Misura della carica elementare per mezzo dello shot noise

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Corso di Metodi di Trattamento del Segnale

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Formule per lo shot noise

$$\langle |I(f)|^2 \rangle = 2eI_0$$

Integrazione sulla banda di frequenza B, assumendo un vero rumore bianco

$$\langle I^2 \rangle = \langle |I(f)|^2 \rangle B = 2eI_0 B$$

$$\langle V^2 \rangle = R^2 \langle |I(f)|^2 \rangle B = 2eI_0 B R^2$$

Calcolando la fluttuazione totale è dunque possibile trovare il valore della carica elementare:

$$e = \frac{\langle V^2 \rangle}{2I_0 B R^2}$$

3. *Über spontane Stromschwankungen in verschiedenen Elektrizitätsleitern;*
von W. Schottky.

Durch Hintereinanderschalten von Glühkathodenverstärkern ist es in den letzten Jahren gelungen, Wechselströme von äußerst geringer Amplitude wahrnehmbar und meßbar zu machen. Viele technische Probleme haben dadurch eine ruckweise Förderung erfahren, aber auch dem Forscher scheint sich ein neues Gebiet zu erschließen; die Verstärkerschaltungen haben für elektrische Untersuchungen sicher dieselbe Bedeutung wie in der Optik das Mikroskop. Da sich bisher noch keine deutliche Grenze für die erreichbare Verstärkung gezeigt hat, konnte man hoffen, durch genügenden Schutz störungsfreie Aufstellung usw. hier sozusagen bis zum unendlich Kleinen vorzudringen; der Traum vom „Gras wachsen hören" stellte sich wieder einmal recht greifbar der Menschheit dar.

Absicht der folgenden Zeilen ist, gewisse unüberschreitbare Grenzen für die Verstärkung mit Glühkathoden- und Gasentladungsröhren nachzuweisen. Das erste unüberwindliche Hindernis ist merkwürdigerweise durch die Größe des Elementarquantums der Elektrizität gegeben. Die Wärmebewegung der Elektrizität bildet eine weitere Grenze; diese scheint aber in den meisten Fällen höher zu liegen als die andere. Doch schicken wir die Untersuchung dieser Erscheinung als der einfacheren und bekannteren unserer Hauptbetrachtung voraus.

W. Schottky, Ann. d. Phys. **57** (1918) 541

6. *Zur Berechnung und Beurteilung des Schroteffektes*

(Bemerkungen zu der Notiz von Herrn J. B. Johnson);

von W. Schottky.

Herr Johnson war so freundlich, mir seine kürzlich (Ann. d. Phys. 67. S. 154—156. 1922) veröffentlichte „Bemerkung zur Bestimmung des elektrischen Elementarquantums aus dem Schroteffekt“ im Manuskript zuzusenden. Es handelt sich dabei im wesentlichen um die Berichtigung eines Rechenfehlers, der mir in meiner ersten Arbeit über diesen Gegenstand (Ann. d. Phys. 57. 541—567. 1918) unterlaufen war. Da diese Korrektur, für die ich Herrn Johnson sehr dankbar bin, sowohl für meine früheren theoretischen Schlußfolgerungen wie für die aus Herrn Hartmanns experimentellen Untersuchungen gezogenen Schlüsse von fundamentaler Bedeutung ist, möchte ich hier zu einigen daran anknüpfenden Bemerkungen zu der veränderten Sachlage Stellung nehmen.

Zunächst noch ein paar Worte zu der mathematischen Seite des Problems. Die Berechnung des Integrals

$$S_1 = \int_0^\infty \frac{dx}{(1-x^2)^2 + r^2 x^2} \qquad \text{(I)}$$

durch Partialbruchzerlegung, wie sie Herr Johnson angibt, und wie sie auch meiner eigenen Rechnung zugrunde lag, ist offenbar nicht ganz leicht zu übersehen, und es werden nur die wenigsten Leser Zeit finden können, sich von der Richtigkeit des Resultats zu überzeugen. Es wird daher vielleicht nicht überflüssig sein, wenn ich noch eine andere ganz einfache Berechnungsweise dieses Integrals hierher setze, die mir mein Vater, F. Schottky, freundlichst mitteilte, ohne daß wir übrigens untersuchen konnten, ob es sich dabei um ein Novum, oder nicht vielmehr um eine den Spezialisten geläufige Umformung handelt.

Die Berechnung beruht darauf, daß man das Integral (I) mit einem anderen Integral

W. Schottky, Ann. d. Phys. **68** (1922) 157

Walter H. Schottky, fisico tedesco che ha lavorato nell'ambito della fisica dello stato solido e in elettronica, e i cui contributi vanno dalla fisica del rumore (shot noise) a diversi processi importanti in elettronica (effetto Schottky, barriera di Schottky, diodo Schottky)

(n. 23/7/1886 a Zurigo, Svizzera, m. 4/3/1976 a Pretzfeld, Germania Ovest)

DETERMINATION OF ELEMENTARY CHARGE E FROM MEASUREMENTS OF SHOT-EFFECT.

By A. W. Hull and N. H. Williams

Abstract

Probability fluctuations in thermionic emission (shot effect).—Schottky first pointed out that if the electrons evaporate independently of each other, probability fluctuations of current are to be expected. These fluctuations were observed and roughly measured by Hartmann. Measurements have now been made, using a much higher frequency (750×10^3) and a reliable arrangement for measuring the small r.m.s. voltages (of order 10^{-4}) due to the effect. The method involves the measurement of the alternating current excited in a tuned circuit by the probability variations in the electron current through a vacuum tube (radiotron, UV 199), the Schottky equation for the mean square current being $\overline{J^2}=i_0e/2RC$ where i_0 is the thermionic current, R and C the resistance and capacity of the tuned circuit and e the electronic charge. To measure the current, it is amplified by a known amount (using a special 4 stage amplifier) and is rectified so that its r.m.s. value may be determined with a d.c. meter. It is important that the rectifier be used only in the range in which it gives a current proportional to the square of the impressed voltage. The tuned circuit picks out a narrow band of frequencies present in the fluctuations, and since the amplification is not the same for frequencies slightly different from the resonance frequency, correction is made for this by a factor F, for which a mathematical expression is derived and values are obtained by integration or summation. Calibration of the amplifier is avoided by substituting, after each reading of the amplified and rectified shot-current, a measured pure sine voltage across the terminals of the tuned circuit, which is adjusted to give the same rectified current as the effect. Calling this v_1, the actual mean square shot voltage $\overline{v_0^2}=v_1^2/F$ and, $\overline{J^2}=Cv_1^2/LF$, where L is the inductance of tuned circuit. It was found that with currents limited by space charge, the observed effect might be only 20 per cent of the theoretical; but with currents limited only by temperature, the agreement was within one per cent. The effect may therefore be used to study the effect of space charge on electronic evaporation.

Determination of elementary charge e, by the shot effect.—Using temperature limited thermionic current, values of e were obtained which lie within two per cent of the mean, and this mean, 4.76×10^{-10} e.s.u., agrees closely with Millikan's value. The precision of these measurements can be greatly increased, so as perhaps to exceed that of the oil-drop measurements.

Hull and Williams, Phys. Rev. **25** (1925) 147

Shot-noise measurements of the electron charge: An undergraduate experiment

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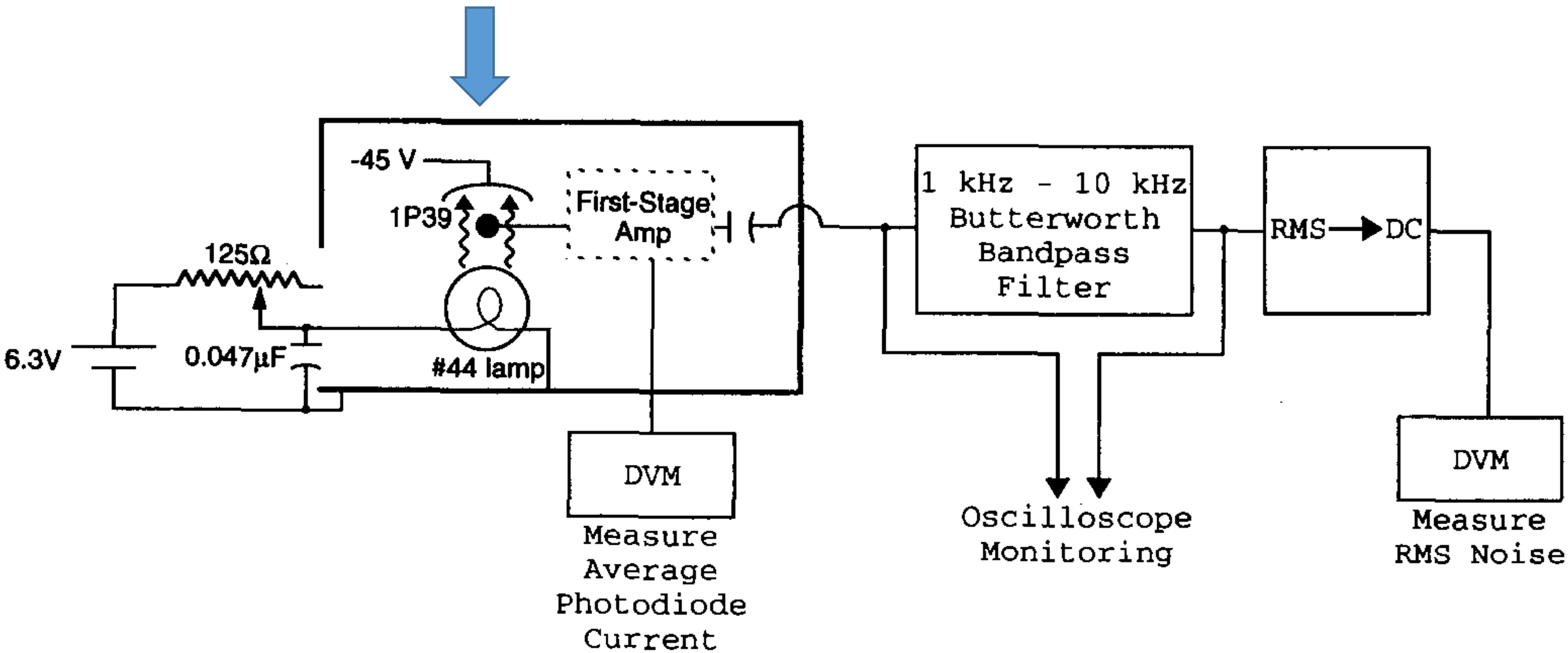
(Received 2 August 1994; accepted 12 October 1994)

