APPENDIX C

GRAPHS AND TABLES

C.1 GAUSSIAN PROBABILITY DISTRIBUTION

The probability function $P_G(x; \mu, \sigma)$ for the Gaussian or normal error distribution is given by

$$P_G(x;\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]$$

If measurements of a quantity x are distributed in this manner around a mean μ with a standard deviation σ, the probability $dQ_G(x; \mu, \sigma)$ for observing a value of x, within an infinitesimally small interval dx, in a random sample measurement is given by

$$dQ_G(x;\mu,\sigma) = P_G(x;\mu,\sigma)\,dx$$

Values of the probability function $P_G(x; \mu, \sigma)$ are tabulated in Table C.1 as a function of the dimensionless deviation

$$z = |x - \mu|/\sigma$$

for z ranging from 0.0 to 3.0 in increments of 0.01 and up to 5.9 in increments

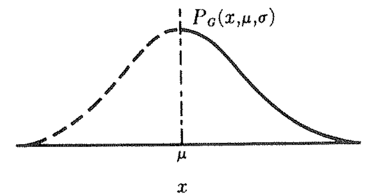

TABLE C.1
Gaussian probability distribution. The Gaussian or normal error distribution $P_G(x;\mu,\sigma)$ vs. $z = |x - \mu| / \sigma$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.39894	0.39892	0.39886	0.39876	0.39862	0.39844	0.39822	0.39797	0.39767	0.39733
0.1	0.39695	0.39654	0.39608	0.39559	0.39505	0.39448	0.39387	0.39322	0.39253	0.39181
0.2	0.39104	0.39024	0.38940	0.38853	0.38762	0.38667	0.38568	0.38466	0.38361	0.38251
0.3	0.38139	0.38023	0.37903	0.37780	0.37654	0.37524	0.37391	0.37255	0.37115	0.36973
0.4	0.36827	0.36678	0.36526	0.36371	0.36213	0.36053	0.35889	0.35723	0.35553	0.35381
0.5	0.35207	0.35029	0.34849	0.34667	0.34482	0.34294	0.34105	0.33912	0.33718	0.33521
0.6	0.33322	0.33121	0.32918	0.32713	0.32506	0.32297	0.32086	0.31874	0.31659	0.31443
0.7	0.31225	0.31006	0.30785	0.30563	0.30339	0.30114	0.29887	0.29659	0.29431	0.29200
0.8	0.28969	0.28737	0.28504	0.28269	0.28034	0.27799	0.27562	0.27324	0.27086	0.26848
0.9	0.26609	0.26369	0.26129	0.25888	0.25647	0.25406	0.25164	0.24923	0.24681	0.24439
1.0	0.24197	0.23995	0.23713	0.23471	0.23230	0.22988	0.22747	0.22506	0.22266	0.22025
1.1	0.21785	0.21546	0.21307	0.21069	0.20831	0.20594	0.20357	0.20122	0.19887	0.19652
1.2	0.19419	0.19186	0.18955	0.18724	0.18494	0.18265	0.18038	0.17811	0.17585	0.17361
1.3	0.17137	0.16915	0.16694	0.16475	0.16256	0.16039	0.15823	0.15609	0.15395	0.15184
1.4	0.14973	0.14764	0.14557	0.14351	0.14147	0.13944	0.13742	0.13543	0.13344	0.13148
1.5	0.12952	0.12759	0.12567	0.12377	0.12189	0.12002	0.11816	0.11633	0.11451	0.11271
1.6	0.11093	0.10916	0.10741	0.10568	0.10397	0.10227	0.10059	0.09893	0.09729	0.09567
1.7	0.09406	0.09247	0.09090	0.08934	0.08780	0.08629	0.08478	0.08330	0.08184	0.08039
1.8	0.07896	0.07755	0.07615	0.07477	0.07342	0.07207	0.07075	0.06944	0.06815	0.06688
1.9	0.06562	0.06439	0.06316	0.06196	0.06077	0.05960	0.05845	0.05731	0.05619	0.05509
2.0	0.05400	0.05293	0.05187	0.05083	0.04981	0.04880	0.04781	0.04683	0.04587	0.04492
2.1	0.04399	0.04307	0.04217	0.04129	0.04041	0.03956	0.03871	0.03788	0.03707	0.03627
2.2	0.03548	0.03471	0.03395	0.03320	0.03247	0.03175	0.03104	0.03034	0.02966	0.02899
2.3	0.02833	0.02769	0.02705	0.02643	0.02582	0.02522	0.02464	0.02406	0.02350	0.02294
2.4	0.02240	0.02187	0.02135	0.02083	0.02033	0.01984	0.01936	0.01889	0.01843	0.01798
2.5	0.01753	0.01710	0.01667	0.01626	0.01585	0.01545	0.01506	0.01468	0.01431	0.01394
2.6	0.01359	0.01324	0.01290	0.01256	0.01224	0.01192	0.01160	0.01130	0.01100	0.01071
2.7	0.01042	0.01015	0.00987	0.00961	0.00935	0.00910	0.00885	0.00861	0.00837	0.00814
2.8	0.00792	0.00770	0.00749	0.00728	0.00707	0.00688	0.00668	0.00649	0.00631	0.00613
2.9	0.00595	0.00578	0.00562	0.00546	0.00530	0.00514	0.00500	0.00485	0.00471	0.00457

