^2 v}{\partial x^a \partial x^b} = 0$. (Details of this work are in the course of publication in *Acta Physica Polonica.*)

BONDI: Can one construct in this way an absorber for gravitational energy by inserting a $\frac{d\eta}{d\tau}$ term, to learn what part of the Riemann tensor would be the energy producing one, because it is that part that we want to isolate to study gravitational waves?

PIRANI: I have not put in an absorption term, but I have put in a "spring." You can invent a system with such a term quite easily.

LICHNEROWICZ: Is it possible to study stability problems for η?

PIRANI: It is the same as the stability problem in classical mechanics, but I haven't tried to see for which kind of Riemann tensor it would blow up.

Chapter 27
An Expanded Version of the Remarks by R.P. Feynman on the Reality of Gravitational Waves[1]

I think it is easy to see that if gravitational waves can be created they can carry energy and can do work. Suppose we have a transverse-transverse wave generated by impinging on two masses close together. Let one mass A carry a stick which runs past touching the other B. I think I can show that the second in accelerating up and down will rub the stick, and therefore by friction make heat. I use coordinates physically natural to A, that is so at A there is flat space and no field (what are they called, "natural coordinates"?). Then Pirani at an earlier section gave an equation for the notion of a nearby particle, vector distance η from origin A, it went like, to 1^{st} order in η

$$\ddot{\eta}^a + R^a_{0b0}\eta^b = 0 \quad (a,b = 1,2,3)$$

R is the curvature tensor calculated at A. Now we can figure R directly, it is not reasonable by coordinate transformation for it is the real curvature. It does not vanish for the transverse-transverse gravity wave but oscillates as the wave goes by. So, η on the RHS is sensibly constant, so the equation says the particle vibrates up and down a little (with amplitude proportional to how far it is from A on the average, and to the wave amplitude.) Hence it rubs the stick, and generates heat.

I heard the objection that maybe the gravity field makes the stick expand and contract too in such a way that there is no relative motion of particle and stick. But this cannot be. Since the amplitude of B's motion is proportional to the distance from A, to compensate it the stick would have to stretch and shorten by certain ratios of its own length. Yet at the center it does no such thing, for it is in natural metric - and that means that the lengths determined by size of atoms etc. are correct and unchanging at the origin. In fact that is the definition of our coordinate system. Gravity does produce strains in the rod, but these are zero at the center for g and its gradients are zero there. I think: any changes in rod lengths would go at least as η^3 and not as η so surely the masses would rub the rod.

Incidentally masses put on opposite side of A go in opposite directions. If I use 4 weights in a cross, the motions at a given phase are as in the figure:

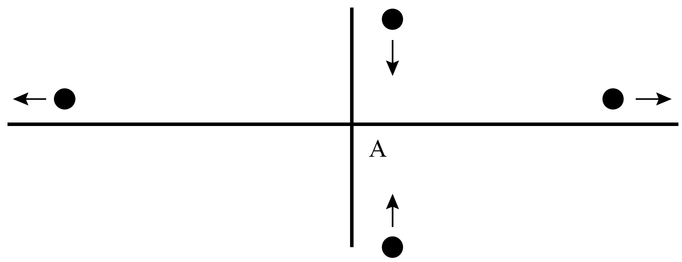

Figure 27.1

Thus a quadrupole moment is generated by the wave.

Now the question is whether such a wave can be generated in the first place. First since it is a solution of the equations (approx.) it can probably be made. Second, when I tried to analyze from the field equations just what happens if we drive 4 masses in a quadrupole motion of masses like the figure above would do - even including the stress-energy tensor of the machinery which drives the weights, it was very hard to see how one could avoid having a quadrupole source and generate waves. Third my instinct is that a device which could draw energy out of a wave acting on it, must if driven in the corresponding motion be able to create waves of the same kind. The reason for this is the following: If a wave impinges on our "absorber" and generates energy - another "absorber" place in the wave behind the first must absorb less because of the presence of the first, (otherwise by using enough absorbers we could draw unlimited energy from the waves). That is, if energy is absorbed the wave must get weaker. How is this accomplished? Ordinarily through interference. To absorb, the absorber parts must move, and *in moving generate a wave* which interferes with the original wave in the so-called forward scattering direction, thus reducing the intensity for a subsequent absorber. In view therefore of the detailed analysis showing that gravity waves can generate heat (and therefore carry energy proportional to R^2 with a coefficient which can be determined from the forward scattering argument). I conclude also that these waves can be generated and are in every respect real.