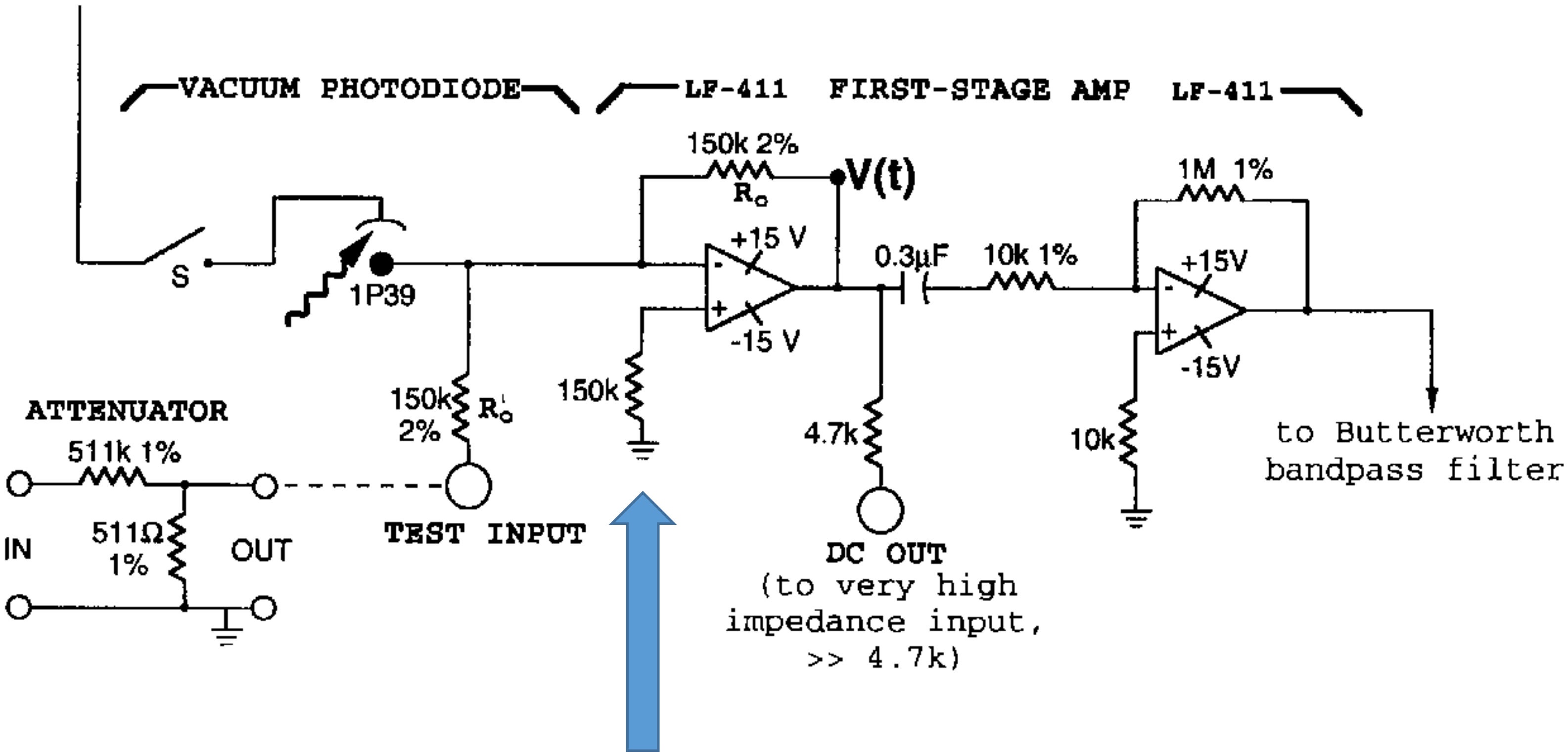
We report the design and performance of a simple apparatus for measurement of shot-noise fluctuations in the current from a vacuum photodiode illuminated with a pilot-lamp light bulb. After calibrating the frequency-dependent gain of the measurement electronics, the charge e of the electron can be obtained by measuring the mean-square shot noise as a function of the dc photodiode current. We employ op-amp circuits and a rms-to-dc integrated circuit to amplify, filter, and detect the shot noise, so that commercial bandpass filters and rms voltmeters are unnecessary. The apparatus is therefore not expensive and can be built using readily available components. Repeated measurements employing different pilot lamps yield a value of $e=(1.581\pm0.015\pm0.032)\times10^{-19}$ C, where the uncertainties represent random ($2\sigma_{\mathrm{mean}}$) and systematic error, respectively. The experiment thus permits reasonably precise measurements of a fundamental constant while allowing the undergraduate student to gain a hands-on understanding of fluctuation phenomena. © *1995 American Association of Physics Teachers.*

American Journal of Physics **63**, 554 (1995); doi: 10.1119/1.17867

Noi usiamo un fotodiodo
semiconduttore al posto
della fotocellula



Alimentazione
fotocellula

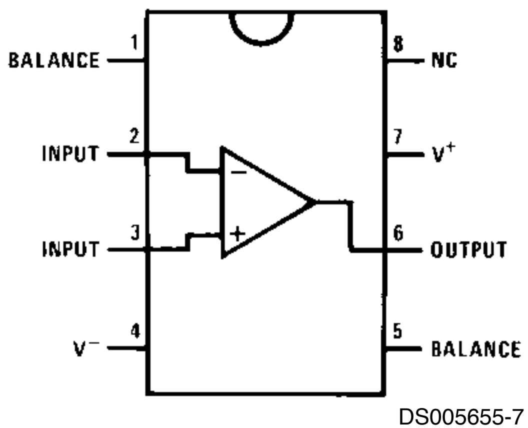


Questa resistenza non c’è nel nostro schema: se presente il circuito auto-oscilla a causa dell’alta capacità di transizione del fotodiodo

Nel circuito mostrato la resistenza da 150k che va a massa dall’input non-invertente è stata inserita per minimizzare il contributo del rumore in corrente dell’OP-AMP, tenendo conto di un’impedenza “infinita” della fotocellula. Questa ipotesi non è valida se c’è un fotodiodo.