z	0.00	0.10	0.20	0.30	0.40
3.0	0.0044318	0.0032668	0.0023841	0.0017226	0.0012322
3.5	0.00087269	0.00061191	0.00042479	0.00029195	0.00019866
4.0	0.00013383	0.000089264	0.000058945	0.000038536	0.000024943
4.5	0.000015984	0.000010141	0.0000063701	0.0000039615	0.0000024391
5.0	0.0000014868	0.00000089730	0.00000053614	0.00000031716	0.00000018575
5.5	0.00000010771	0.00000006183	0.00000003514	0.00000001978	0.00000001102

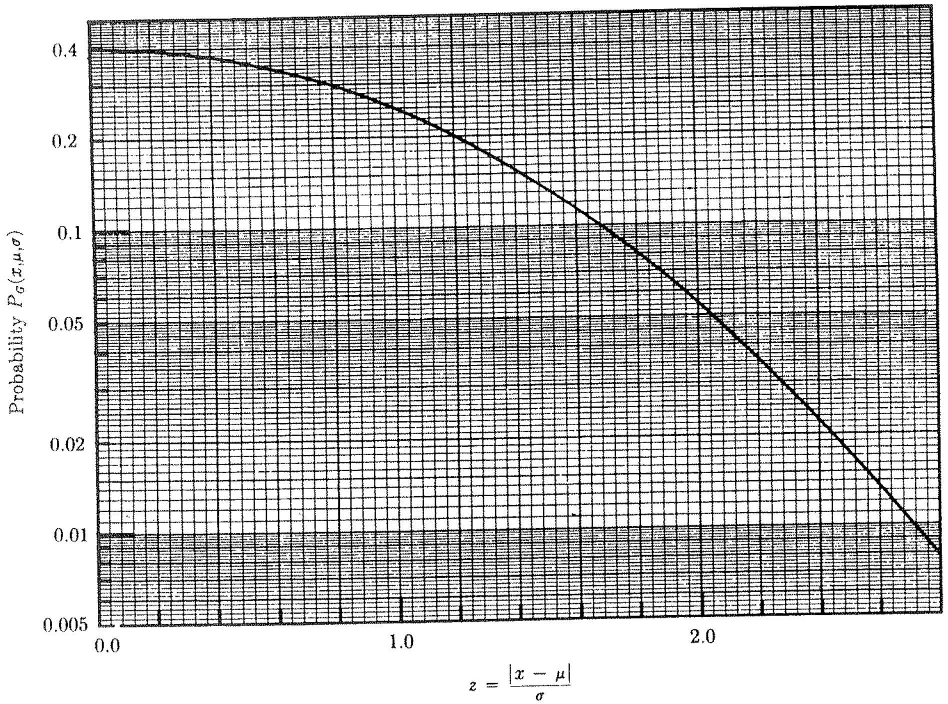

FIGURE C.1
The Gaussian probability function $P_G(x;\mu,\sigma)$ vs. $z = |x-\mu|/\sigma$.

of 0.1. This function is graphed on a semilogarithmic scale as a function of z in Figure C.1.

The function that is tabulated and graphed is $P_G(z;0,1)$, which gives the probability that $x = \mu \pm z\sigma$. It is the curve of Figure 2.5 tabulated only for positive values of z as indicated.

C.2 INTEGRAL OF GAUSSIAN DISTRIBUTION

The integral $A_G(x,\mu,\sigma)$ of the probability function $P_G(x,\mu,\sigma)$ for the Gaussian or normal error distribution is given by

$$A_G(x,\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}}\int_{\mu-z\sigma}^{\mu+z\sigma} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right] dx$$

