Gravitational Waves course: The key principles of GR

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Einstein formulated three key principles which are at the very basis of GR

- Principle of general covariance
- Principle of consistency
- Principle of equivalence

Principle of general covariance

One of the postulates of special relativity is the principle of relativity, i.e., that the laws of physics are the same in any inertial frame of reference.

The principle of general covariance extends that requirement to say that the form of the laws of physics should be the same in all – inertial and accelerating – frames.

In other words, physical phenomena shouldn't depend on the choice of coordinate systems used to describe them, and therefore all frames are equally valid.

To paraphrase Foster and Nightingale:

A physical equation of general relativity is generally true in all coordinate systems if (a) the equation is a tensor equation (i.e., it preserves its form under general coordinate transformations) (b) the equation is true in special relativity.

Taken together, these two conditions mean that if we have a valid tensor equation that is true in special relativity we can, with a little bit of twiddling, transform it into an equation that is true in general relativity.

Principle of consistency

The **principle of consistency** requires that a new scientific theory must be able to account for the successful predictions of the old theories it replaces.

This means that, given the appropriate conditions, general relativity should reduce to both the laws of Newtonian mechanics and, in the absence of gravity, to the formulations of special relativity.

Example of consistency, the general-relativistic continuity equation

$$rac{\partial n}{\partial t} +
abla \cdot (n\mathbf{v}) = 0$$
 $n = n(t, \mathbf{x})$ number density in the rest frame

$$\Rightarrow \quad \frac{\partial nc\gamma}{\partial (ct)} + \nabla \cdot (n\gamma \mathbf{v})) = 0$$

$$\Rightarrow \quad \partial_{\alpha}(nU^{\alpha}) = 0 \quad \Rightarrow \quad \nabla_{\alpha}(nU^{\alpha}) = \nabla_{\alpha}(J^{\alpha})$$
SR
GR

Principle of equivalence

(Text from C. Will, "The Confrontation between General Relativity and Experiment", *Living Rev. Relativ.* **17**, 4 (2014). <u>https://doi.org/10.12942/lrr-2014-4</u>)

The principle of equivalence has historically played an important role in the development of gravitation theory. Newton regarded this principle as such a cornerstone of mechanics that he devoted the opening paragraph of the Principia to it.

In 1907, Einstein used the principle as a basic element in his development of general relativity (GR). We now regard the principle of equivalence as the foundation, not of Newtonian gravity or of GR, but of the broader idea that spacetime is curved. Much of this viewpoint can be traced back to Robert Dicke, who contributed crucial ideas about the foundations of gravitation theory between 1960 and 1965. These ideas were summarized in his influential Les Houches lectures of 1964, and resulted in what has come to be called the Einstein equivalence principle (EEP).

One elementary equivalence principle is the kind Newton had in mind when he stated that the property of a body called "mass" is proportional to the "weight", and is known as the weak equivalence principle (WEP).

An alternative statement of WEP is that the trajectory of a freely falling "test" body (one not acted upon by such forces as electromagnetism and too small to be affected by tidal gravitational forces) is independent of its internal structure and composition.

In the simplest case of dropping two different bodies in a gravitational field, WEP states that the bodies fall with the same acceleration (this is often termed the Universality of Free Fall, or UFF).

The Einstein equivalence principle (EEP) is a more powerful and far-reaching concept; it states that:

- 1. WEP is valid.
- 2. The outcome of any local non-gravitational experiment is independent of the velocity of the freely-falling reference frame in which it is performed.
- 3. The outcome of any local non-gravitational experiment is independent of where and when in the universe it is performed.

The second piece of EEP is called *local Lorentz invariance* (LLI), and the third piece is called *local position invariance* (LPI).

For example, a measurement of the electric force between two charged bodies is a local non-gravitational experiment; a measurement of the gravitational force between two bodies (Cavendish experiment) is not.

The Einstein equivalence principle is the heart and soul of gravitational theory, for it is possible to argue convincingly that if EEP is valid, then gravitation must be a "curved spacetime" phenomenon, in other words, the effects of gravity must be equivalent to the effects of living in a curved spacetime.

As a consequence of this argument, the only theories of gravity that can fully embody EEP are those that satisfy the postulates of "metric theories of gravity", which are:

- 1. Spacetime is endowed with a symmetric metric.
- 2. The trajectories of freely falling test bodies are geodesics of that metric.
- 3. In local freely falling reference frames, the non-gravitational laws of physics are those written in the language of special relativity.