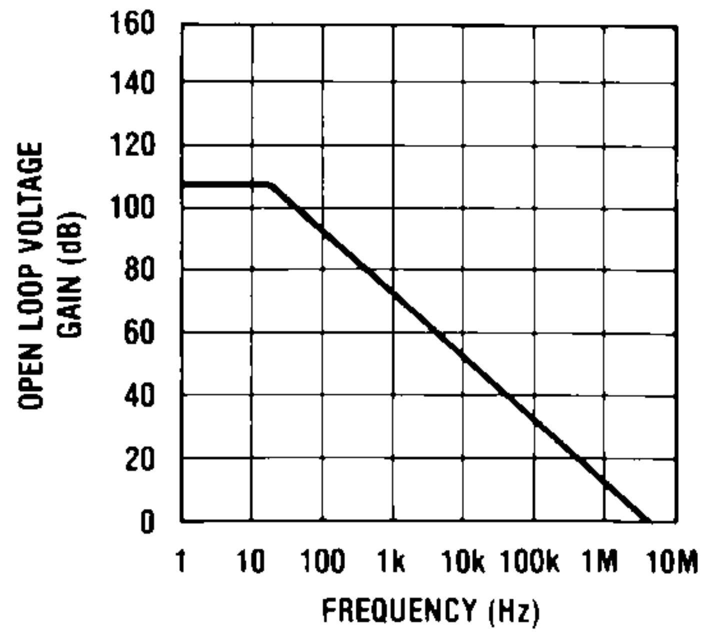
OP-AMP LF411

Dual-In-Line Package



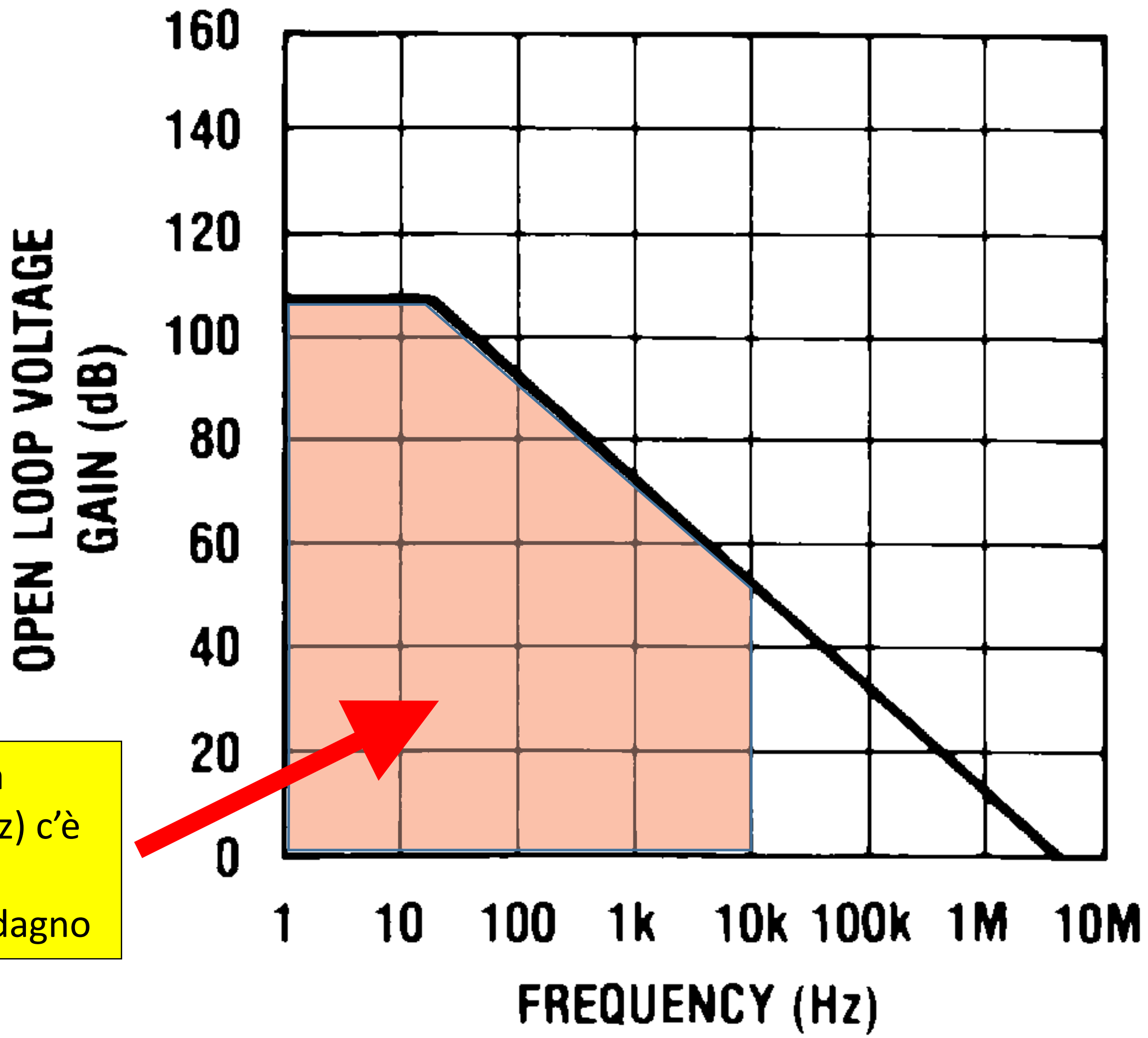
Top View
Order Number LF411ACN,
LF411CN or LF411MJ/883 (Note 1)

Open Loop Frequency Response



La scelta dell’LF411 non è ottimale. Ha una risposta in frequenza piuttosto scarsa, e ha il problema di un offset (posizione dello zero della tensione in uscita) che non è piccolo.

Open Loop Frequency Response



Nella regione a bassa frequenza (f < 10 kHz) c'è una significativa diminuzione del guadagno

Il filtro passa-banda (un filtro passa alto in cascata con un filtro passa-basso) limita la regione utile ad un intervallo ben definito

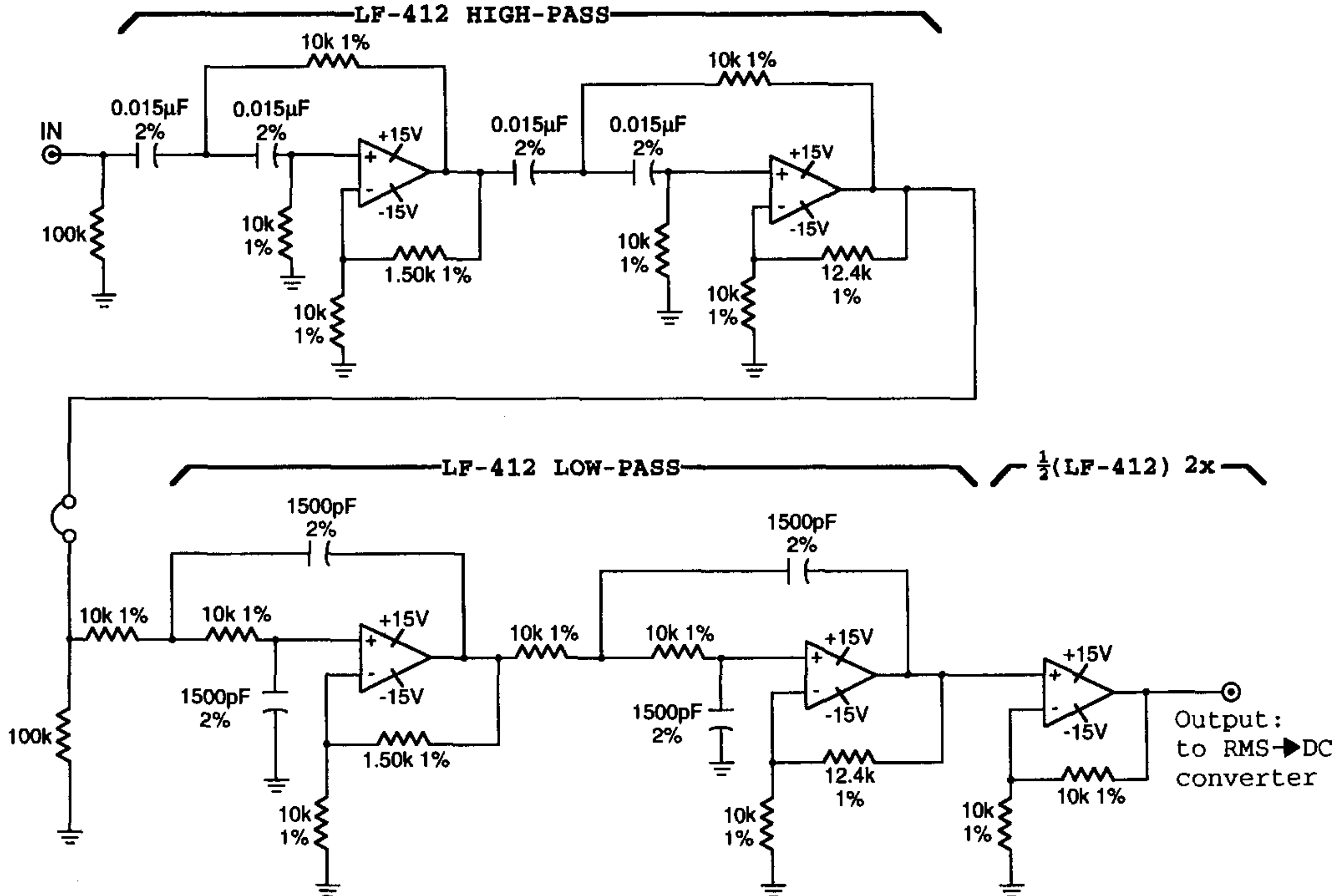


Fig. 4. Circuit schematic of the Butterworth bandpass filter designed to pass frequencies between 1 and 10 kHz. The high- and low-pass sections were constructed with dual LF-412 op-amps. The overall gain of the filter at 3 kHz, including the final 2× amplification, is about 13×.

La conversione a RMS viene realizzata dal circuito integrato AD637JQ

ANALOG DEVICES

High Precision, Wideband RMS-to-DC Converter

Data Sheet **AD637**

FEATURES

- High accuracy
  - 0.02% maximum nonlinearity, 0 V to 2 V rms input
  - 0.1% additional error to crest factor of 3
- Wide bandwidth
  - 8 MHz at 2 V rms input
  - 600 kHz at 100 mV rms
- Computes
  - True rms
  - Square
  - Mean square
  - Absolute value
- dB output (60 dB range)
- Chip select/power-down feature allows
  - Analog three-state operation
  - Quiescent current reduction from 2.2 mA to 350 µA

FUNCTIONAL BLOCK DIAGRAM

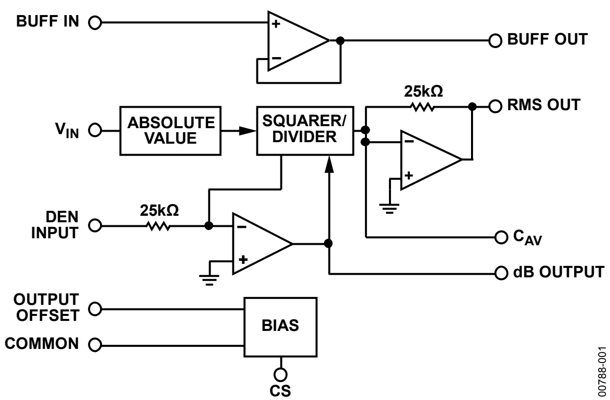


Figure 1.