$$z = \frac{|x-\mu|}{\sigma}$$

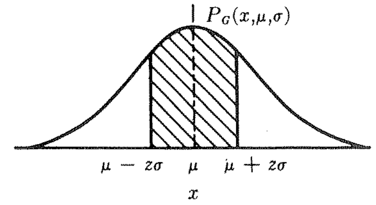

TABLE C.2
Integral of Gaussian distribution. The integral of the Gaussian probability distribution $A_G(x;\mu,\sigma)$ vs. $z = |x-\mu|/\sigma$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0	0.00798	0.01596	0.02393	0.03191	0.03988	0.04784	0.05581	0.06376	0.07171
0.1	0.07966	0.08759	0.09552	0.10343	0.11134	0.11924	0.12712	0.13499	0.14285	0.15069
0.2	0.15852	0.16633	0.17413	0.18191	0.18967	0.19741	0.20514	0.21284	0.22052	0.22818
0.3	0.23582	0.24344	0.25103	0.25860	0.26614	0.27366	0.28115	0.28862	0.29605	0.30346
0.4	0.31084	0.31819	0.32551	0.33280	0.34006	0.34729	0.35448	0.36164	0.36877	0.37587
0.5	0.38292	0.38995	0.39694	0.40389	0.41080	0.41768	0.42452	0.43132	0.43809	0.44481
0.6	0.45149	0.45814	0.46474	0.47131	0.47783	0.48431	0.49075	0.49714	0.50350	0.50981
0.7	0.51607	0.52230	0.52847	0.53461	0.54070	0.54674	0.55274	0.55870	0.56461	0.57047
0.8	0.57629	0.58206	0.58778	0.59346	0.59909	0.60467	0.61021	0.61570	0.62114	0.62653
0.9	0.63188	0.63718	0.64243	0.64763	0.65278	0.65789	0.66294	0.66795	0.67291	0.67783
1.0	0.68269	0.68750	0.69227	0.69699	0.70166	0.70628	0.71085	0.71538	0.71985	0.72428
1.1	0.72866	0.73300	0.73728	0.74152	0.74571	0.74985	0.75395	0.75799	0.76199	0.76595
1.2	0.76985	0.77371	0.77753	0.78130	0.78502	0.78869	0.79232	0.79591	0.79945	0.80294
1.3	0.80639	0.80980	0.81316	0.81647	0.81975	0.82298	0.82616	0.82930	0.83240	0.83546
1.4	0.83848	0.84145	0.84438	0.84727	0.85012	0.85293	0.85570	0.85843	0.86112	0.86377
1.5	0.86638	0.86895	0.87148	0.87397	0.87643	0.87885	0.88123	0.88358	0.88588	0.88816
1.6	0.89039	0.89259	0.89476	0.89689	0.89898	0.90105	0.90308	0.90507	0.90703	0.90896
1.7	0.91086	0.91272	0.91456	0.91636	0.91813	0.91987	0.92158	0.92326	0.92491	0.92654
1.8	0.92813	0.92969	0.93123	0.93274	0.93422	0.93568	0.93711	0.93851	0.93988	0.94123
1.9	0.94256	0.94386	0.94513	0.94638	0.94761	0.94882	0.95000	0.95115	0.95229	0.95340
2.0	0.95449	0.95556	0.95661	0.95764	0.95864	0.95963	0.96059	0.96154	0.96247	0.96338
2.1	0.96426	0.96513	0.96599	0.96682	0.96764	0.96844	0.96922	0.96999	0.97074	0.97147
2.2	0.97219	0.97289	0.97358	0.97425	0.97490	0.97555	0.97617	0.97679	0.97739	0.97797
2.3	0.97855	0.97911	0.97965	0.98019	0.98071	0.98122	0.98172	0.98221	0.98268	0.98315
2.4	0.98360	0.98404	0.98448	0.98490	0.98531	0.98571	0.98610	0.98648	0.98686	0.98722
2.5	0.98758	0.98792	0.98826	0.98859	0.98891	0.98922	0.98953	0.98983	0.99012	0.99040
2.6	0.99067	0.99094	0.99120	0.99146	0.99171	0.99195	0.99218	0.99241	0.99264	0.99285
2.7	0.99306	0.99327	0.99347	0.99366	0.99385	0.99404	0.99422	0.99439	0.99456	0.99473
2.8	0.99489	0.99504	0.99520	0.99534	0.99549	0.99563	0.99576	0.99589	0.99602	0.99615
2.9	0.99627	0.99638	0.99650	0.99661	0.99672	0.99682	0.99692	0.99702	0.99712	0.99721

z	0.00	0.10	0.20	0.30	0.40
3.0	0.9973002	0.9980648	0.9986257	0.99903315	0.99932614
3.5	0.99953474	0.99968178	0.99978440	0.99985530	0.999903805
4.0	0.999936656	0.999958684	0.999973308	0.999982920	0.999989174
4.5	0.9999932043	0.9999957748	0.9999973982	0.9999984132	0.99999904149
5.0	0.99999942657	0.99999966024	0.99999980061	0.99999988410	0.99999993327
5.5	0.99999996193	0.99999997847	0.99999998793	.99999999328	0.99999999627

If measurements of a quantity x are distributed according to the Gaussian distribution around a mean μ with a standard deviation σ, $A_G(x; \mu, \sigma)$ is equal to the probability for observing a value of x in a random sample measurement that is between $\mu - z\sigma$ and $\mu + z\sigma$; that is, it is the probability that $|x - \mu| < z\sigma$.

Values of the integral $A_G(x; \mu, \sigma)$ are tabulated in Table C.2 as a function of z for z ranging from 0.0 to 3.0 in increments of 0.01 and up to 5.9 in increments of 0.1. This function is graphed on a probability scale as a function of z in Figure C.2.