The argument that leads to this conclusion simply notes that, if EEP is valid, then in local freely falling frames, the laws governing experiments must be independent of the velocity of the frame (local Lorentz invariance), with constant values for the various atomic constants (in order to be independent of location).

The only laws we know of that fulfill this are those that are compatible with special relativity, such as Maxwell's equations of electromagnetism, and the standard model of particle physics.

Furthermore, in local freely falling frames, test bodies appear to be unaccelerated, in other words they move on straight lines; but such "locally straight" lines simply correspond to "geodesics" in a curved spacetime.

General relativity is a metric theory of gravity, but then so are many others, including the Brans–Dicke theory and its generalizations.

Theories in which varying non-gravitational constants are associated with dynamical fields that couple to matter directly are not metric theories. Neither, in this narrow sense, is superstring theory, which, while based fundamentally on a spacetime metric, introduces additional fields (dilatons, moduli) that can couple to material stress-energy in a way that can lead to violations, say, of WEP.

It is important to point out, however, that there is some ambiguity in whether one treats such fields as EEP-violating gravitational fields, or simply as additional matter fields, like those that carry electromagnetism or the weak interactions.

Still, the notion of curved spacetime is a very general and fundamental one, and therefore it is important to test the various aspects of the Einstein equivalence principle thoroughly. ...

"Fundamental Ideas and Methods of the Theory of Relativity, Presented in Their Development"

Albert Einstein (after 22 January 1920) https://einsteinpapers.press.princeton.edu/vol7-trans/129

15. The Basic Idea of the Theory of General Relativity in Its Original Form

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When I was busy (in 1907) writing a summary of my work on the theory of special relativity for the Jahrbuch für Radioaktivität und Elektronik [Yearbook for Radioactivity and Electronics], I also had to try to modify the Newtonian theory of gravitation such as to fit its laws into the theory. While attempts in this direction showed the practicability of this enterprise, they did not satisfy me because they would have had to be based upon unfounded physical hypotheses. At that moment I got the happiest thought of my life in the following form:

In an example worth considering, the gravitational field has a relative existence only in a manner similar to the electric field generated by magneto-electric induction. Because for an observer in free-fall from the roof of a house there is during the fall—at least in his immediate vicinity—no gravitational field. Namely, if the observer lets go of any bodies, they remain relative to him, in a state of rest or uniform motion, independent of their special chemical or physical nature. The observer, therefore, is justified in interpreting his state as being "at rest."

The extremely strange and confirmed experience that all bodies in the same gravitational field fall with the same acceleration immediately attains, through this idea, a deep physical meaning. Because if there were just one single thing to fall in a gravitational field in a manner different from all others, the observer could recognize from it that he is in a gravitational field and that he is falling. But if such a thing does not exist—as experience has shown with high precision—then there is no objective reason for the observer to consider himself as falling in a gravitational field. To the contrary, he has every right to consider himself in a state of rest and his vicinity as free of fields as far as gravitation is concerned.

The experimental fact that the acceleration in free-fall is independent of the material, therefore, is a powerful argument in favor of expanding the postulate of relativity to coordinate systems moving nonuniformly relative to each other.

On the other hand, one can also start with a space that has no gravitational field. A material point in this space, when sufficiently distant from other masses, behaves free of acceleration relative to an inertial system K. However, if one introduces a uniformly accelerated coordinate system K' relative to K (uniformly accelerated parallel translation), then K' is no inertial system in the sense of classical mechanics or the theory of special relativity. Every mass point sufficiently distant from others is uniformly accelerated relative to K'. When seen from K, the acceleration of the system K' is of course the cause of the relative acceleration of the mass point relative to K' and on the basis of classical mechanics, as understood up to the present day, it is the only possible interpretation.

However, we can also view as K' an admissible system ("at rest") and attribute the acceleration of masses relative to K' to a static gravitational field that fills the entire space that is under consideration. This interpretation again is possible based upon the experimental fact that in a gravitational field (such as that relative to K') all bodies fall in the same manner.

If we know the laws of nature with respect to a (gravitation-free) system K, then we can by mere transformation learn the laws relative to K', i.e., we learn about the physical properties of a gravitational field by means of a purely speculative method. At its basis here is the hypothesis that the principle of relativity also holds in reference to coordinate systems that are mutually accelerated to each other, and that the physical properties of space that rule relative to K' are completely equivalent to a gravitational field (hypothesis of equivalence).

The generalization of the principle of relativity, therefore, points to a speculative way of investigating the properties of the gravitational field.