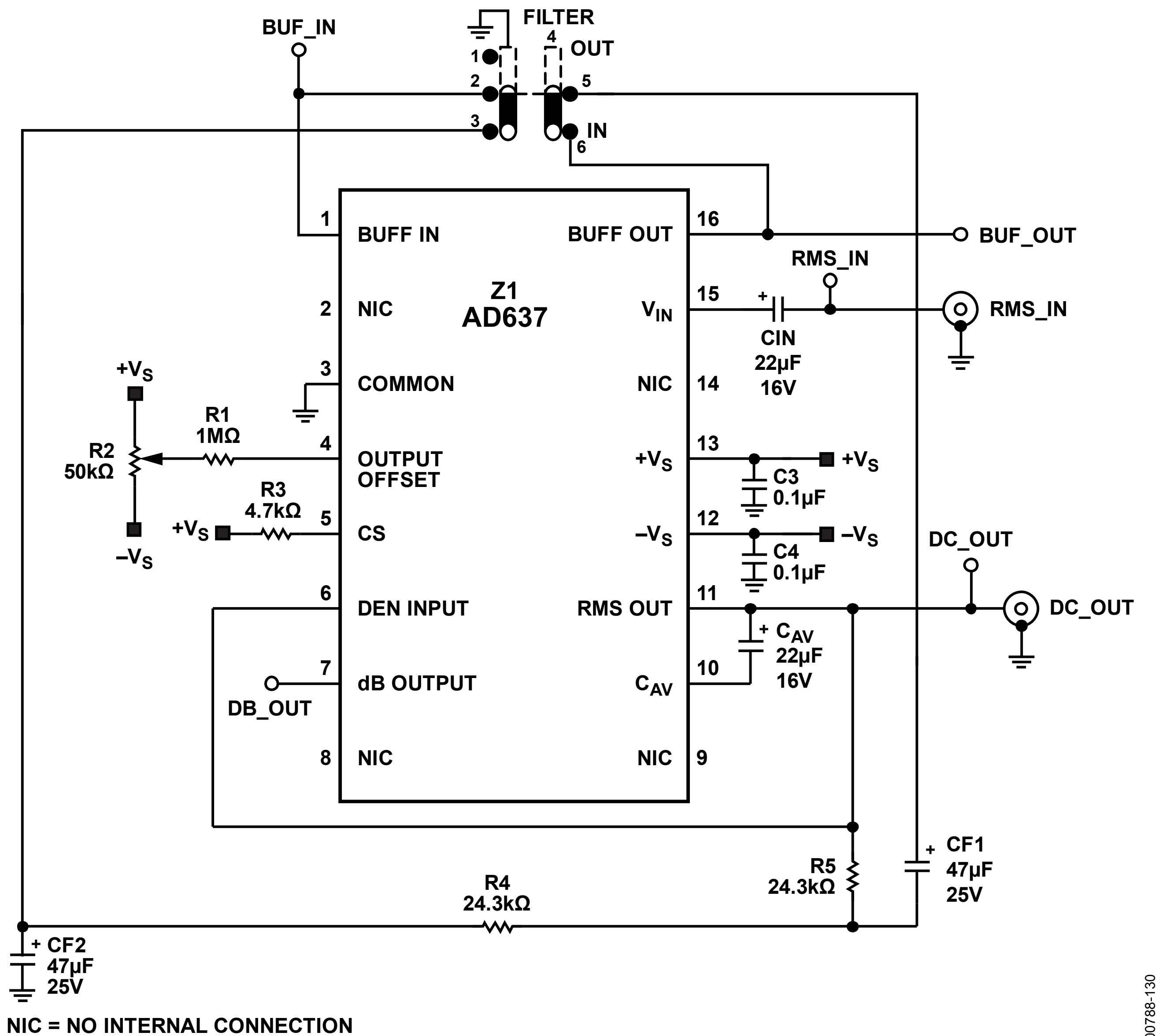


Figure 30. Evaluation Board Schematic

A causa del guadagno variabile in funzione della frequenza, si deve modificare la formula per la carica elementare

$$I = I_0 + \int_{f>0} I(f) e^{2\pi i f t} dt$$

- Trasformata di Fourier della corrente

$$V_0 = R_0 I_0$$

- Amplificazione DC in transimpedenza del sistema (R0 guadagno del primo stadio)

$$V(f) = R_0 I(f)$$

- Amplificazione in transimpedenza delle componenti di Fourier

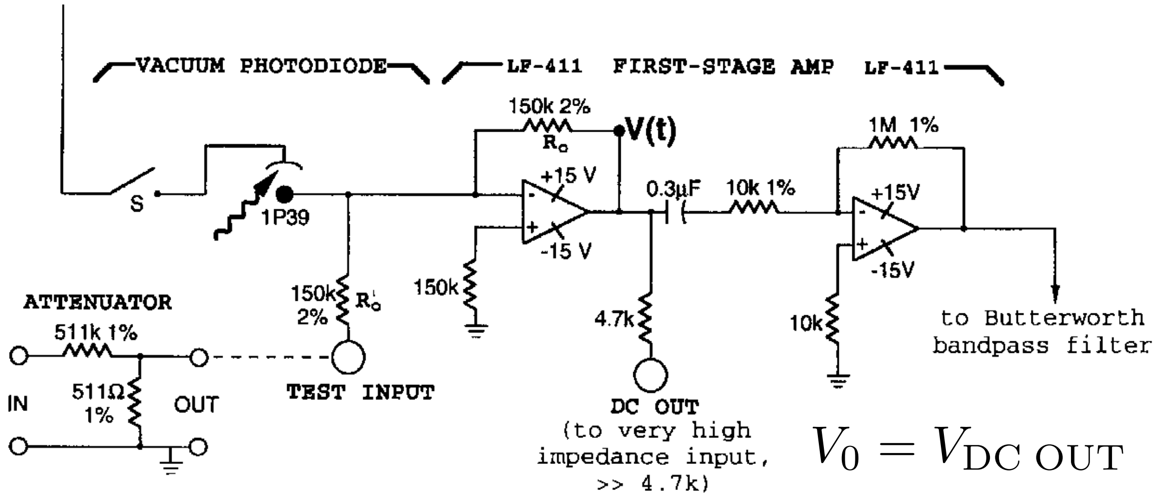
$$\langle |V(f)|^2 \rangle = 2 e I_0 R_0^2$$

- Shot noise in tensione

$$\begin{aligned} V_{\mathrm{RMS}}^2 &= \int_B \langle |V(f)|^2 \rangle df \\ &= \int_B 2 e I_0 R_0^2 df = 2 e I_0 R_0^2 B \end{aligned}$$

- Tensione RMS rilevata in uscita

Introduciamo ora un fattore che tiene conto sia del guadagno variabile in frequenza del primo stadio che del guadagno variabile in frequenza di tutti gli stadi successivi



$$V_0 = V_{\text{DC OUT}}$$

$$V_{\text{RMS}}(f) = g(f)V_{\text{test}}(f)$$

$$V_{\text{RMS}}^2 = \int_B \langle g^2(f)|V(f)|^2\rangle df$$

$$= \int_B 2eI_0R_0^2g^2(f)df = 2eV_0R_0\int_B g^2(f)df$$

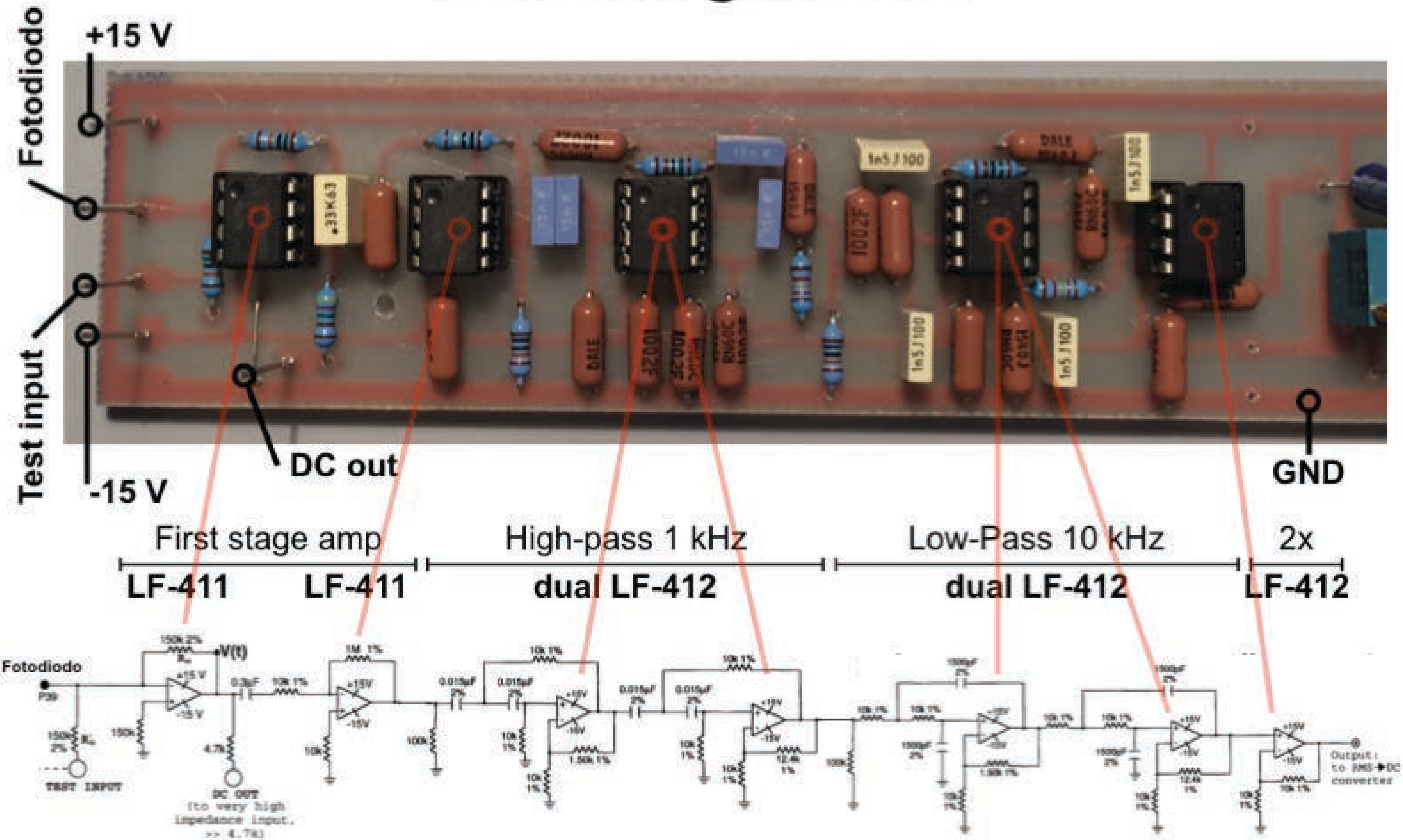
La misura richiede due passaggi

1. Misura di *g(f)*, utilizzando un generatore di funzioni nel test input
2. Misura dello shot noise

$$V_{\mathrm{RMS}}^2 = \langle V_{\mathrm{amp}}^2 \rangle + 2eV_0R_0 \int_B g^2(f)df$$