A related function is the error function erf Z:

$$\text{erf } Z = \frac{1}{\sqrt{\pi}} \int_{-Z}^{Z} e^{-z^2} dz = A_G(z\sqrt{2}, 0, 1)$$

The function that is tabulated and graphed is the shaded area between the limits $\mu \pm z\sigma$ as indicated.

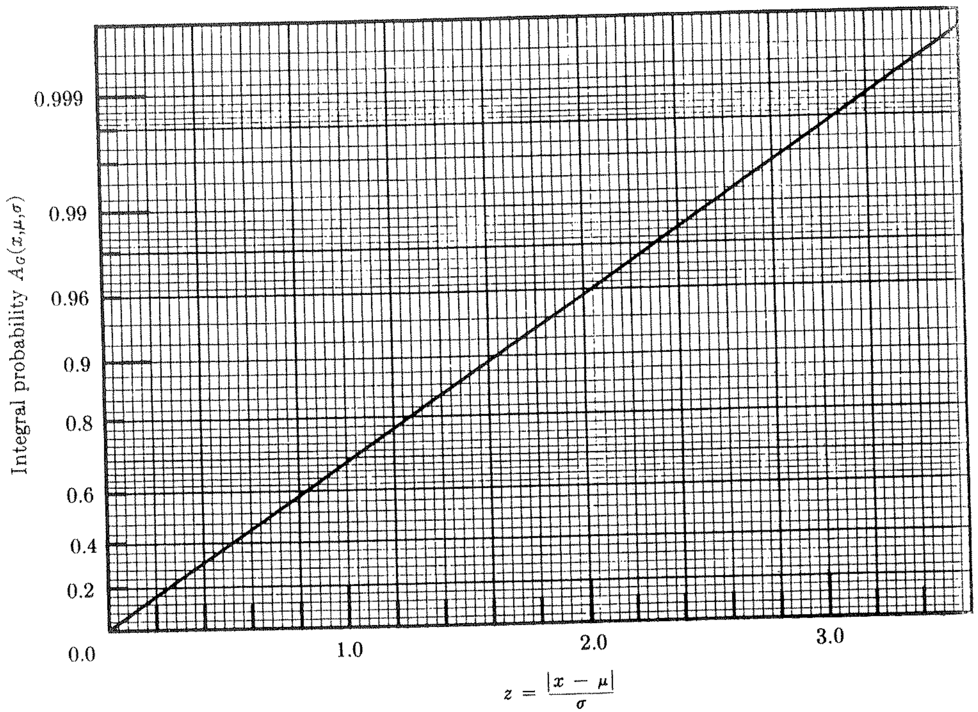

FIGURE C.2
The integral of the Gaussian probability distributions $A_G(x; \mu, \sigma)$ vs. $z = |x - \mu|/\sigma$.

C.3 LINEAR-CORRELATION COEFFICIENT

The probability distribution $P_r(r, \nu)$ for the linear-correlation coefficient r for ν degrees of freedom is given by

$$P_r(r; \nu) = \frac{1}{\sqrt{\pi}} \frac{\Gamma[(\nu + 1)/2]}{\Gamma(\nu/2)} (1 - r^2)^{(\nu-2)/2}$$

The probability of observing a value of the correlation coefficient larger than r for a random sample of N observations with ν degrees of freedom is the integral of this probability $P_c(r; N)$:

$$P_c(r; N) = \frac{1}{\sqrt{\pi}} \frac{\Gamma[(\nu + 1)/2]}{\Gamma(\nu/2)} \int_{|r|}^{1} (1 - x^2)^{(\nu-2)/2} dx \qquad \nu = N - 2$$

If two variables of a parent population are uncorrelated, the probability that a random sample of N observations will yield a correlation coefficient for those two variables greater in magnitude than $|r|$ is given by $P_c(r; N)$.

Values of the coefficient $|r|$ corresponding to various values of the probability $P_c(r; N)$ are tabulated in Table C.3 for N ranging from 3 to 100, and values of $P_c(r; N)$ ranging from 0.001 to 0.5. The functional dependence of r corresponding to representative values of $P_c(r, N)$ is graphed on a semilogarithmic scale as a smooth variation with the number of observations N in Figure C.3.

The function that is tabulated and graphed is the shaded area under the tails of the probability curve for values larger than $|r|$ as indicated.