Because all bodies in a gravitational field have the same fall, a stimulus arose that pointed with irresistible force toward a generalization of the principle of relativity. (Consequently, it is necessary to point out that this result (of the equivalence hypothesis) is supported with extraordinary precision, in particular by the tests made by Eötvös. This is based upon the following consideration.)

This experimental fact can also be phrased in a second especially remarkable form. According to Newton's law of motion, the fall of a body occurs according to the equation

(inertial mass) X (acceleration in fall) = (gravitational force of the earth).

On the other hand,

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(gravitational force of the earth) = (intensity of the gravitational field) X (gravitational mass).
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In these equations "inertial mass" means the mass that is responsible for the inertial reaction of the body, "gravitational mass" is the constant responsible for the influence of the gravitational field on the same body—two constants which by definition are completely independent of each other. From both equations together follows

(inertial mass) X (acceleration in fall) = (gravitational mass) X (intensity of the gravitational field).

In order to keep the experimentally confirmed law

(acceleration in fall) = (intensity of the gravitational field)

valid, it must also be true that

inertial mass = gravitational mass.

The experimental fact of the same fall of all bodies therefore can, in the spirit of Newtonian mechanics, also be viewed as the equality of the inertial and gravitational mass, which from the point of view of Newtonian mechanics is by no means self-evident.

This theorem has been confirmed with extraordinary precision by the tests of Eötvös, which are based upon the following. A body on the surface of the earth is under the influence of the gravitational force of the earth and of the centrifugal force of the earth's rotation. The first force is proportional to the gravitational mass, the latter to the inertial mass. The resultant of both forces is independent of the material only if the ratio of inertial and gravitational mass is independent of the material. Eötvös attached masses of different material to the ends of the horizontal balance beam of a torsion scale. In case of an incomplete proportionality of inertial and gravitational mass, the resulting forces acting upon the two masses could not be exactly parallel; i.e., there should have been a torsion moment acting upon the system when the balance beam was oriented in the east-west direction. The negative outcome was registered with such precision that the relative difference between inertial and gravitational mass had to be smaller than 10⁻⁷.



Image Credit: NASA

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At the end of the last Apollo 15 moon walk, Commander David Scott (pictured above) performed a live demonstration for the television cameras. He held out a geologic hammer and a feather and dropped them at the same time. Because they were essentially in a vacuum, there was no air resistance and the feather fell at the same rate as the hammer, as Galileo had concluded hundreds of years before - all objects released together fall at the same rate regardless of mass. Mission Controller Joe Allen described the demonstration in the "Apollo 15 Preliminary Science Report":

"During the final minutes of the third extravehicular activity, a short demonstration experiment was conducted. A heavy object (a 1.32-kg aluminum geological hammer) and a light object (a 0.03-kg falcon feather) were released simultaneously from approximately the same height (approximately 1.6 m) and were allowed to fall to the surface. Within the accuracy of the simultaneous release, the objects were observed to undergo the same acceleration and strike the lunar surface simultaneously, which was a result predicted by well-established theory, but a result nonetheless reassuring considering both the number of viewers that witnessed the experiment and the fact that the homeward journey was based critically on the validity of the particular theory being tested." - Joe Allen, NASA SP-289, Apollo 15 Preliminary Science Report, Summary of Scientific Results, p. 2-11

From <u>https://moon.nasa.gov/resources/331/the-apollo-15-hammer-feather-drop/</u>



EARLY GRAVITY EXPERIMENTS were performed by dropping weights (Galileo's famous experiment was not the first of its kind) or by observing the period of pendulums. Two pendulums of the same length should swing in synchrony regardless of the nature of the suspended masses. Isaac Newton observed the periods of pendulums supporting many different substances to show that all reacted in the same fashion to the force of gravitation. In the diagrams the broken lines inside the colored circles indicate that a dense substance, such as lead, has been hollowed out to make it equal in mass to a lighter substance (gray circles).

The Eötvös experiment





EÖTVÖS EXPERIMENT, performed in two series, first in 1889 and again in 1908, demonstrated the constancy of gravitational acceleration with great accuracy. Eötvös observed the effect on a torsion balance when two weights affixed to a beam were acted on simultaneously by two forces: the gravitational force of the earth and the centrifugal force created by the earth's rotation. Conceivably two masses of different composition might react differently to these two forces and produce a torque resulting in a slight rotation of the balance. By observing the balance in different orientations Eötvös verified that no significant rotation took place.