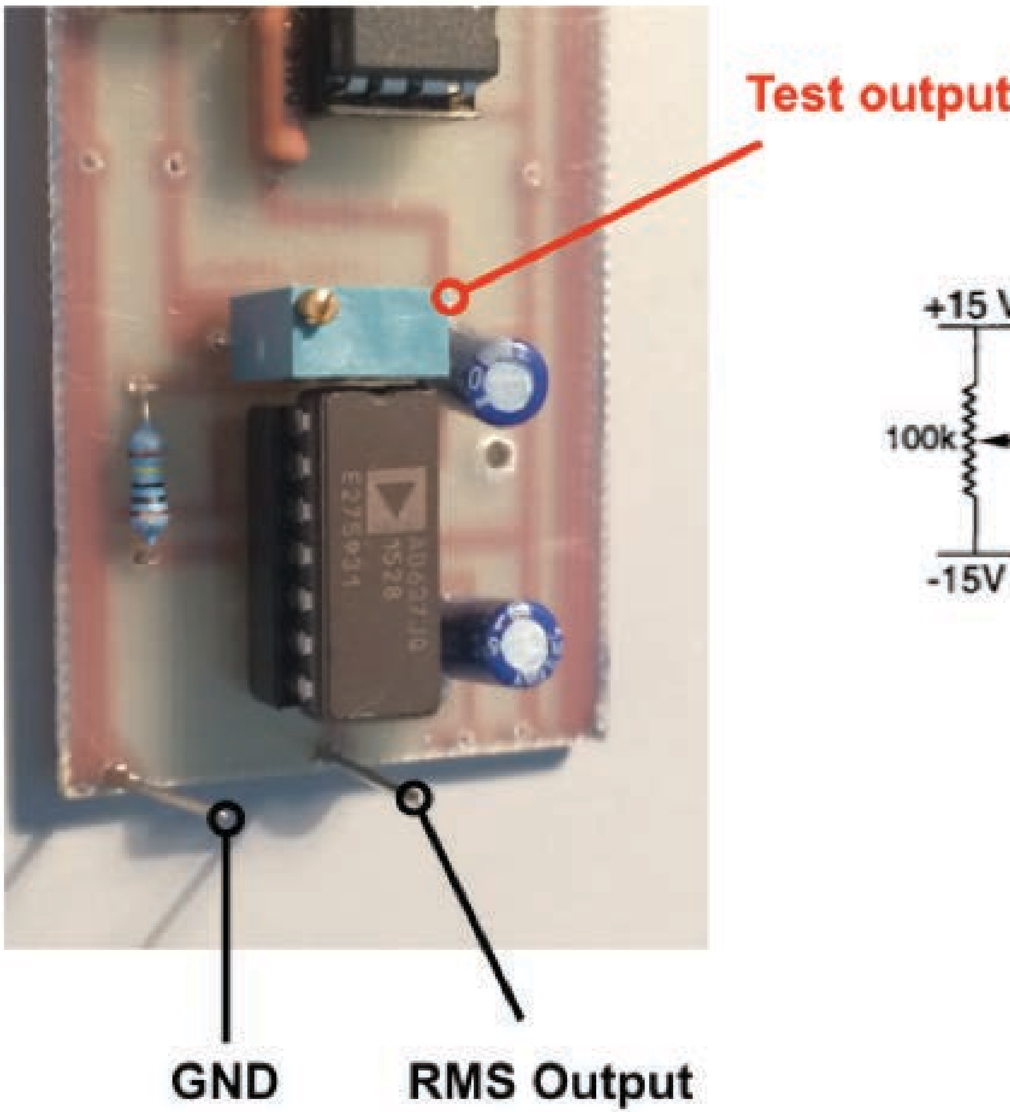
e = pendenza della retta nel piano $(V_0, V_{\mathrm{RMS}}^2)$

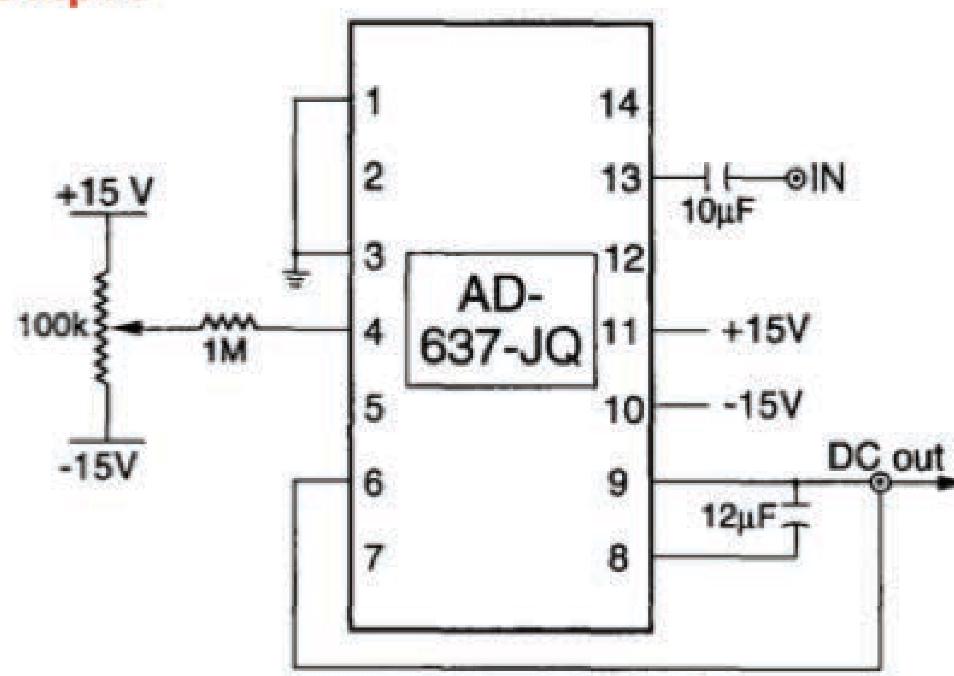
Schema generale



(slide di Alessandro Sala)

AD-637-JQ





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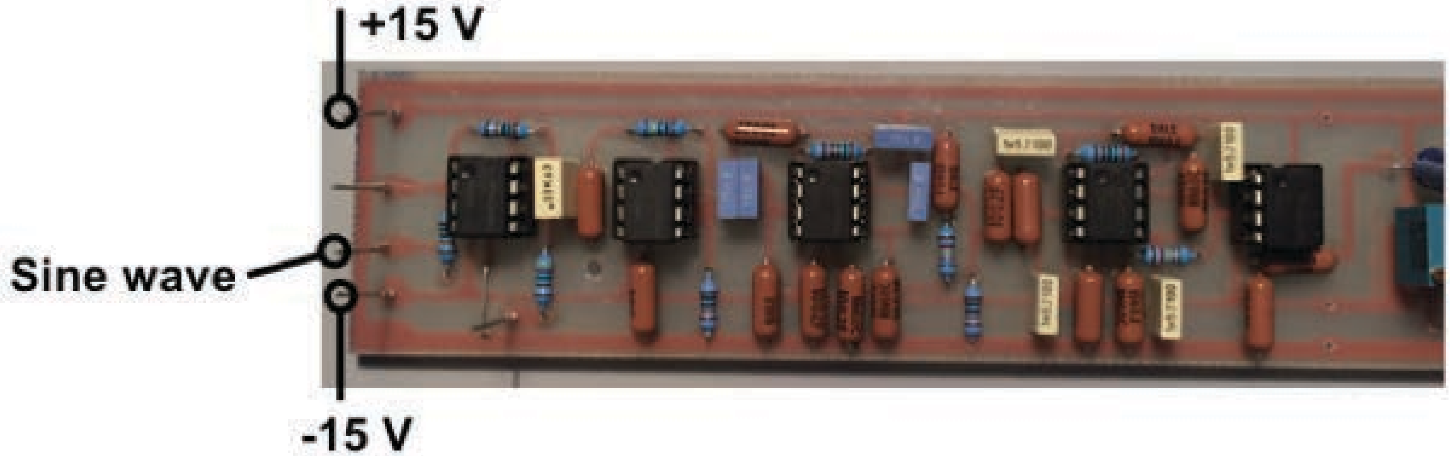
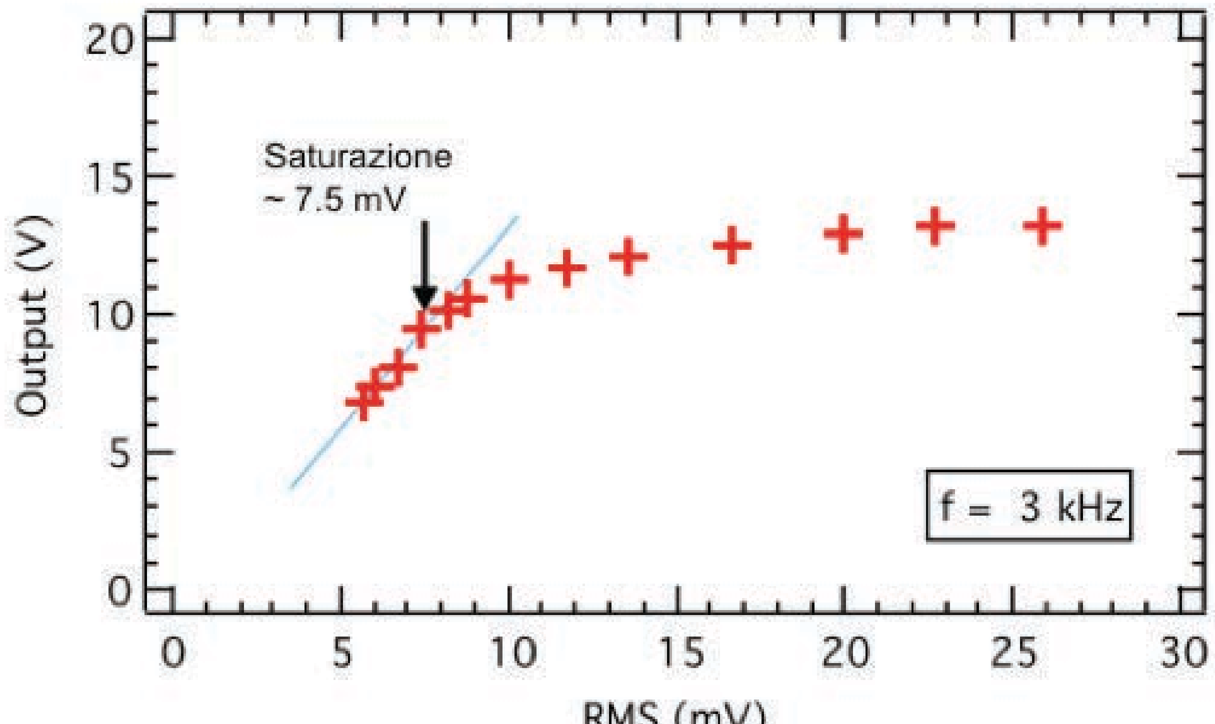
Calibrazione