C.4 χ^2 DISTRIBUTION

The probability distribution $P_x(x^2; \nu)$ for χ^2 is given by

$$P_x(x^2; \nu) = \frac{1}{2^{\nu/2}\Gamma(\nu/2)} (x^2)^{(\nu-2)/2} e^{-x^2/2}$$

The probability of observing a value of chi-square larger than χ^2 for a random sample of N observations with ν degrees of freedom is the integral of this

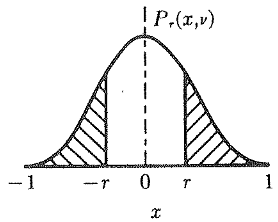

TABLE C.3

Linear-correlation coefficient. The linear-correlation coefficient r vs. the number of observations N and the corresponding probability $P_c(r;N)$ of exceeding r in a random sample of observations taken from an uncorrelated parent population ($\rho = 0$)

N	0.50	0.20	0.10	0.050	0.020	0.010	0.005	0.002	0.001
3	0.707	0.951	0.988	0.997	1.000	1.000	1.000	1.000	1.000
4	0.500	0.800	0.900	0.950	0.980	0.990	0.995	0.998	0.999
5	0.404	0.687	0.805	0.878	0.934	0.959	0.974	0.986	0.991
6	0.347	0.608	0.729	0.811	0.882	0.917	0.942	0.963	0.974
7	0.309	0.551	0.669	0.754	0.833	0.875	0.906	0.935	0.951
8	0.281	0.507	0.621	0.707	0.789	0.834	0.870	0.905	0.925
9	0.260	0.472	0.582	0.666	0.750	0.798	0.836	0.875	0.898
10	0.242	0.443	0.549	0.632	0.715	0.765	0.805	0.847	0.872
11	0.228	0.419	0.521	0.602	0.685	0.735	0.776	0.820	0.847
12	0.216	0.398	0.497	0.576	0.658	0.708	0.750	0.795	0.823
13	0.206	0.380	0.476	0.553	0.634	0.684	0.726	0.772	0.801
14	0.197	0.365	0.458	0.532	0.612	0.661	0.703	0.750	0.780
15	0.189	0.351	0.441	0.514	0.592	0.641	0.683	0.730	0.760
16	0.182	0.338	0.426	0.497	0.574	0.623	0.664	0.711	0.742
17	0.176	0.327	0.412	0.482	0.558	0.606	0.647	0.694	0.725
18	0.170	0.317	0.400	0.468	0.543	0.590	0.631	0.678	0.708
19	0.165	0.308	0.389	0.456	0.529	0.575	0.616	0.662	0.693
20	0.160	0.299	0.378	0.444	0.516	0.561	0.602	0.648	0.679
22	0.152	0.284	0.360	0.423	0.492	0.537	0.576	0.622	0.652
24	0.145	0.271	0.344	0.404	0.472	0.515	0.554	0.599	0.629
26	0.138	0.260	0.330	0.388	0.453	0.496	0.534	0.578	0.607
28	0.133	0.250	0.317	0.374	0.437	0.479	0.515	0.559	0.588
30	0.128	0.241	0.306	0.361	0.423	0.463	0.499	0.541	0.570
32	0.124	0.233	0.296	0.349	0.409	0.449	0.484	0.526	0.554
34	0.120	0.225	0.287	0.339	0.397	0.436	0.470	0.511	0.539
36	0.116	0.219	0.279	0.329	0.386	0.424	0.458	0.498	0.525
38	0.113	0.213	0.271	0.320	0.376	0.413	0.446	0.486	0.513
40	0.110	0.207	0.264	0.312	0.367	0.403	0.435	0.474	0.501
42	0.107	0.202	0.257	0.304	0.358	0.393	0.425	0.463	0.490
44	0.104	0.197	0.251	0.297	0.350	0.384	0.416	0.453	0.479
46	0.102	0.192	0.246	0.291	0.342	0.376	0.407	0.444	0.469
48	0.100	0.188	0.240	0.285	0.335	0.368	0.399	0.435	0.460
50	0.098	0.184	0.235	0.279	0.328	0.361	0.391	0.427	0.451
60	0.089	0.168	0.214	0.254	0.300	0.330	0.358	0.391	0.414
70	0.082	0.155	0.198	0.235	0.278	0.306	0.332	0.363	0.385
80	0.077	0.145	0.185	0.220	0.260	0.286	0.311	0.340	0.361
90	0.072	0.136	0.174	0.207	0.245	0.270	0.293	0.322	0.341
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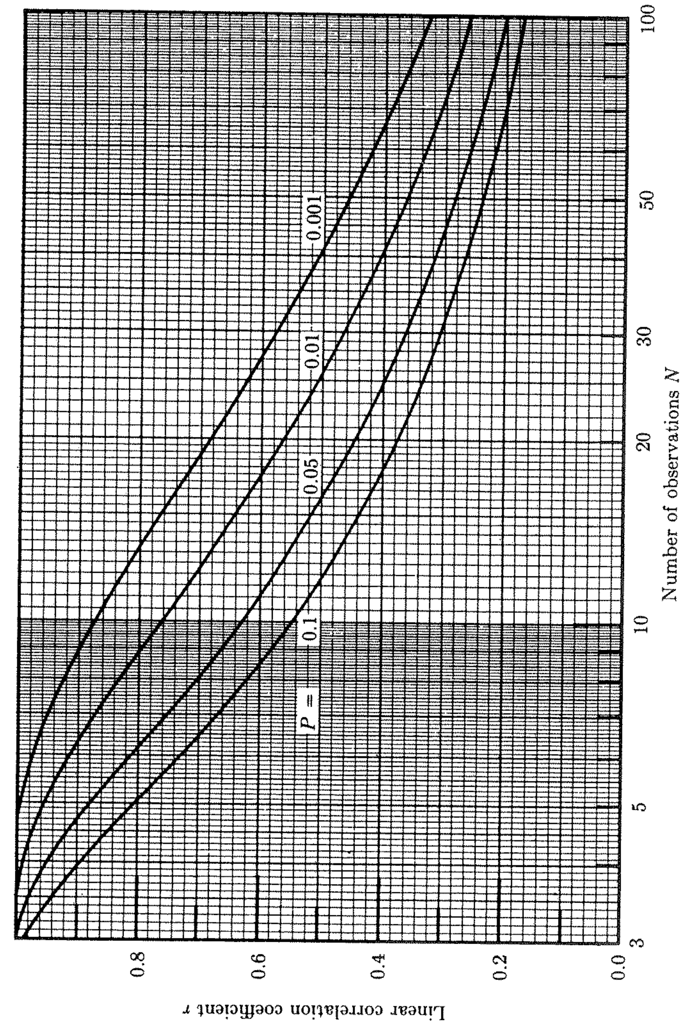