FIG. 1. Reproduction of a drawing of a single torsion balance used by Eötvös for some of his measurements (11). The scale below the drawing is one meter in length.

The Dicke experiment



NEW TEST OF GRAVITATION, being conducted at Princeton University, attempts to learn whether or not there is any discrepancy in the rate at which masses of different composition fall toward the sun. In the idealized form of the experiment diagramed, two weights are suspended at the North Pole and revolve with the earth as it turns on its axis. At "6 a.m." (*left*), when the beam is perpendicular to a line drawn to the sun, the colored weight is being carried toward the sun by the earth's rotation. Twelve hours later (*right*) the same weight is being carried away from the sun. As a consequence, if the colored weight should tend to fall toward the sun faster than the other, it will alternately make the beam rotate slightly faster than the earth for 12 hours and then slightly slower for the next 12 hours. This hypothetical positive result is illustrated below.



HYPOTHETICAL POSITIVE RESULT of the author's experiment would lead to the asymmetry diagramed. The suspended weights, as viewed from above, are carried around in a full circle every 24 hours by the earth's rotation. If both weights experience the same gravitational acceleration toward the sun, their rate of rotation will be absolutely uniform. If, however, one weight (*color*) should tend to fall slightly faster, it would speed up the rotation rate while it was being carried toward the sun (e.g., at "6 a.m.") and slow it down while being carried away from the sun (e.g., at "6 p.m."). In actual fact, no asymmetry has been observed in the rotation.



THE PRINCETON APPARATUS uses an electrooptical system to monitor and record any slight rotation in the suspended triangle, whether due to gravitation or some other disturbing force. One leg of the triangle is silvered so that it serves as a mirror in the optical system. A slit placed in the beam of a flashlight bulb is reflected by the mirror and is brought to a focus on a wire oscillating at

3,000 cycles per second. Because of this oscillation the intensity of the light striking the photocell varies with time. If the mirror turns slightly, the signal from the photocell changes and gives rise to a direct-current voltage, which, applied to the electrodes, exerts a restoring force on one of the copper weights. The magnitude of this force is logged continuously on a strip recorder.



FIG. 7. Details of the oscillating wire light modulator. (a) Top view of the oscillating wire device, showing the magnet and pole piece assembly, prism, and light pipe. (b) Side view, showing the method of mounting the oscillating wire between the pole pieces. (c) Block diagram of the balanced bridge oscillator which drives the oscillating wire. (d) Sketch of the diffraction image of the slit focused and centered on the equilibrium position of the oscillating wire. As the wire oscillates about the position illustrated, the light received by the photomultiplier is modulated at the second harmonic of the wire frequency. Only when the diffraction image shifts off-center from the equilibrium position of the wire is the fundamental wire frequency detected by the photomultiplier. (e) Calculated fractional light intensity received by the photomultiplier as a function of displacement of the diffraction image of the slit from the center of the wire.

The MICROSCOPE space mission

At the core of the satellite there are two pairs of nested cylinders. In one pair, both cylinders are made of the same material, an alloy of the heavy metals platinum and rhodium. In the other pair, the inner cylinder is again platinum–rhodium, while the outer one is an alloy of light metals: titanium, aluminum and vanadium.

Microscope is orbiting in Earth's gravitational pull. According to the equivalence principle, all four cylinders should feel the same force regardless of the density of the metals, and therefore they should move along the same orbit.

If, however, the experiment were to measure tiny differences in the way that the two pairs of cylinders move, this could point to a breakdown of the principle and call for a revision of our current theory of gravity.

As they orbit Earth inside the satellite, the relative positions of the cylinders is measured by electrostatic sensors.

To make such precise measurements in space, the satellite must be extremely still with respect to the cylinders floating freely within it. To achieve this, Microscope will fire micro thrusters to compensate for tiny disturbances to its trajectory caused by the pressure of sunlight or impacts of micrometeoroids, for example.



The MicroSCOPE (Micro-Satellite à traînée Compensée pour l'Observation du Principe d'Equivalence) mission, a test of the perfect proportionality between the inertial mass and the gravitational mass of a body





FIG. 1. Left: The four test mass orbiting around Earth (credit CNES, Virtual-IT 2017). Right: Test masses and satellite frames; the $(X_{sat}, Y_{sat}, Z_{sat})$ triad defines the satellite frame; the reference frames $(X_k, Y_k, Z_k, k = 1, 2)$ are attached to the test masses (black for the inner mass k = 1, red for the outer mass k = 2); the X_k axes are the test-mass cylinders' longitudinal axis and define the direction of WEP test measurement; the Y_k axes are normal to the orbital plane, and define the rotation axis when the satellite spins; the Z_k axes complete the triads. The 7 μ m gold wires connecting the test masses to the common Invar sole plate are shown as yellow lines. $\vec{\Delta}$ represents the test-mass off-centering. The centers of mass correspond to the origins of the sensor-cage-attached reference frames.