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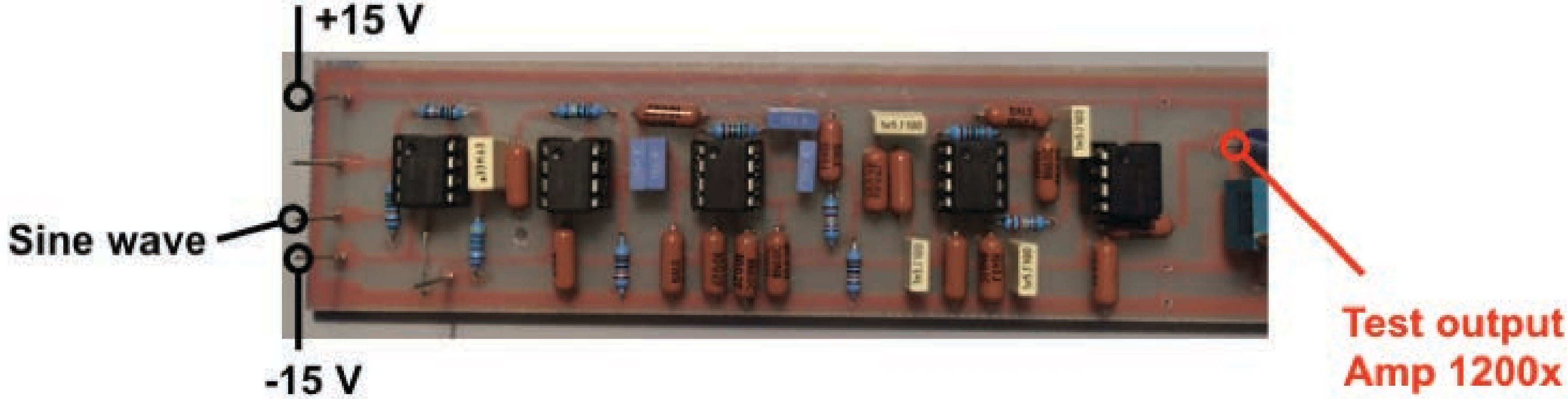
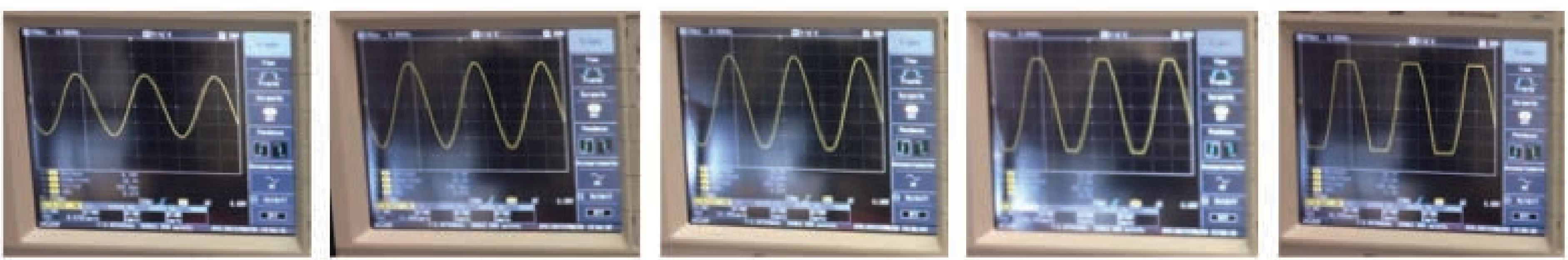


Output RMS
f = 3 kHz

6.53 V	9.20 V	10.13 V	10.75 V	11.71 V



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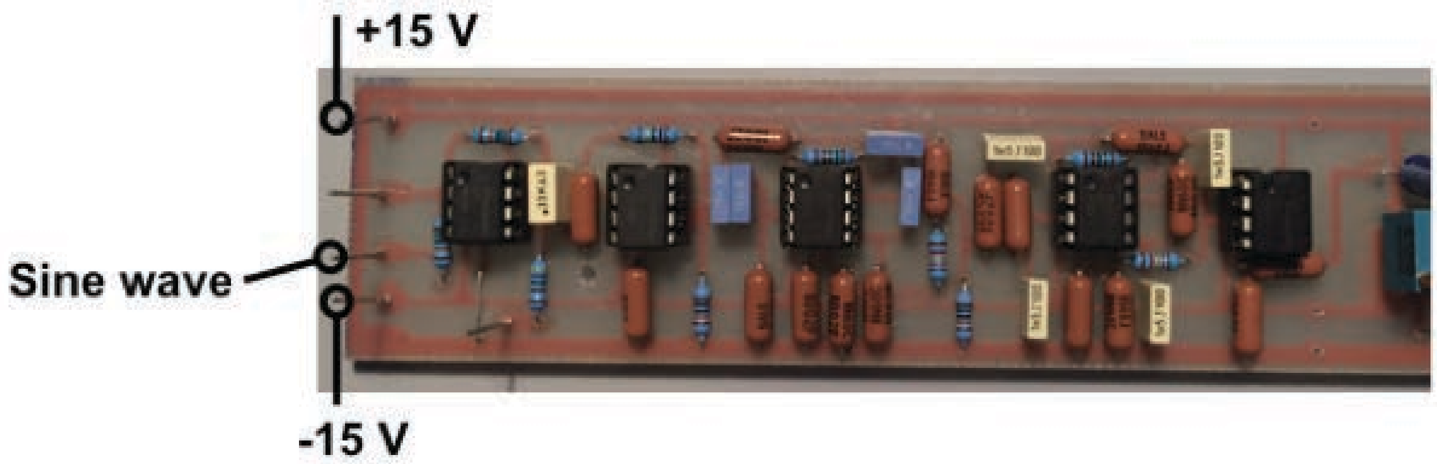