FIGURE C.3

The linear-correlation coefficient r vs. the number of observations N and the corresponding probability $P_c(r;N)$ that the variables are not correlated.

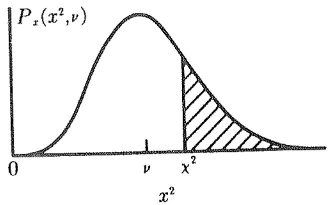

TABLE C.4
χ^2 **distribution. Values of the reduced chi-square $\chi_\nu^2 = \chi^2 / \nu$ corresponding to the probability $P_\chi(\chi^2; \nu)$ of exceeding χ^2 vs. the number of degrees of freedom ν**

ν	P: 0.99	0.98	0.95	0.90	0.80	0.70	0.60	0.50
1	0.00016	0.00063	0.00393	0.0158	0.0642	0.148	0.275	0.455
2	0.0100	0.0202	0.0515	0.105	0.223	0.357	0.511	0.693
3	0.0383	0.0617	0.117	0.195	0.335	0.475	0.623	0.789
4	0.0742	0.107	0.178	0.266	0.412	0.549	0.688	0.839
5	0.111	0.150	0.229	0.322	0.469	0.600	0.731	0.870
6	0.145	0.189	0.273	0.367	0.512	0.638	0.762	0.891
7	0.177	0.223	0.310	0.405	0.546	0.667	0.785	0.907
8	0.206	0.254	0.342	0.436	0.574	0.691	0.803	0.918
9	0.232	0.281	0.369	0.463	0.598	0.710	0.817	0.927
10	0.256	0.306	0.394	0.487	0.618	0.727	0.830	0.934
11	0.278	0.328	0.416	0.507	0.635	0.741	0.840	0.940
12	0.298	0.348	0.436	0.525	0.651	0.753	0.848	0.945
13	0.316	0.367	0.453	0.542	0.664	0.764	0.856	0.949
14	0.333	0.383	0.469	0.556	0.676	0.773	0.863	0.953
15	0.349	0.399	0.484	0.570	0.687	0.781	0.869	0.956
16	0.363	0.413	0.498	0.582	0.697	0.789	0.874	0.959
17	0.377	0.427	0.510	0.593	0.706	0.796	0.879	0.961
18	0.390	0.439	0.522	0.604	0.714	0.802	0.883	0.963
19	0.402	0.451	0.532	0.613	0.722	0.808	0.887	0.965
20	0.413	0.462	0.543	0.622	0.729	0.813	0.890	0.967
22	0.434	0.482	0.561	0.638	0.742	0.823	0.897	0.970
24	0.452	0.500	0.577	0.652	0.753	0.831	0.902	0.972
26	0.469	0.516	0.592	0.665	0.762	0.838	0.907	0.974
28	0.484	0.530	0.605	0.676	0.771	0.845	0.911	0.976
30	0.498	0.544	0.616	0.687	0.779	0.850	0.915	0.978
32	0.511	0.556	0.627	0.696	0.786	0.855	0.918	0.979
34	0.523	0.567	0.637	0.704	0.792	0.860	0.921	0.980
36	0.534	0.577	0.646	0.712	0.798	0.864	0.924	0.982
38	0.545	0.587	0.655	0.720	0.804	0.868	0.926	0.983
40	0.554	0.596	0.663	0.726	0.809	0.872	0.928	0.983
42	0.563	0.604	0.670	0.733	0.813	0.875	0.930	0.984
44	0.572	0.612	0.677	0.738	0.818	0.878	0.932	0.985
46	0.580	0.620	0.683	0.744	0.822	0.881	0.934	0.986
48	0.587	0.627	0.690	0.749	0.825	0.884	0.936	0.986
50	0.594	0.633	0.695	0.754	0.829	0.886	0.937	0.987
60	0.625	0.662	0.720	0.774	0.844	0.897	0.944	0.989
70	0.649	0.684	0.739	0.790	0.856	0.905	0.949	0.990
80	0.669	0.703	0.755	0.803	0.865	0.911	0.952	0.992
90	0.686	0.718	0.768	0.814	0.873	0.917	0.955	0.993
100	0.701	0.731	0.779	0.824	0.879	0.921	0.958	0.993
120	0.724	0.753	0.798	0.839	0.890	0.928	0.962	0.994
140	0.743	0.770	0.812	0.850	0.898	0.934	0.965	0.995
160	0.758	0.784	0.823	0.860	0.905	0.938	0.968	0.996