Figure 2. MICRO-SCOPE's measurement principle. A WEP violation is detected if the two cylindrical test-masses experience different accelerations (red arrows) as the satellite orbits the Earth; the difference in those accelerations is measured by the difference in the voltages applied to the test-masses to keep them in equilibrium. Black arrows show the sensitive axis along which a WEP violation is looked for.

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Editors' Suggestion Featured in Physics

MICROSCOPE Mission: Final Results of the Test of the Equivalence Principle

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The *MICROSCOPE* mission was designed to test the weak equivalence principle (WEP), stating the equality between the inertial and the gravitational masses, with a precision of 10^{-15} in terms of the Eötvös ratio η . Its experimental test consisted of comparing the accelerations undergone by two collocated test masses of different compositions as they orbited the Earth, by measuring the electrostatic forces required to keep them in equilibrium. This was done with ultrasensitive differential electrostatic accelerometers onboard a drag-free satellite. The mission lasted two and a half years, cumulating five months worth of science free-fall data, two-thirds with a pair of test masses of different composition—platinum. We summarize the data analysis, with an emphasis on the characterization of the systematic uncertainties due to thermal instabilities and on the correction of short-lived events which could mimic a WEP violation signal. We found no violation of the WEP, with the Eötvös parameter of the titanium and platinum pair constrained to $\eta(\text{Ti}, \text{Pt}) = [-1.5 \pm 2.3(\text{stat}) \pm 1.5(\text{syst})] \times 10^{-15}$ at 1σ in statistical errors.

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THE EQUIVALENCE PRINCIPLE UNDER A MICROSCOPE



PHYSICS HAS TWO KINDS OF MASS:



WEP SAYS THESE TWO TYPES OF MASS ARE EQUIVALENT--A RADICAL IDEA, SINCE IT IMPLIES A SPECIAL RELATION BETWEEN GRAVITY AND MASS THAT OTHER FORCES DON'T HAVE,







Physics magazine



Eötvös parameter

$$\eta(2,1) = 2\frac{a_2 - a_1}{a_2 + a_1} = 2\frac{m_{g2}/m_{i2} - m_{g1}/m_{i1}}{m_{g2}/m_{i2} + m_{g1}/m_{i1}}$$

Figure 1. Tests of WEP throughout the 20th century. The arrow on the lower right corner shows the expectation for MICRO-SCOPE. Figure adapted from [14].

Tests of the WEP

Year	Investigator	Sensitivity	Method
500?	Philoponus ^[14]	"small"	Drop tower
1585	Stevin ^[15]	5×10 ⁻²	Drop tower
1590?	Galileo ^[16]	2×10 ⁻²	Pendulum, drop tower
1686	Newton ^[17]	10 ⁻³	Pendulum
1832	Bessel ^[18]	2×10 ⁻⁵	Pendulum
1908 (1922)	Eötvös ^[19]	2×10 ⁻⁹	Torsion balance
1910	Southerns ^[20]	5×10 ⁻⁶	Pendulum
1918	Zeeman ^[21]	3×10 ⁻⁸	Torsion balance
1923	Potter ^[22]	3×10 ⁻⁶	Pendulum
1935	Renner ^[23]	2×10 ⁻⁹	Torsion balance
1964	Dicke, Roll, Krotkov ^[10]	3x10 ⁻¹¹	Torsion balance
1972	Braginsky, Panov ^[24]	10 ⁻¹²	Torsion balance
1976	Shapiro, et al. ^[25]	10 ⁻¹²	Lunar laser ranging
1981	Keiser, Faller ^[26]	4×10 ⁻¹¹	Fluid support
1987	Niebauer, et al. ^[27]	10 ⁻¹⁰	Drop tower
1989	Stubbs, et al. ^[28]	10 ⁻¹¹	Torsion balance
1990	Adelberger, Eric G.; et al. ^[29]	10 ⁻¹²	Torsion balance
1999	Baessler, et al. ^{[30][31]}	5x10 ⁻¹⁴	Torsion balance
2017	MICROSCOPE ^[32]	10 ⁻¹⁵	Earth orbit