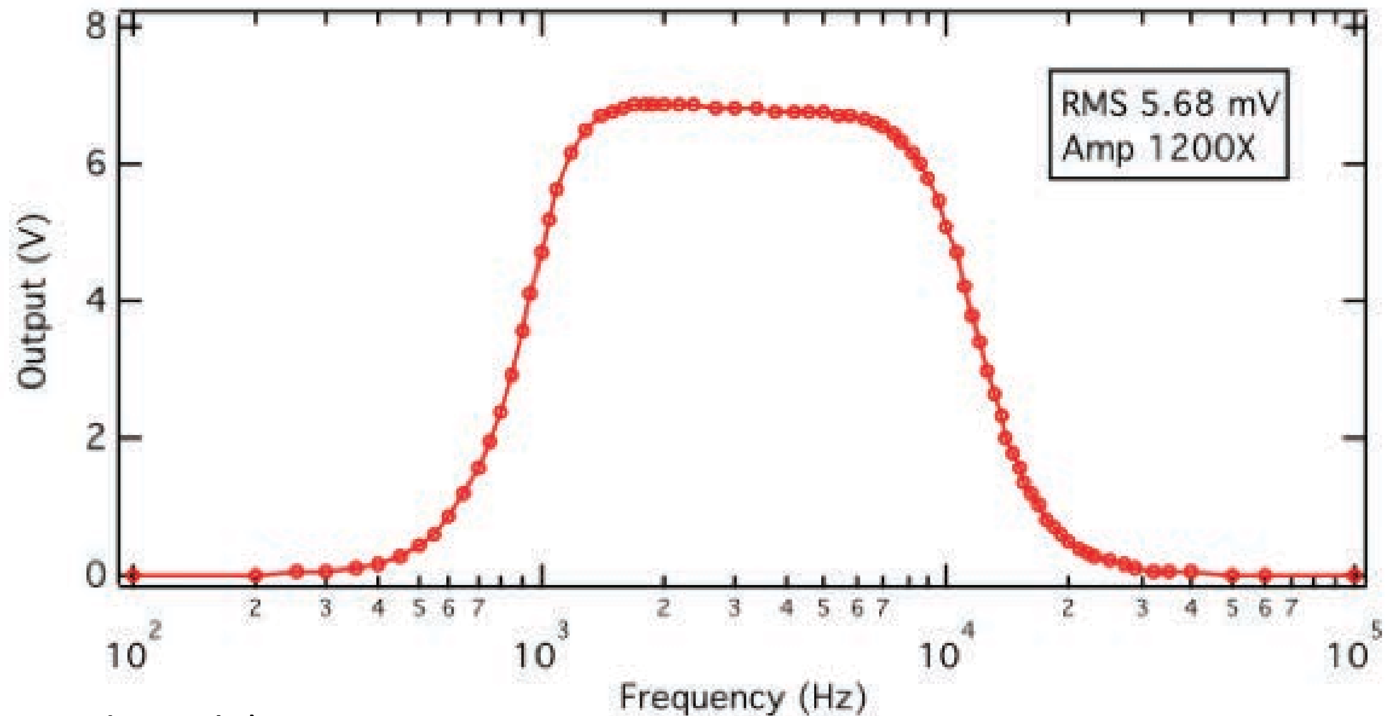

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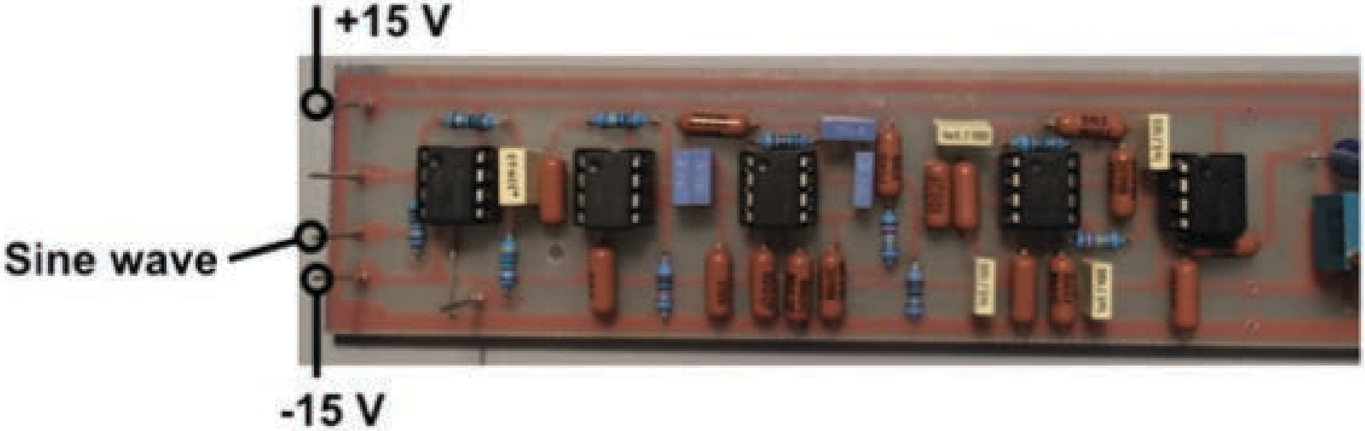
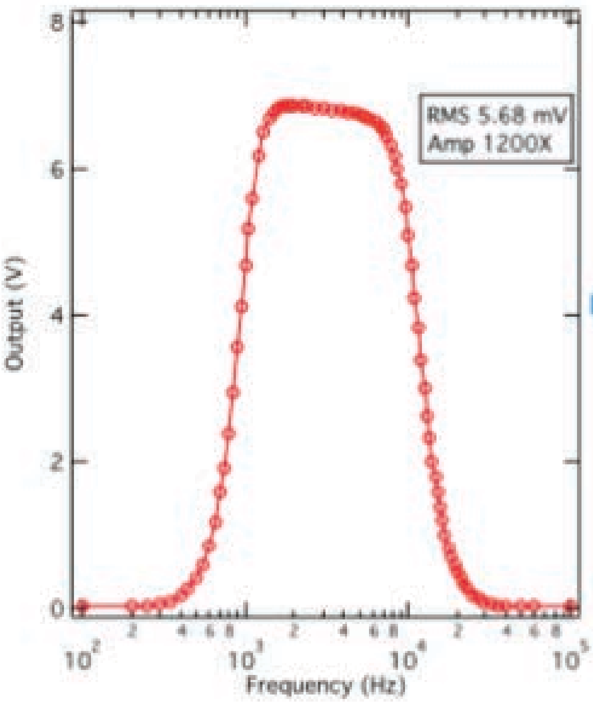
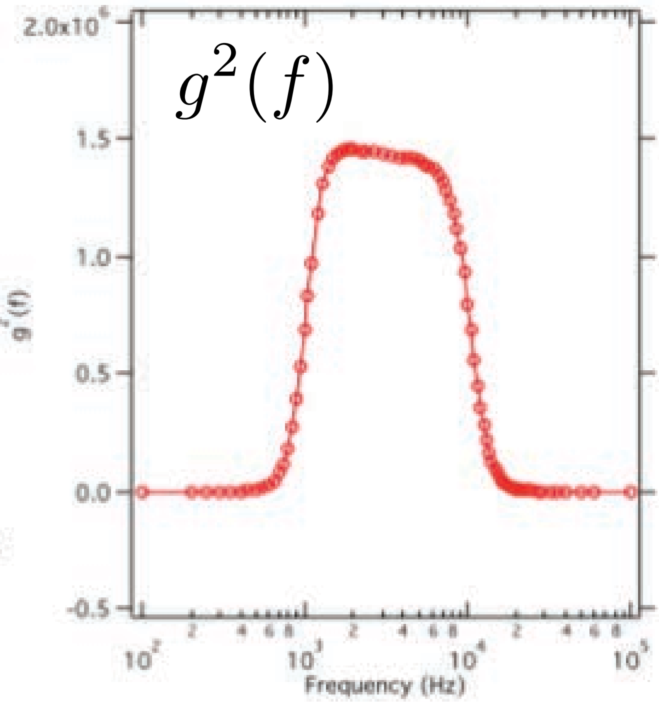




$$\rightarrow g^2(f) = \frac{v_{out}^2 - v_{out\ base}^2}{v_{in\ rms}^2}. \rightarrow$$

$$\int_0^\infty g^2(f)\,df = (1.36\pm0.02)\times10^{10}\ \text{Hz}$$



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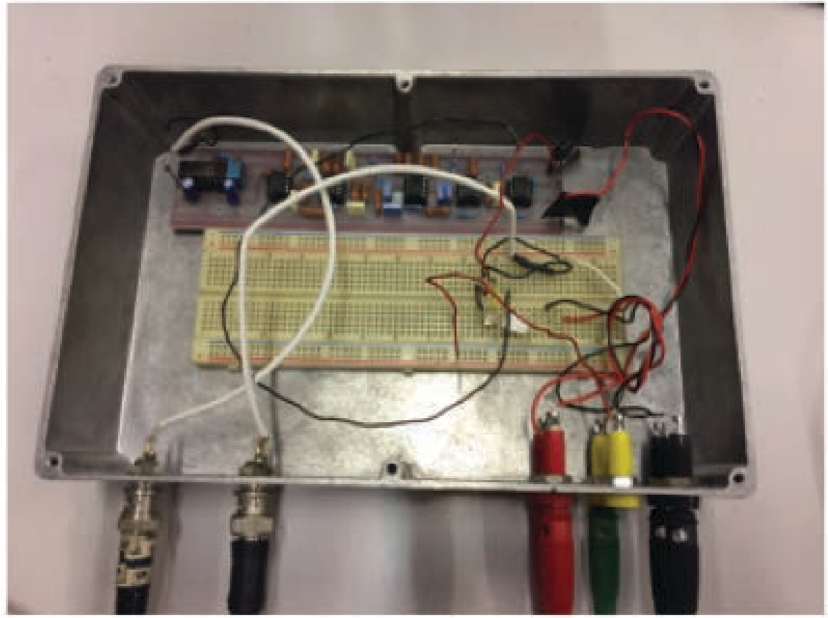
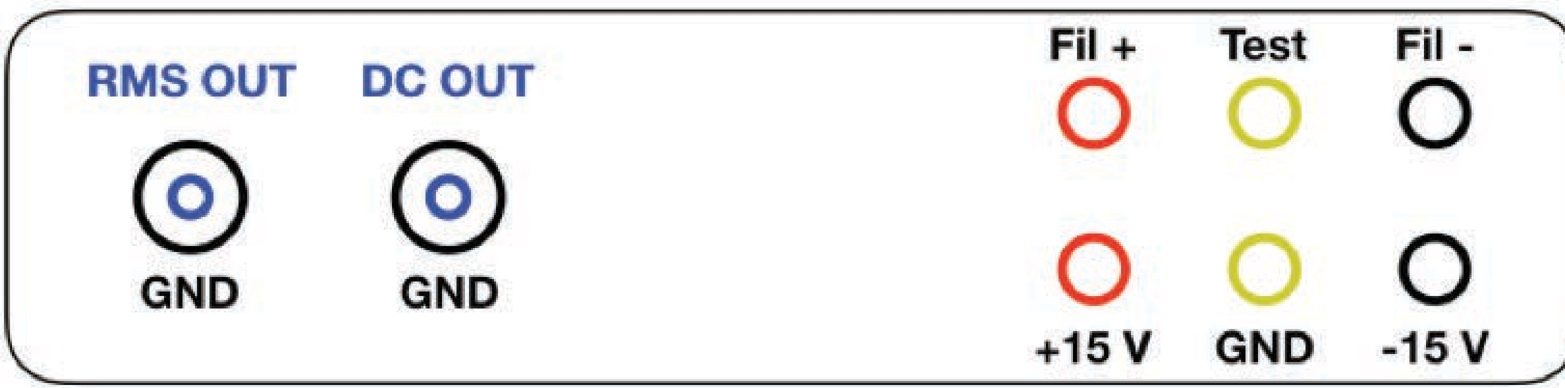
Scatola