TABLE C.4
χ^2 **distribution** (*continued*)

ν	P: 0.40	0.30	0.20	0.10	0.05	0.02	0.01	0.001
1	0.708	1.074	1.642	2.706	3.841	5.412	6.635	10.827
2	0.916	1.204	1.609	2.303	2.996	3.912	4.605	6.908
3	0.982	1.222	1.547	2.084	2.605	3.279	3.780	5.423
4	1.011	1.220	1.497	1.945	2.372	2.917	3.319	4.617
5	1.026	1.213	1.458	1.847	2.214	2.678	3.017	4.102
6	1.035	1.205	1.426	1.774	2.099	2.506	2.802	3.743
7	1.040	1.198	1.400	1.717	2.010	2.375	2.639	3.475
8	1.044	1.191	1.379	1.670	1.938	2.271	2.511	3.266
9	1.046	1.184	1.360	1.632	1.880	2.187	2.407	3.097
10	1.047	1.178	1.344	1.599	1.831	2.116	2.321	2.959
11	1.048	1.173	1.330	1.570	1.789	2.056	2.248	2.842
12	1.049	1.168	1.318	1.546	1.752	2.004	2.185	2.742
13	1.049	1.163	1.307	1.524	1.720	1.959	2.130	2.656
14	1.049	1.159	1.296	1.505	1.692	1.919	2.082	2.580
15	1.049	1.155	1.287	1.487	1.666	1.884	2.039	2.513
16	1.049	1.151	1.279	1.471	1.644	1.852	2.000	2.453
17	1.048	1.148	1.271	1.457	1.623	1.823	1.965	2.399
18	1.048	1.145	1.264	1.444	1.604	1.797	1.934	2.351
19	1.048	1.142	1.258	1.432	1.586	1.773	1.905	2.307
20	1.048	1.139	1.252	1.421	1.571	1.751	1.878	2.266
22	1.047	1.134	1.241	1.401	1.542	1.712	1.831	2.194
24	1.046	1.129	1.231	1.383	1.517	1.678	1.791	2.132
26	1.045	1.125	1.223	1.368	1.496	1.648	1.755	2.079
28	1.045	1.121	1.215	1.354	1.476	1.622	1.724	2.032
30	1.044	1.118	1.208	1.342	1.459	1.599	1.696	1.990
32	1.043	1.115	1.202	1.331	1.444	1.578	1.671	1.953
34	1.042	1.112	1.196	1.321	1.429	1.559	1.649	1.919
36	1.042	1.109	1.191	1.311	1.417	1.541	1.628	1.888
38	1.041	1.106	1.186	1.303	1.405	1.525	1.610	1.861
40	1.041	1.104	1.182	1.295	1.394	1.511	1.592	1.835
42	1.040	1.102	1.178	1.288	1.384	1.497	1.576	1.812
44	1.039	1.100	1.174	1.281	1.375	1.485	1.562	1.790
46	1.039	1.098	1.170	1.275	1.366	1.473	1.548	1.770
48	1.038	1.096	1.167	1.269	1.358	1.462	1.535	1.751
50	1.038	1.094	1.163	1.263	1.350	1.452	1.523	1.733
60	1.036	1.087	1.150	1.240	1.318	1.410	1.473	1.660
70	1.034	1.081	1.139	1.222	1.293	1.377	1.435	1.605
80	1.032	1.076	1.130	1.207	1.273	1.351	1.404	1.560
90	1.031	1.072	1.123	1.195	1.257	1.329	1.379	1.525
100	1.029	1.069	1.117	1.185	1.243	1.311	1.358	1.494
120	1.027	1.063	1.107	1.169	1.221	1.283	1.325	1.446
140	1.026	1.059	1.099	1.156	1.204	1.261	1.299	1.410
160	1.024	1.055	1.093	1.146	1.191	1.243	1.278	1.381
180	1.023	1.052	1.087	1.137	1.179	1.228	1.261	1.358
200	1.022	1.050	1.083	1.130	1.170	1.216	1.247	1.338

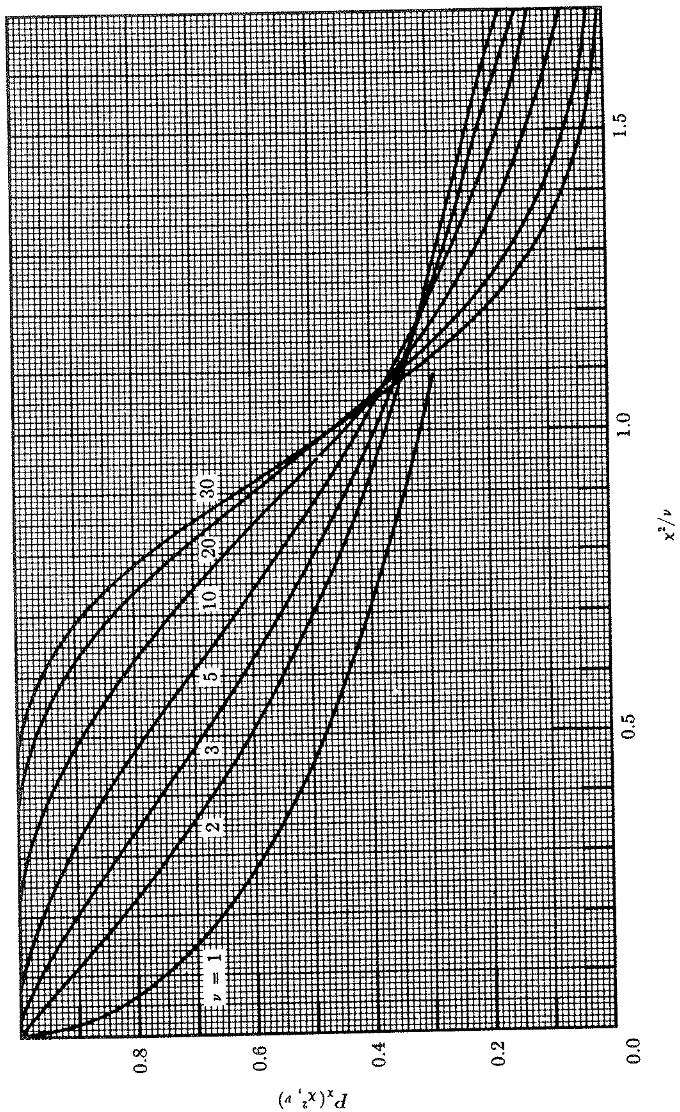

FIGURE C.4
The probability $P_\chi(\chi^2;\nu)$ of exceeding χ^2 vs. the reduced chi-square $\chi_\nu^2 = \chi^2/\nu$ and the number of degrees of freedom ν.

probability $P_\chi(\chi^2;\nu)$:

$$P_\chi(\chi^2;\nu) = \frac{1}{2^{\nu/2}\Gamma(\nu/2)} \int_{\chi^2}^{\infty} (x^2)^{(\nu-2)/2} e^{-x^2/2}\, d(x^2)$$

Values of the reduced chi-square $\chi_\nu^2 = \chi^2/\nu$ corresponding to various values of the integral probability $P_\chi(\chi^2;\nu)$ of exceeding χ^2 in a measurement with ν degrees of freedom are tabulated in Table C.4 for ν ranging from 1 to 200. The functional dependence of $P_\chi(\chi^2;\nu)$ corresponding to representative values of ν is graphed in Figure C.4 as a smooth variation with the reduced chi-square χ_ν^2.

The function that is tabulated and graphed is the shaded area under the tail of the probability curve for values larger than χ^2 as indicated.

C.5 *F* DISTRIBUTION

The probability distribution for F is given by

$$P_f(f,\nu_1,\nu_2) = \frac{\Gamma[(\nu_1+\nu_2)/2]}{\Gamma(\nu_1/2)\Gamma(\nu_2/2)} \left(\frac{\nu_1}{\nu_2}\right)^{\nu_1/2} \frac{f^{(\nu_1-2)/2}}{(1+f\nu_1/\nu_2)^{1/2(\nu_1+\nu_2)}}$$

The probability of observing a value of F test larger than F for a random sample with ν_1 and ν_2 degrees of freedom is the integral of this probability:

$$P_F(F;\nu_1,\nu_2) = \int_F^{\infty} P_f(f;\nu_1,\nu_2)\, df$$

Values of F corresponding to various values of the integral probability $P_F(F;\nu_1,\nu_2)$ of exceeding F in a measurement are tabulated in Table C.5 for $\nu_1 = 1$ and graphed in Figure C.5 as a smooth variation with the probability. Values of F corresponding to various values of ν_1 and ν_2 ranging from 1 to ∞ are listed in Table C.6 and graphed in Figure C.6 for $P_F(F;\nu_1,\nu_2) = 0.05$ and in Table C.7 and Figure C.7 for $P_F(F;\nu_1,\nu_2) = 0.01$. These values were adapted by permission from Dixon and Massey.

The function that is tabulated and graphed is the shaded area under the tail of the probability curve for values larger than F as indicated.