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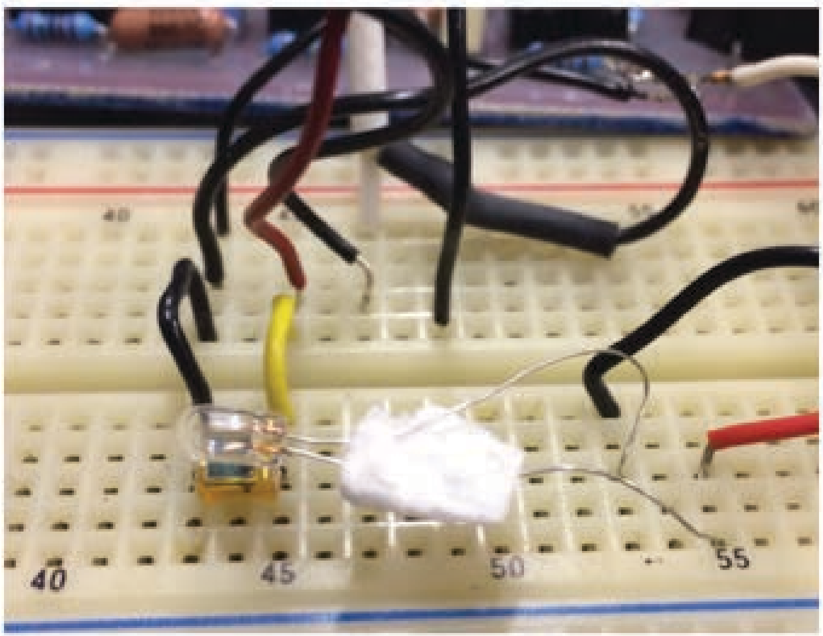
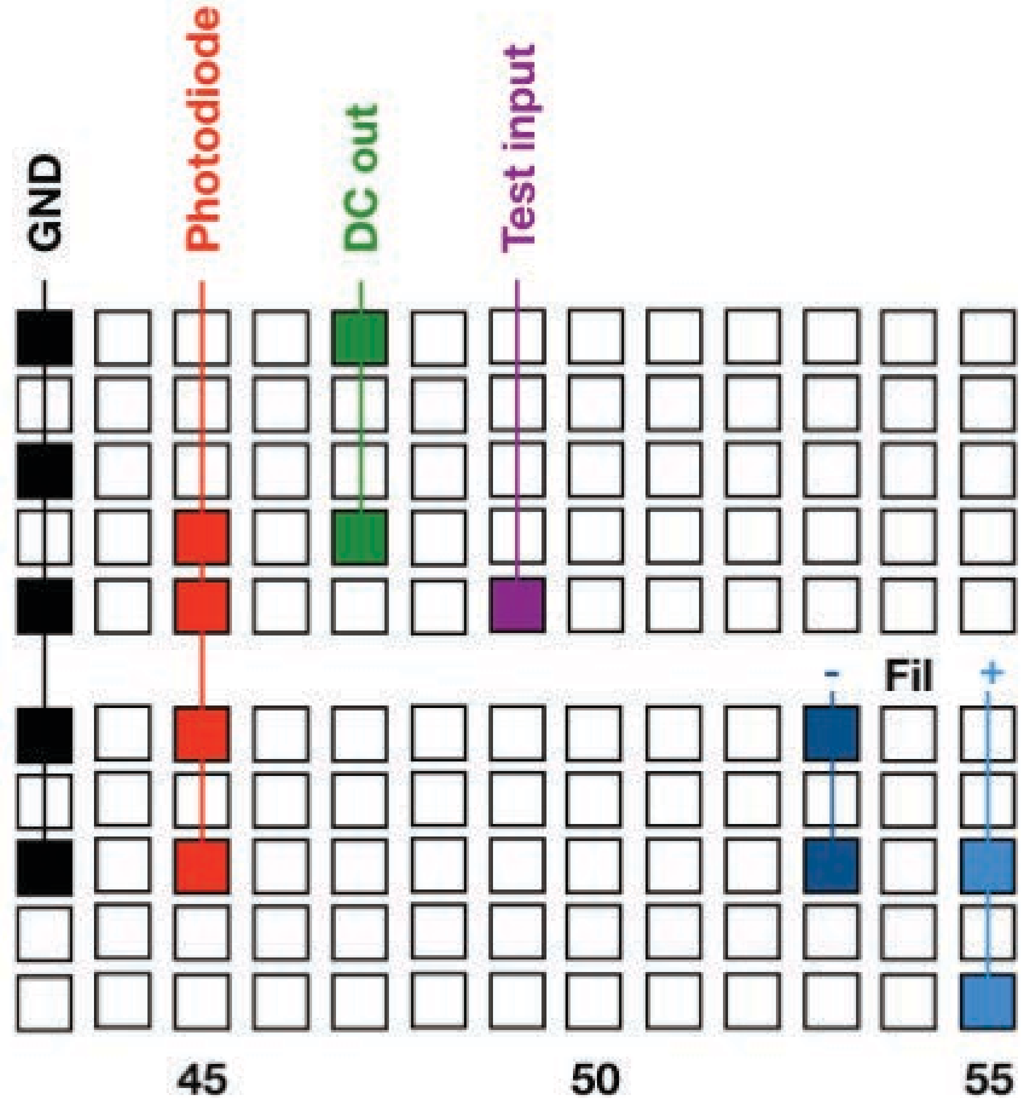
Scatola





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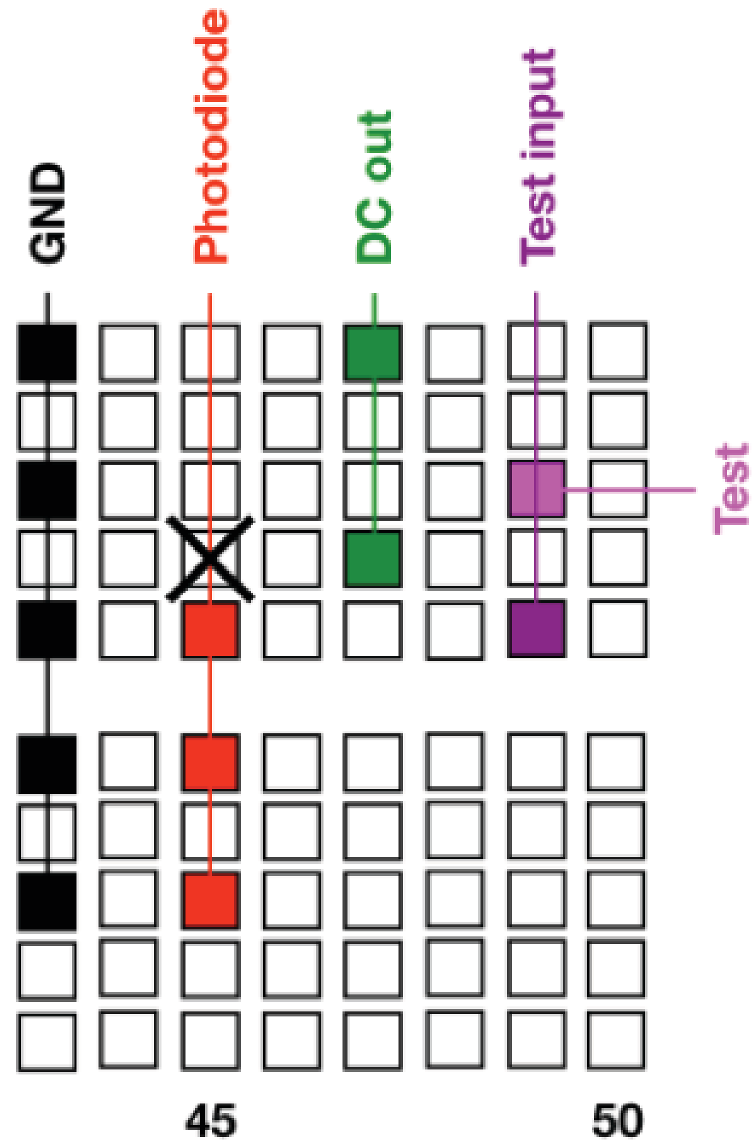
Test con sinusoide

1. Staccare il cavo che va dal circuito al fotodiodo
2. Collegare l'ingresso a banana Test a Test Input
3. Mandare il sinusoidale su Test



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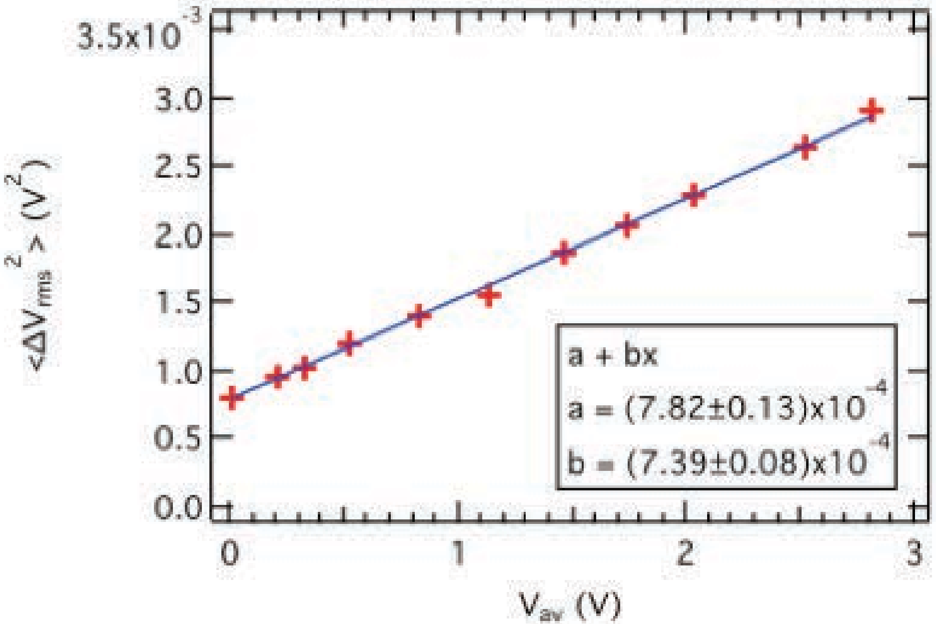
Misura



$$(\Delta V_{\rm rms}^{\rm out})^2 = (\Delta V_{\rm amp,rms})^2 + 2eR_0V_{\rm av}\int_0^\infty g^2(f)\,df.$$

$e = (1.82 \pm 0.05) \times 10^{-19}$ C

$\Delta V_{amp,rms}$ = 27 mV
a filamento staccato ~ 15 mV,
rumore extra dal generatore
del voltaggio per il filamento!

(slide di Alessandro Sala)